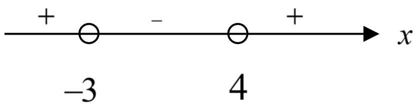
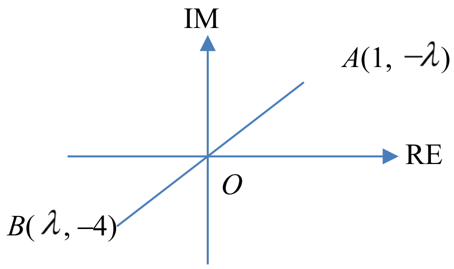
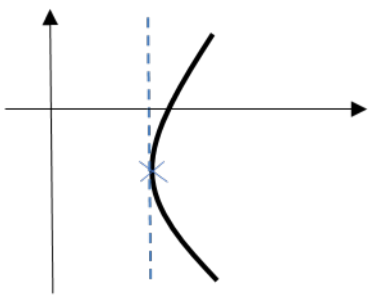
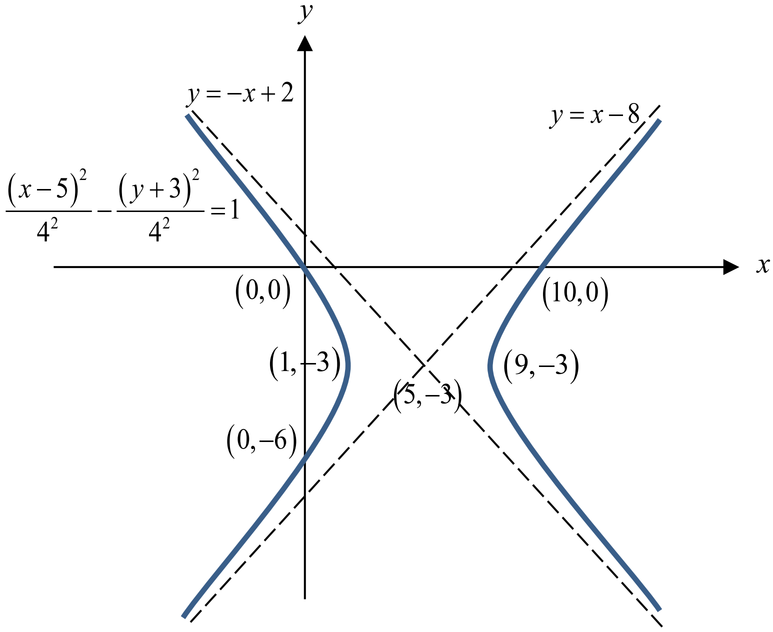
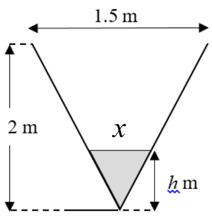
Qn	Suggested solution	Comments
1	<u>Method 1</u> As $x^2+x+3=\left(x+\frac{1}{2}\right)^2+\frac{11}{4}\geq\frac{11}{4}>0$ for all real values of x. OR <u>Method 2</u> As (coefficient of x^2) $=1>0$ and discriminant $=(1)^2-4(1)(3)=-11<0$, we have that $x^2+x+3>0$ all real values of x. $\frac{x^2+2x+6}{x^2-x-12}\geq\frac{1}{x-4}$ $\frac{x^2+2x+6}{(x-4)(x+3)}-\frac{1}{x-4}\geq 0,\qquad x\neq 4 \text{ and } x\neq -3$ $\frac{x^2+2x+6-(x+3)}{(x-4)(x+3)}\geq 0$ $\frac{x^2+x+3}{(x-4)(x+3)}\geq 0$ This implies $\frac{1}{(x-4)(x+3)}\geq 0$ since $x^2+x+3>0$ all real values of x. $(x-4)(x+3)>0$  $x<-3$ or $x>4$	
2	$z=\frac{\lambda-4\text{i}}{1-\lambda\text{i}}\times\frac{1+\lambda\text{i}}{1+\lambda\text{i}}$ $=\frac{\left(\lambda+\lambda^2\text{i}\right)-4\text{i}+4\lambda}{1+\lambda^2}$ $=\frac{(5\lambda)+\left(\lambda^2-4\right)\text{i}}{1+\lambda^2}$ $\arg(z)=\pi\ \Rightarrow\ z\in\mathbb{R},\ z<0$ and $\text{Im}(z)=0$ $\lambda^2-4=0\quad\Rightarrow\quad\lambda=\pm 2$ When $\lambda=2,\quad z=\frac{10}{5}=2$ (Rejected since $z<0$) When $\lambda=-2,\quad z=\frac{-10}{5}=-2$	

	Alternatively,  Sketch indicates that $\lambda < 0$. $OA // OB \quad \Rightarrow \quad \frac{-\lambda}{1} = \frac{-4}{\lambda}$ $-\lambda^2 = -4$ $\lambda = -2$ (since $\lambda < 0$) Hence, $z = \frac{-2-4\text{i}}{1+2\text{i}} = \frac{-2(1+2\text{i})}{1+2\text{i}} = -2$	
3i	$PQ^2 = PR^2 + QR^2 - 2PR \cdot QR \cos \angle PRQ$ $= 5^2 + 6^2 - 2(5)(6)\cos(\angle PRS + \theta)$ $= 25 + 36 - 60[\cos(\angle PRS)\cos\theta - \sin(\angle PRS)\sin\theta]$ $= 61 - 60\left(\frac{3}{5}\cos\theta - \frac{4}{5}\sin\theta\right)$ $= 61 - 36\cos\theta + 48\sin\theta$ $\therefore PQ = (61 - 36\cos\theta + 48\sin\theta)^{\frac{1}{2}}$ (shown)	

3ii	$PQ=\left(61-36\cos\theta+48\sin\theta\right)^{\frac{1}{2}}$ $\approx\left(61-36\left(1-\frac{\theta^2}{2}\right)+48\theta\right)^{\frac{1}{2}}$ $=\left(25+48\theta+18\theta^2\right)^{\frac{1}{2}}$ $=5\left(1+\frac{48}{25}\theta+\frac{18}{25}\theta^2\right)^{\frac{1}{2}}$ $\approx5\left(1+\frac{1}{2}\left(\frac{48}{25}\theta+\frac{18}{25}\theta^2\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{48}{25}\theta\right)^2\right)$ $=5\left(1+\frac{24}{25}\theta+\frac{9}{25}\theta^2-\frac{288}{625}\theta^2\right)$ $=5\left(1+\frac{24}{25}\theta-\frac{63}{625}\theta^2\right)$ $=5+\frac{24}{5}\theta-\frac{63}{125}\theta^2,\ p=\frac{24}{5},\ q=-\frac{63}{125}$	
4i	$\frac{dy}{dx}+xy=e^{-x}$ $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=-e^{-x}$ $\frac{d^3y}{dx^3}+x\frac{d^2y}{dx^2}+\frac{dy}{dx}+\frac{dy}{dx}=e^{-x}$ When $x=0$, $y=1$, $\frac{dy}{dx}=1$, $\frac{d^2y}{dx^2}=-2$, $\frac{d^3y}{dx^3}=-1$. Let $y = f(x)$. The Maclaurin series for y is: $y=f(0)+xf'(0)+\frac{x^2}{2!}f''(0)+\frac{x^3}{3!}f'''(0)+...$ $y=1+x-x^2-\frac{1}{6}x^3+...$	$f(0) = 1\left(\because y=1 \text{ when } x=0\right)$
4ii	Since $y=1+x-x^2-\frac{1}{6}x^3+...$, differentiating y w.r.t. x, $\frac{dy}{dx}=1-2x-\frac{1}{2}x^2+...$	
5ai	$\int\frac{1}{x^2+2x+5}\,dx$ $=\int\frac{1}{\left(x+1\right)^2+2^2}\,dx$ $=\frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right)+C$	

5aii	$\int \frac{(\ln x)^3}{x}\,\mathrm{d}x$ $=\frac{(\ln x)^4}{4}+C$	$\mathrm{f}(x)=\ln x,\ \mathrm{f}'(x)=\frac{1}{x}$ $\int \mathrm{f}'(x)(\mathrm{f}(x))^3\,\mathrm{d}x$ $=\frac{(\mathrm{f}(x))^4}{4}+C$ Note that $(\ln x)^3 \neq 3\ln x$
5b	$\int_0^{\frac{\pi}{2}} x\cos x\,\mathrm{d}x$ $=[x(\sin x)]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}\sin x\,\mathrm{d}x$ $=\frac{\pi}{2}+\int_0^{\frac{\pi}{2}}-\sin x\,\mathrm{d}x$ $=\frac{\pi}{2}+[\cos x]_0^{\frac{\pi}{2}}$ $=\frac{\pi}{2}+(0-1)$ $=\frac{\pi}{2}-1$	$\int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x=uv-\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x$ Take note the correct choices of function to be $\frac{\mathrm{d}u}{\mathrm{d}x}$ (integrated) and v (to be differentiated) choose $\frac{\mathrm{d}u}{\mathrm{d}x}=\sin x$ and $v=x$
6a	$x^2+ay^2+bx+cy=0$ Since C passes through $(5,-3)$ and $\left(\frac{1}{2},-\frac{3}{2}\right)$, $5^2+a(-3)^2+5b+(-3)c=0$ $\Rightarrow 9a+5b-3c=-25 \quad -(1)$ $\left(\frac{1}{2}\right)^2+a\left(-\frac{3}{2}\right)^2+\frac{1}{2}b+\left(-\frac{3}{2}\right)c=0$ $\frac{1}{4}+\frac{9}{4}a+\frac{1}{2}b-\frac{3}{2}c=0$ $\Rightarrow 9a+2b-6c=-1 \quad -(2)$ At $\left(\frac{1}{2},-\frac{3}{2}\right)$, tangent is parallel to the y-axis. $2x+2ay\frac{\mathrm{d}y}{\mathrm{d}x}+b+c\frac{\mathrm{d}y}{\mathrm{d}x}=0$ $(2ay+c)\frac{\mathrm{d}y}{\mathrm{d}x}=-2x-b$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-2\left(\frac{1}{2}\right)-b}{2a\left(-\frac{3}{2}\right)+c}$ $=\frac{-1-b}{-3a+c}$ Since tangent is parallel to the y-axis. $-3a+c=0 \quad -(3)$ By GC, $a=-1$, $b=-5$ and $c=-3$.	Note that $\left(\frac{1}{2},-\frac{3}{2}\right)$ is also a point on the curve and needs to be substituted to get another equation  Tangent parallel to the y-axis is vertical, so $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-1-b}{-3a+c}$ is undefined, meaning that denominator is zero and **NOT** numerator. (See diagram)

6b	$x^2-y^2-10x-6y=0$ $(x-5)^2-(5)^2-\left[(y+3)^2-(3)^2\right]=0$ $(x-5)^2-(y+3)^2=4^2$ $\frac{(x-5)^2}{4^2}-\frac{(y+3)^2}{4^2}=1$ Asymptote: $y+3=x-5,\quad y+3=-x+5$ $\Rightarrow y=x-8,\quad y=-x+2$ 	Use brackets when completing the square. This is hyperbola. Refer to conics App on GC to see the various forms for various conic sections. Always draw asymptotes first **before** the curve in order for your curve to approach the asymptotes. Watch for "tail-end behaviour". Curve should not "steer away" from asymptote. We want a relatively proportional curve here with correct labels – centre, vertices, asymptotes, passing through the origin. A rather common mistake is that many students drew a circle instead. Question: What is the key difference between a hyperbola and ellipse (and circles)?
7i	Let x be the length as denoted in the diagram: By similar triangles, $\frac{x}{h}=\frac{1.5}{2}$ $x=\frac{3}{4}h$  $V=\frac{1}{2}(h)\left(\frac{3}{4}h\right)(8)$ $=3h^2$	Be clear about the usage of similar triangles as this is a "show" question – provide clear justification.
7ii	$\frac{\mathrm{d}V}{\mathrm{d}h}=6h$ When volume of water in the drain = 7.2 m^3, the water level is $7.2=3h^2$ $h^2=2.4$ $h=\sqrt{2.4}\ (\because h>0)$	

	$\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}$ $=\frac{1}{6\left(\sqrt{2.4}\right)}\times(0.03-0.02)$ $=0.0010759$ $=0.00108$ The rate at which the water level is rising is 0.00108 m s^{-1}.	No need to make h the subject to find $\frac{dh}{dV}$. Recall that $\frac{dh}{dV}=\frac{1}{\frac{dV}{dh}}=\frac{1}{6h}$.
8i	Let u_n be the n^{th} term of the geometric series. $u_n=ar^{n-1}$ $u_3=36$ $ar^2=36$ ----------(1) $S_\infty=243$ $\frac{a}{1-r}=243$ $a=243(1-r)$ -------(2) Substitute (2) into (1), $243(1-r)r^2=36$ $243r^2-243r^3-36=0$ Using GC to solve for the roots of the cubic equation, $r=-\frac{1}{3}$ (reject $\because r>0$) or $\frac{2}{3}$ $\therefore$ Common ratio, $r=\frac{2}{3}$. Substitute $r=\frac{2}{3}$ into (2), $a=243\left(1-\frac{2}{3}\right)$ $=81$ $\therefore$ First term, $a=81$.	The deadly mistake in question 7 was the incorrect usage of formulas. Correct formula for nth term of geometric series is $u_n=ar^{n-1}$ Correct formula for sum to infinity is $S_\infty=\frac{a}{1-r}$. No need to solve this equation by algebraic means – just use GC! **Always read question to see if GC can/should be used.**
8ii	$\frac{6}{2}\left[2(1)+(6-1)d\right]=\frac{81\left[1-\left(\frac{2}{3}\right)^3\right]}{1-\frac{2}{3}}$ $3(2+5d)=243\left[1-\left(\frac{2}{3}\right)^3\right]$ $6+15d=243-243\left(\frac{2}{3}\right)^3$ $6+15d=171$ $15d=165$ $d=11$	Correct formula for sum of n terms of a **geometric** series is $\frac{a(1-r^n)}{1-r}$. Correct formula for sum of n terms of a arithmetic series is $\frac{n}{2}[2a+(n-1)d]$. Make sure you know what the "letters" in the formula mean.

8iii

$$1+(n-1)(11)>\frac{81\left[1-\left(\frac{2}{3}\right)^{2n}\right]}{1-\frac{2}{3}}$$

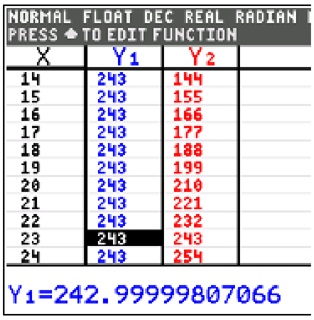
$$1+11n-11>243\left[1-\left(\frac{2}{3}\right)^{2n}\right]$$

$$11n-10>243\left[1-\left(\frac{2}{3}\right)^{2n}\right]$$

Using GC, $n\geq 23$.

n	$11n-10$	$243\left[1-\left(\frac{2}{3}\right)^{2n}\right]$
22	232	242.999996
23	243	242.999998
24	254	242.999999

The least value of n is 23.


NORMAL FLOAT DEC REAL RADIAN
PRESS ◆ TO EDIT FUNCTION

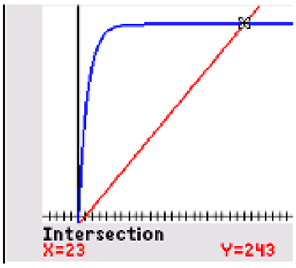
X	Y1	Y2
14	243	144
15	243	155
16	243	166
17	243	177
18	243	188
19	243	199
20	243	210
21	243	221
22	243	232
23	243	243
24	243	254

Y1=242.99999807066

Do not be tricked by GC.
Always do some verification if in doubt.

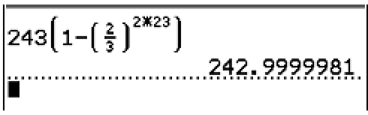
$y=243\left[1-\left(\frac{2}{3}\right)^{2x}\right]$

$y=11x-10$



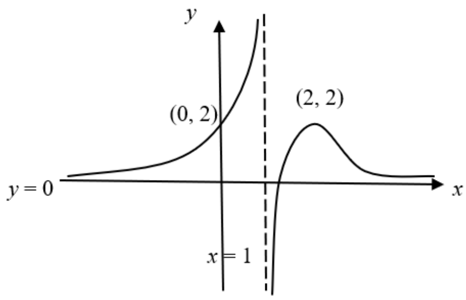
By evaluating / checking the answer when it seems too unnatural that

$$243\left[1-\left(\frac{2}{3}\right)^{2\times 23}\right]=243$$



A verification (see above) shows that this isn't the case.
If you are using tables, place your selection over the value to see its unrounded value.

9a



There are 2 ways to do this
Method 1

$y=\mathrm{f}(x)\to -y=\mathrm{f}(x)\Rightarrow y=-\mathrm{f}(x)$

Reflect graph in x-axis

$y=-\mathrm{f}(x)\to y-2=-\mathrm{f}(x)\Rightarrow y=2-\mathrm{f}(x)$

Translate resultant graph 2 units in positive y-direction.

Method 2

$y=\mathrm{f}(x)\to y+2=\mathrm{f}(x)\Rightarrow y=-2+\mathrm{f}(x)$

Translate graph 2 units in negative y-direction

$y=-2+\mathrm{f}(x)\to -y=-2+\mathrm{f}(x)\Rightarrow y=2-\mathrm{f}(x)$

Reflect resultant graph in x-axis

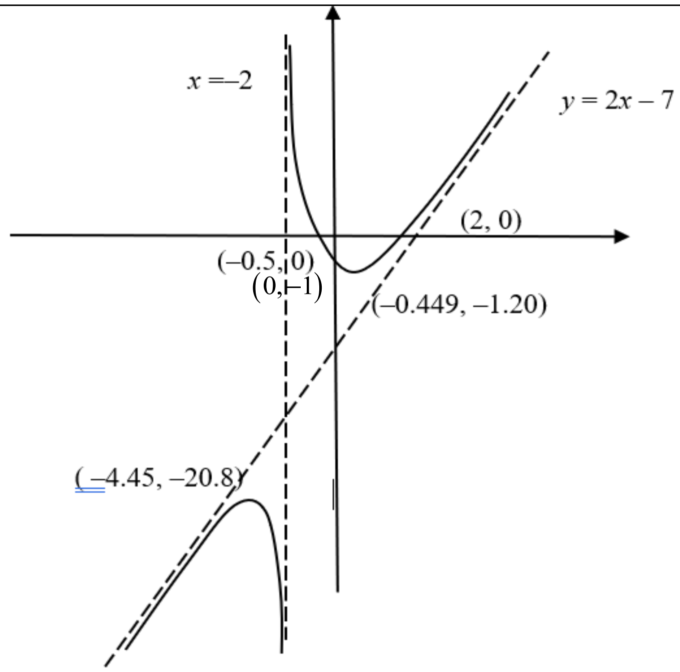
Read question carefully, label all the required features in the graph.

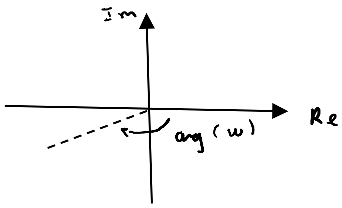
The y-intercept and stationary point should be at the same level.

Generally, when we reflect a graph in the x-axis, the y-coordinates will change. Vice versa for a reflection in the y-axis. Do note that this applies to the asymptotes as well.

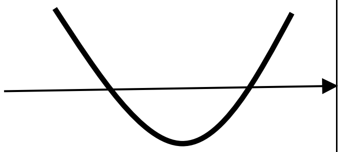
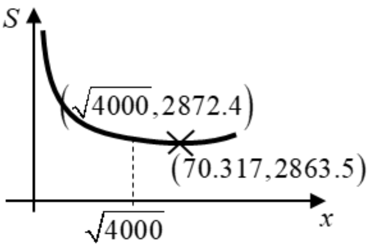
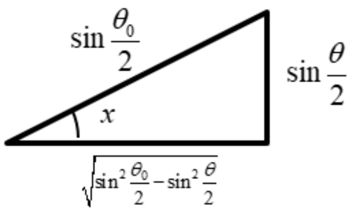
It is a good habit to write out the intermediate equations and trace the position of important points.

9bi	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x+2)(4x+k)-(2x^2+kx-2)}{(x+2)^2}$ $=\frac{4x^2+kx+8x+2k-2x^2-kx+2}{(x+2)^2}$ $=\frac{2x^2+8x+2k+2}{(x+2)^2}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ Discriminant >0 $8^2-4(2)(2k+2)>0$ $64-16k-16>0$ $16k<48$ $k<3$ Alternatively, $2(x^2+4x)+2k+2$ $=2\left[(x+2)^2-2^2\right]+2k+2$ $=2(x+2)^2-6+2k$ $-6+2k<0$ $k<3$ Set of values of $k=\{k\in\mathbb{R}:k<3\}$.	We need to find $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ (stationary points) before applying $b^2-4ac>0$ (2 stationary points)
9bii	Long division of $2x^2-3x-2$ by $x+2$: quotient $2x-7$ $2x^2-3x-2$ $-(2x^2+4x)$ $-7x-2$ $-(-7x-14)$ 12 $\therefore y=2x-7+\frac{12}{x+2}$ Alternatively, $2x^2-3x-2=(ax+b)(x+2)+c$ Comparing coefficients, $a=2$ $2a+b=-3\Rightarrow b=-3-4=-7$ $2b+c=-2\Rightarrow c=-2+2(-7)=-12$ $\therefore y=2x-7+\frac{12}{x+2}$ Equation of asymptotes are $y=2x-7$, $x=-2$	When in the form $y=ax+b+\frac{c}{x+d}$, $y=ax+b$ will be an asymptote as $x\to\pm\infty$ because $\frac{c}{x+d}\to 0$ $x=-d$ will be the other asymptote as $y\to\pm\infty$. Equations of asymptotes MUST start with "x =" or "y ="

9b iii		Read question carefully, for all the required features in the graph. There is no need to give exact coordinates for the stationary points. Asymptotes are like the "skeleton" of our graphs, they will guide the shape and position of the graph. We should draw them in first. We will need to adjust the window settings to see the left half of the graph. We can either look at the table of values (ys) or use the trace function (r) to see what are the y-values we are working with.
9b iv	Hence $y = x + \frac{6}{x+2}$ ↓ First, **stretch** parallel to y-axis by **factor 2**, with x-axis invariant $\frac{y}{2} = x + \frac{6}{x+2}$ $y = 2x + \frac{12}{x+2}$ Or: Translate 3.5 units in the negative y-direction, followed by a stretch parallel to y-axis, factor 2, with x-axis invariant. ↓ Next, **translate** 7 units in the negative y-direction $y + 7 = 2x + \frac{12}{x+2}$ $y = 2x - 7 + \frac{12}{x+2}$	Read question carefully, we want to transform $y = x + \frac{6}{x+2}$ to C, not the other way round. It is a good habit to write out the intermediate equations. Pay attention to the words used to describe the transformation. "Scale" and "move" are not accepted.
10a	$\lvert w \rvert = \sqrt{\left(\sqrt{3}\right)^2 + 1^2} = 2$	
10b	$\lvert w + a\mathrm{i} \rvert = \left\lvert \left(-\sqrt{3}\right) + (a-1)\mathrm{i} \right\rvert$ $= \sqrt{\left(-\sqrt{3}\right)^2 + (a-1)^2}$ $= \sqrt{3 + a^2 - 2a + 1}$ $= \sqrt{a^2 - 2a + 4}$	In general, $\lvert a+b \rvert \neq \lvert a \rvert + \lvert b \rvert$. Also, $\lvert p + q\mathrm{i} \rvert = \sqrt{p^2 + q^2}$ only when p and q are both real numbers. w is not a real number in this case.

10c	$w+\frac{b\mathrm{i}}{w}=-\sqrt{3}-\mathrm{i}+\frac{b\mathrm{i}}{-\sqrt{3}-\mathrm{i}}$ $=-\sqrt{3}-\mathrm{i}+\frac{b\mathrm{i}\left(-\sqrt{3}+\mathrm{i}\right)}{\left(-\sqrt{3}-\mathrm{i}\right)\left(-\sqrt{3}+\mathrm{i}\right)}$ $=-\sqrt{3}-\mathrm{i}+\frac{b\mathrm{i}}{4}\left(-\sqrt{3}+\mathrm{i}\right)$ $=-\sqrt{3}-\frac{b}{4}-\mathrm{i}-\frac{\sqrt{3}b\mathrm{i}}{4}$ $\therefore \mathrm{Im}\left(w+\frac{b}{w}\right)=-1-\frac{\sqrt{3}b}{4}$	
10d	Since the cubic equation has real coefficients, then the roots are 4, *w*, *w**. Hence a cubic equation is $(z-w)(z-w^*)(z-4)=0$ $\left[z-\left(-\sqrt{3}-\mathrm{i}\right)\right]\left[z-\left(-\sqrt{3}+\mathrm{i}\right)\right](z-4)=0$ $\left[\left(z+\sqrt{3}\right)+\mathrm{i}\right]\left[\left(z+\sqrt{3}\right)-\mathrm{i}\right](z-4)=0$ $\left(z^2+2\sqrt{3}z+3-(-1)\right)(z-4)=0$ $\left(z^2+2\sqrt{3}z+4\right)(z-4)=0$ $z^3+\left(2\sqrt{3}-4\right)z^2+\left(4-8\sqrt{3}\right)z-16=0$	For expansion of factors in complex numbers, it is usual to expand $(z-w)(z-w^*)$ first. $z^3+\left(2\sqrt{3}-4\right)z^2+$ $\left(4-8\sqrt{3}\right)z-16$ is an expression. There is no "=", so it is not an equation.
10ei	$\arg w=-\frac{5\pi}{6}$  	Good habit to sketch an Argand diagram and find out which quadrant the point representing *w* is in
10e ii	$\arg\left(w^n w^*\right)=n\arg w+\arg w^*=-\frac{5\pi}{6}n+\frac{5\pi}{6}$ $w^n w^*$ is real $\Rightarrow \arg\left(w^n w^*\right)=k\pi$, where $k\in\mathbb{Z}$ $\Rightarrow -\frac{5\pi}{6}n+\frac{5\pi}{6}=k\pi$ $\Rightarrow (-n+1)=\frac{6k}{5}$ $\Rightarrow n=1-\frac{6k}{5}$ For the three smallest positive whole number values of *n*, we take $k=0,-5,-10,$ $\Rightarrow n=1,7,13$	Read question carefully, "*without using a calculator*" means trial and error methods will not be credited. Remember that the polar form of complex numbers is very useful for raising to high powers and multiplication.

11i	Floor area $xy = 2000 \Rightarrow y = \dfrac{2000}{x}$ Area $S = 2(4x) + 2x\sqrt{\left(0.01y^2\right)^2 + \left(\dfrac{y}{2}\right)^2}$ $= 8x + 2x\sqrt{\dfrac{y^2}{4}\left(4\left(0.01y\right)^2 + 1\right)}$ $= 8x + 2x\left(\dfrac{y}{2}\right)\sqrt{4\left(0.01\left(\dfrac{2000}{x}\right)\right)^2 + 1}$ $= 8x + x\left(\dfrac{2000}{x}\right)\sqrt{4\left(0.01\left(\dfrac{2000}{x}\right)\right)^2 + 1}$ $= 8x + 2000\sqrt{\dfrac{1600}{x^2} + 1}$	In general, $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$
11ii	$\dfrac{dS}{dx} = 8 + 2000\left(\dfrac{1}{2}\right)\left(\dfrac{1600}{x^2}+1\right)^{-1/2}\left(-\dfrac{3200}{x^3}\right)$ $\dfrac{dS}{dx} = 0 \Rightarrow 8 - 1000\left(\dfrac{1600}{x^2}+1\right)^{-1/2}\left(\dfrac{3200}{x^3}\right) = 0$ $1000\left(\dfrac{1600}{x^2}+1\right)^{-1/2}\left(\dfrac{3200}{x^3}\right) = 8$ $\dfrac{400000}{x^3} = \sqrt{\dfrac{1600}{x^2}+1}$ $\dfrac{1.6\times10^{11}}{x^6} = \dfrac{1600}{x^2}+1$ $x^6 + 1600x^4 - 1.6\times10^{11} = 0$	Be very careful with the number of zeroes in your working. This is a show question. For $\sqrt{a} - b = 0$, to express *a* in terms of *b*, we write $\sqrt{a} = b$ before squaring both sides to get *a* = *b*2.
11 iii	By GC, *x* = 70.317 (since *x* > 0). From GC, $\dfrac{d^2S}{dx^2} = 0.31350 > 0$ when *x* = 70.317 OR <table><tr><td>x</td><td>70.2</td><td>70.317</td><td>70.4</td></tr><tr><td>Value of $\dfrac{dS}{dx}$</td><td>-0.0368</td><td>0</td><td>0.0259</td></tr><tr><td>Sign of $\dfrac{dS}{dx}$</td><td>-</td><td>0</td><td>+</td></tr></table> Hence *S* is minimum when *x* = 70.317. Minimum $S = \left[8(70.317) + 2000\sqrt{\dfrac{1600}{70.317^2}+1}\right]$ = 2863.486 = 2860 (to 3 s.f.)	$x^6 + 1600x^4 - 1.6\times10^{11} = 0$ is an equation satisfied by the stationary point. But, $x^6 + 1600x^4 - 1.6\times10^{11} \neq \dfrac{dS}{dx}$ Show values for the minimum point testing. Read the question carefully. We need to find the minimum value of *S*, not just the value of *x* that gives minimum *S*.

11iv	Length at most two times its width $\Rightarrow$ $x \le 2y$. $x \le 2y$ $x \le 2\left(\frac{2000}{x}\right)$ $x^2 - 4000 \le 0 \quad \because x > 0$ $\left(x+\sqrt{4000}\right)\left(x-\sqrt{4000}\right) \le 0$ $-\sqrt{4000} \le x \le \sqrt{4000}$  Since $x > 0$, we have $0 < x \le \sqrt{4000}$ Consider the graph of $S = 8x + 2000\sqrt{\frac{1600}{x^2}+1}, 0 < x \le \sqrt{4000}$. The smallest value of S occurs at $x = \sqrt{4000}$, Sub $x = \sqrt{4000}$ into S, smallest value of S = 2870 (3 sf) 	At most implies inequality, not an equation. The base area condition in the original question is not changed. This part is solved like a range of functions question. Sketch the graph on the required domain, and we find the smallest value of S from there. <u>Smallest</u> value need not occur at a stationary point, while a <u>minimum</u> value is a stationary point.
12i	$\sin x = \frac{\sin\frac{\theta}{2}}{\sin\frac{\theta_0}{2}}$  $\cos x = \frac{\sqrt{\sin^2\frac{\theta_0}{2}-\sin^2\frac{\theta}{2}}}{\sin\frac{\theta_0}{2}}$ $= \frac{\sqrt{\frac{1}{2}\left(1-\cos\left(2\times\frac{\theta_0}{2}\right)\right)-\frac{1}{2}\left(1-\cos\left(2\times\frac{\theta}{2}\right)\right)}}{\sin\frac{\theta_0}{2}}$ $= \frac{\sqrt{\cos\theta-\cos\theta_0}}{\left(\sqrt{2}\right)\sin\frac{\theta_0}{2}}$	This method uses trigonometry and Pythagoras Theorem from secondary school days. We have also used this method in integration by substitution tutorial. An alternative is to use $\sin^2 x + \cos^2 x = 1$.

	$\frac{\mathrm{d}}{\mathrm{d}\theta}(\sin x)=\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\frac{\sin\frac{\theta}{2}}{\sin\frac{\theta_0}{2}}\right)$ $(\cos x)\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)=\frac{\frac{1}{2}\left(\cos\frac{\theta}{2}\right)}{\sin\frac{\theta_0}{2}}$ $\left(\frac{\sqrt{\cos\theta-\cos\theta_0}}{(\surd 2)\sin\frac{\theta_0}{2}}\right)\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)=\frac{\left(\cos\frac{\theta}{2}\right)}{2\sin\frac{\theta_0}{2}}$ $\frac{\mathrm{d}x}{\mathrm{d}\theta}=\frac{\left(\cos\frac{\theta}{2}\right)}{2\sin\frac{\theta_0}{2}}\times\frac{(\surd 2)\sin\frac{\theta_0}{2}}{\sqrt{\cos\theta-\cos\theta_0}}$ $\frac{\mathrm{d}x}{\mathrm{d}\theta}=\frac{\cos\frac{\theta}{2}}{(\surd 2)\sqrt{\cos\theta-\cos\theta_0}}$	We need to understand that x and θ are variables, while θ_0 is a constant. It is easier to discuss this question in terms of $\frac{\mathrm{d}}{\mathrm{d}\theta}(\sin x)$, rather than $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos x)$.
12ii	When $\theta=\theta_0,\ \ \sin x=1\ \Rightarrow\ x=\frac{\pi}{2}$. $\theta=0,\ \ \sin x=0\ \Rightarrow\ x=0$. $\frac{\mathrm{d}x}{\mathrm{d}\theta}=\frac{\cos\frac{\theta}{2}}{(\surd 2)\sqrt{\cos\theta-\cos\theta_0}}\ \Rightarrow\ \frac{\mathrm{d}\theta}{\mathrm{d}x}=\frac{\surd 2}{\cos\frac{\theta}{2}}\cdot\sqrt{\cos\theta-\cos\theta_0}$ Hence, $T=4\sqrt{\frac{L}{2g}}\int_0^{\theta_0}\frac{1}{\sqrt{\cos\theta-\cos\theta_0}}\,\mathrm{d}\theta$ $=4\sqrt{\frac{L}{2g}}\int_0^{\frac{\pi}{2}}\frac{\sqrt{2}}{\cos\frac{\theta}{2}}\,\mathrm{d}x$ $=4\sqrt{\frac{L}{g}}\int_0^{\frac{\pi}{2}}\frac{1}{\cos\frac{\theta}{2}}\,\mathrm{d}x$ $=4\sqrt{\frac{L}{g}}\int_0^{\frac{\pi}{2}}\frac{1}{\sqrt{1-\sin^2\frac{\theta}{2}}}\,\mathrm{d}x$ $=4\sqrt{\frac{L}{g}}\int_0^{\frac{\pi}{2}}\frac{1}{\sqrt{1-\sin^2\frac{\theta_0}{2}\sin^2 x}}\,\mathrm{d}x$ $=4\sqrt{\frac{L}{g}}\int_0^{\frac{\pi}{2}}\frac{1}{\sqrt{1-k^2\sin^2 x}}\,\mathrm{d}x$ where $k=\sin\left(\frac{\theta_0}{2}\right)$	This is an integration by substitution question. Hence, we need to show clearly how we obtained the new limits (answer is given) and how we substituted the integrand. You need to use the given results in (i) in this part.

12 iii	$\frac{1}{\sqrt{1-y}}=(1-y)^{-\frac{1}{2}}$ $=1+\left(-\frac{1}{2}\right)(-y)+...$ $=1+\frac{1}{2}y+...$	This portion is unlinked to the previous parts. Do not give up on free marks like this.
12iv	When k is small, $k\sin x$ is small $(\because -1\le \sin x\le 1)$. This implies $k^2\sin^2 x$ is small, and $k^n\sin^n x$ for $n\ge 4$ can be ignored. Replace y with $k^2\sin^2 x$ in (iii), we have $T=4\sqrt{\frac{L}{g}}\int_0^{\frac{\pi}{2}}\frac{1}{\sqrt{1-k^2\sin^2 x}}\,dx$ $\approx 4\sqrt{\frac{L}{g}}\int_0^{\frac{\pi}{2}}\left(1+\frac{1}{2}k^2\sin^2 x\right)dx$ $\therefore a=2$	This part is linked to (iii).
12v	$T\approx 4\sqrt{\frac{L}{g}}\int_0^{\frac{\pi}{2}}\left(1+\frac{1}{2}k^2\sin^2 x\right)dx$ $=4\sqrt{\frac{L}{g}}\left(\int_0^{\frac{\pi}{2}}1\,dx+\frac{1}{2}k^2\int_0^{\frac{\pi}{2}}\sin^2 x\,dx\right)$ $=4\sqrt{\frac{L}{g}}\left(\frac{\pi}{2}+\frac{1}{2}k^2\int_0^{\frac{\pi}{2}}\frac{1-\cos 2x}{2}\,dx\right)$ $=2\pi\sqrt{\frac{L}{g}}+k^2\sqrt{\frac{L}{g}}\left[x-\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}}$ $=2\pi\sqrt{\frac{L}{g}}+k^2\sqrt{\frac{L}{g}}\left[\frac{\pi}{2}-0\right]$ $=2\pi\sqrt{\frac{L}{g}}\left(1+\frac{k^2}{4}\right)$	This part is linked to (iv). The key ideas are how to solve $\int_0^{\frac{\pi}{2}}1\,dx$ and $\int_0^{\frac{\pi}{2}}\sin^2 x\,dx$. This question is not about applying small angle approximation as x is not necessarily small. x can take values as large as $\frac{\pi}{2}$.

Overall comments:

1. **Algebraic skills needs to be improved** by more practices and positive reinforcement by deliberately taking note of mistakes and to not repeat them, *rather than brushing them off as careless*. Poor knowledge of laws of logarithm were common. Take note that $(\ln x)^3\ne 3\ln x$ (see Q2a(ii))! Manipulation of modulus function was also done badly. Take note that $\left|3e^{-x}-8\right|\ne\left|3e^{-x}\right|+8$ (see Q6(ii)), $|w+ai|\ne|w|+|ai|$ (see Q10 b). Similar issue with square roots is also observed. $\sqrt{a+b}\ne\sqrt{a}+\sqrt{b}$.

2. **Question needs to be read thoroughly**. Misreads were rather common (e.g. in Q7(iii), "*n*th term" was misread into "sum of first *n* terms", 9iii find the minimum value of *S*, but significant number of you stopped at *x* that gave minimum value of *S* instead)

3. **Understand command words in the question**. "Show" requires very clear and thorough workings, "exact" refers to non-calculator usage (see Q6(i)).

4. **Be aware of what the GC can or cannot do as well as how to use its function.** GC can be used to solve system of linear equations (Q4a), inequalities (Q7(iii)), obtain the general shape of curve (Q4(b), etc). More

practice needed on adjusting window settings to see required portions of the graph (see Q8biii). We can actually use the GC to help find the nature of a stationary point. (see Q9iii). Examples can be found here: https://youtu.be/FQs22CcANsM