internal bureau of
examinations

a department of HCA-SC

MARKING SCHEME

for the assessment referenced by the code

4049-S4-PR-1-01 **PRELIMINARY EXAMINATION ONE** *Paper One*

(**JHPES** June Holiday Preliminary Examination Series)

Marking scheme type: MS / EH (underline one)

Security classification: **Restricted**

DESIGNATED FOR PRINTING

ANNOTATIONS & ABBREVIATIONS

Marking Annotations

These annotations are to be used when marking. These may also appear in the marking scheme.

Annotation	**Meaning**
Tick & Cross	
BOD	Benefit of doubt
ecf	Error carry forward
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
‸	Omission
MR	Misread
BP	Blank page
Seen	
iow	In other words

Scheme Abbreviations

These abbreviations appear in the marking scheme. These are not valid abbreviations for use in marking.

Abbreviation	**Meaning**
dep*	Mark is dependent on a previous mark, suffixed by *. The * may be omitted if there is only one previous **M** mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given in the question paper
awrt	Anything which rounds to
BC	By calculator
DR	Detailed Reasoning; candidate's proof should be of sufficient rigour
EX	The candidate's answer must be left in exact form. If this is not seen, penalize for leaving in exact form.

MARKING FOR MATHEMATICS & THE NATURAL SCIENCES

Skip this page if the cover page of the marking scheme does not have **MS** underlined (in other words, **EH** is underlined).

Mark type	Full name	Meaning & Remarks
M	Method mark	A suitable method has been selected and *applied* in a manner that demonstrates their understanding of the question's required concept or method. Method marks are not lost for numerical or algebraic errors, nor are they lost for errors in units. The candidate must specialize the relevant method to be awarded the mark, e.g. by substituting relevant quantities into formulae. A method mark may be implied by a correct answer unless the question has a **DR** abbreviation requiring that the method must be explicitly stated.
A	Accuracy mark	Awarded for a correct answer or intermediate step correctly obtained. Accuracy marks are not awarded unless its associated method mark is earned (or implied). Hence **M0 A1** can never be awarded.
B	Independent mark	Awarded for a correct result or statement which does not require any method mark.

Positive marking. Unless otherwise stated, marks once earned cannot subsequently be deducted (e.g. wrong working following a correct answer is ignored). This is occasionally emphasized in the marking scheme by the abbreviation **ISW**. However, if the candidate passes through the correct answer as part of an **incorrect** argument, the mark can be lost.

Annotation requirement. **Annotations must be used during marking.** For a response awarded either zero or full marks, a single appropriate annotation (cross, tick, M0 or ⌒) is sufficient. However, for responses that are not awarded either of the marks, you must make it clear how you have arrived at the mark you have awarded. All responses must have sufficient annotation for a reviewer to check whether the mark awarded is correct, without the reviewer's independent marking.

The **NR** *mark type.* Award **NR** (no response)

- If there is nothing written at all in the answer space
- OR if there is a comment totally unrelated to the question (e.g. “can’t do” or “idk”)
- OR if there is a mark (e.g. a dash, a question mark, or a picture) which is not an attempt at answering the question

Award 0 marks only for an attempt that earns no credit (this includes copying out the question, as this is *related* to the question).

The mark scheme is designed to assist in marking incorrect answers. Correct solutions leading to correct answers are awarded full marks, but work must not be judged on the answer alone. Answers that are given must be correctly obtained; key steps of the working must always be checked and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method, and such work must be carefully assessed. If a candidate uses a method which does not correspond to the mark scheme (e.g. the method is outside the scope of the syllabus), escalate the question to the relevant persons in HCA.

Any errata in the mark scheme must be reported to the setter of the Assessment.

For every three errors in sf/dp/units, deduct one mark from the entire paper.

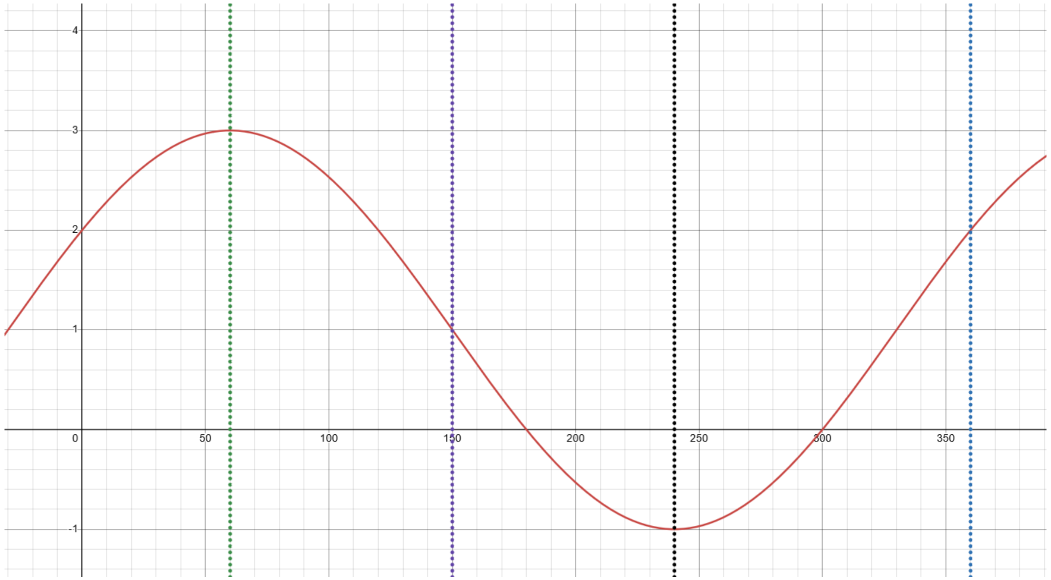
SPECIFICATION GRID

	Topic	Item	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**TOTAL**
A1	Quadratic functions							2						2		4
A2	Equations and inequalities							3							3	6
A3	Surds											3				3
A4	Polynomials and partial fractions								8		5					13
A5	Binomial expansions									7						7
A6	Exponential and logarithmic functions				6											6
G1	Trigonometric functions, identities, and equations		3	5									8			16
G2	2D coordinate geometry					9										9
G3	Proofs in plane geometry						10									10
C1	Differentiation and Integration		7		2						3			4		16
	Total marks for item (maximum 10 marks)		10	5	8	9	10	5	8	7	8	3	8	6	3	**90**

The maximum mark for this paper is 90 marks.

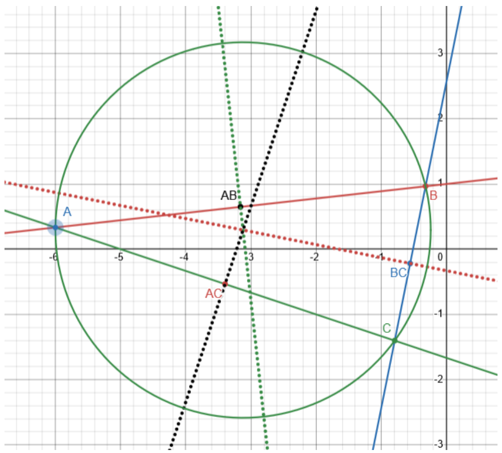
Question	Working	Marks	Remarks
1a	$\tan\ 3\theta\ = \tan\ \theta + 2\theta$		
	$= \dfrac{\tan\ \theta\ + \tan\ 2\theta}{1 - \tan\ \theta\ \tan\ 2\theta}$	**M1**	Candidate applies identity **2.6**.
	$= \dfrac{\tan\ \theta\ + \dfrac{2\tan\ \theta}{1-\tan^2\ \theta}}{1 - \dfrac{2\ \tan^2\ \theta}{1-\tan^2\ \theta}}$		Candidate applies identity **2.9** (which is itself a result of identity **2.6**).
	$= \dfrac{\tan\ \theta\ - \tan^3\ \theta\ + 2\tan\ \theta}{1 - \tan^2\ \theta\ - 2\tan^2\ \theta}$	**M1**	Require candidate to simplify by multiplying by $1 - tan^2\ \theta$ throughout.
	$= \dfrac{3\tan\ \theta}{1 - 3\tan^2\ \theta}$	**A1**	**WWW**, require **DR**.
1b	$\dfrac{d}{d\theta}\big(\ln\big(1+\tan^2(k\theta)\big)\big) = \dfrac{d}{d\theta}(\ln(\sec^2(k\theta)))$	**M1**	Insist on derivative operator. Condone differentiating with respect to the incorrect variable.
	$= \dfrac{d}{d\theta}\ 2\ln\ \sec\ k\theta$		
	$= \dfrac{d}{d\theta}(-2\ln\ \cos\ k\theta\ \ )$		
	$= \dfrac{2k\sin\ \theta}{\cos\ \theta}$		Candidates must recognize that $\tan\ \theta\ = \frac{\sin\ \theta}{\cos\ \theta}$.
	$= 2k\tan\ k\theta$	**A1**	Do not award if differentiated with respect to the wrong variable.
1c	$\displaystyle\int_0^{\frac{\pi}{9}} \frac{4\tan^3\ \theta\ - 12\tan\ \theta}{4 - 3\sec^2\ \theta}\ d\theta = \int_0^{\frac{\pi}{9}} \frac{4\tan^3\ \theta\ - 12\tan\ \theta}{4 - 3\ 1 + \tan^2\ \theta}\ d\theta$	**M1**	Require the usage of the identity $\sec^2\ \theta\ = 1 + \tan^2\ \theta$.

	$= \int_0^{\frac{\pi}{9}} \frac{4\tan^3\theta - 12\tan\theta}{4-3-3\tan^2\theta}\,\mathrm{d}\theta = \int_0^{\frac{\pi}{9}} \frac{4\tan^3\theta - 12\tan\theta}{1-3\tan^2\theta}\,\mathrm{d}\theta$		
	$= 4\int_0^{\frac{\pi}{9}} \frac{\tan^3\theta - 3\tan\theta}{1-3\tan^2\theta}\,\mathrm{d}\theta = -4\int_0^{\frac{\pi}{9}} \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\,\mathrm{d}\theta = -4\int_0^{\frac{\pi}{9}} \tan 3\theta\ \mathrm{d}\theta$	**M1**	The candidate must be seen successfully transforming the integrand into terms of *tan* 3θ . Do not award the mark if the candidate integrates with respect to the wrong variable.
	$\frac{d}{d\theta} 2\ln(\sec(k\theta)) = 2k\tan(k\theta) \Longrightarrow \frac{d}{d\theta}\ln(\sec(k\theta)) = k\tan(k\theta)$		Permit preparation for integration by substitution. **ISW** if the candidate prepares to integrate by parts or applies any other technique which does not make appropriate use of the result obtained in **1(b)**.
	$-4\int_0^{\frac{\pi}{9}} \tan 3\theta\ d\theta = -\frac{4}{3}\int_0^{\frac{\pi}{9}} 3\tan 3\theta\ d\theta$	**M1**	**SOI**. Require the candidate to divide by $k = 3$.
	$= -\frac{4}{3}[\ln \sec 3\theta\ ]_0^{\frac{\pi}{9}}$	**M1**	Award for successful reversal of differentiation in **1(b)**.
	$= -\frac{4}{3}\left(\ln\left(\sec\left(\frac{\pi}{3}\right)\right) - \ln \sec \mathbf{0}\ \right) = -\frac{4}{3}\ \ln 2\ - \ln 1$		
	$= -\frac{4}{3}\ln 2$	**A1**	**EX**.

2a	$\sqrt{3}\sin\ x\ \ + \cos\ x\ \ = R\sin\ x + \alpha$		
	$R = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2$	**M1**	
	$\alpha = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$	**M1**	**EX.**
	$\sqrt{3}\sin\ x\ \ + \cos\ x\ \ + 1 = 2\sin\ x + 30°\ \ + 1$	**A1**	**EX.**
2b		**B1** **B1**	Candidate labels the axes and makes the appropriate markings (see the dotted lines for the x-axis part). Candidate draws graph correctly with good shape and smoothness.

3a	$\log_5\ 4-\omega\ \ -2\log_5\ \omega\ \ =1$		
	$\log_5\ 4-\omega\ \ -\log_5\ \omega^2\ \ =1$		
	$\log_5\left(\frac{4-\omega}{\omega^2}\right)=1$		
	$4-\omega=5\omega^2$		
	$5\omega^2+\omega-4=0$		
	$5\omega-4\ \ \omega+1\ \ =0\Longrightarrow\omega_1=\frac{4}{5},\ \omega_2=-1$	**M1**	Require candidate to display working when solving the quadratic equation. Else, do not award.
	$\omega=\frac{4}{5}$, reject $\omega=-1$ because $\omega>0$	**A1**	
3b	$k=\frac{6\log_2\ 6\ \ -6\log_2\ 3}{6-3}$	**M1**	Candidate applies formula for gradient of a straight line correctly.
	$k=2$	**A1**	Final answer obtained.
	$6\log_2\ x\ \ -\log_2\ 7\ \ =\frac{\ln\ x^6\ \ -\ln\ 7}{\ln\ 2}$	**M1**	Candidate converts the logarithm into a form suitable for applying the derivative of ln x , applying the change of base rule. **SOI**.

	$\frac{d}{dx}\left(\frac{\ln\ x^6\ \ -\ln\ 7}{\ln\ 2}\right) = \frac{6}{x\ln\ 2}$	**M1**	Award the above **M1** if the candidate correctly completes this step.
	$\frac{6}{x\ln(2)} = 2 \Longrightarrow x\ln(2) = 3$	**M1**	Candidate equates k and the above derivative.
	$x = \frac{3}{\ln\ 2} \approx 4.33$	**A1**	
4	$AB\colon\ y = \frac{1}{9}x + 1$		
	$BC\colon\ 7y = 35x + 18 \Longrightarrow y = 5x + \frac{18}{7}$		
	$AC\colon\ 3y = -x - 5 \Longrightarrow y = -\frac{1}{3}x - \frac{5}{3}$		
	$A\left(-6, \frac{1}{3}\right)$ $B\left(-\frac{9}{28}, \frac{27}{28}\right)$ $C\left(-\frac{89}{112}, -\frac{157}{112}\right)$	**M2***	Award **M1** for finding two points, and award the full **M2** for finding all three points. Condone missing y-coordinate, candidate will have to use it later.
	$M_{AB}\left(-\frac{177}{56}, \frac{109}{168}\right)$ $M_{AC}\left(-\frac{761}{224}, -\frac{359}{672}\right)$ $M_{BC}\left(-\frac{125}{224}, -\frac{7}{32}\right)$	**M2** dep	Award **M1** for each midpoint found, up to a maximum of **M2** (two midpoints). If **M2** is not achieved, **ISW**.
	$y_{\perp AB} = -9x - \frac{2335}{84}$	**M2**	Candidate finds <u>two</u> (or more) equations of perpendicular

	$y_{\perp AC} = 3x + \dfrac{3245}{336}$ $y_{\perp BC} = -\dfrac{1}{5}x - \dfrac{37}{112}$		bisectors.
	$O\left(-\dfrac{4195}{1344}, \dfrac{395}{1344}\right)$	**M1**	Candidate finds the coordinates of the point of intersection of **<u>any</u>** two perpendicular bisectors. Condone missing y -coordinate here, as it may be **SOI** by the **A1** mark.
	$r = OA = \sqrt{\left(-\dfrac{4195}{1344} + 6\right)^2 + \left(\dfrac{395}{1344} - \dfrac{1}{3}\right)^2} = \sqrt{\dfrac{7485985}{903168}} \approx 2.87899033$	**M1**	Accept rounding.
	$\left(x + \dfrac{4195}{1344}\right)^2 + \left(y - \dfrac{395}{1344}\right)^2 = \dfrac{7485985}{903168}$	**A1**	**OE**; also accept general form. Allow the radius to be rounded off to three significant figures.
			*For the marker's visualization.*

5a	reflex $\angle COA = 112° \times 2 = 224°$ (angle at center is twice angle at circumference)	**M1**	**DR.** For this entire question, **require correct reasons, or else the mark should not be awarded.**
	$\angle COA = 360° - 224° = 136°$ (angles at a point)	**A1**	
5b	$\angle CEA = 180° - 112° = 68°$ (angles in opposite segments)	**B1**	
5c	$\angle ACE = 36°$ (alternate segment theorem)	**M1**	Accept ‘tangent chord theorem’ and ‘angles in alternate segments.’
	$CO = OA$ (radii.)	**M1**	Require candidates to explain why COA is isosceles.
	$\angle OCA = \frac{180° - 136°}{2} = 22°$ (base angles in isosceles triangle)		
	$\angle ECF = 36° - 22° = 14°$	**A1**	
5d	$5 = 6 - y \implies y = -1$ $A\ -1,\ 6$	**B1**	A is directly below O.
5e	$OF = \sqrt{\ 2-8\ ^{2} +\ 6-6\ ^{2}} = \sqrt{6^2} = 6$	**M1**	
	$OF \neq OA$ hence OF is not a radius.		
	Thus, the circle does not pass through F.	**A1**	

6a	$\varphi^4 - 16\varphi^2 < 0, \varphi > 0$	**M1**	
	$\varphi^2 \ \varphi^2 - 16 \ < 0$		
	$\varphi \neq 0$ so $\varphi^2 - 16 < 0$		No working is necessary to solve this inequality; candidates should easily be able to find the solution by inspection.
	$-4 < \varphi < 4$	**A1**	
6b	$4y = 9 - x$		Allow candidate to prepare $x = f \ y$ and substitute x with $f \ y$.
	$\dfrac{9-x}{4} = \dfrac{x^2 - 3x + 2}{x + 3}$	**M1**	Allow direct substitution.
	$9 - x \quad x + 3 \ = 4x^2 - 12x + 8$	**M1**	
	$-x^2 + 6x + 27 = 4x^2 - 12x + 8$		
	$5x^2 - 18x - 19 = 0$	**M1**	
	$x = \dfrac{9 \pm 4\sqrt{11}}{5}$, $y = \dfrac{9 \mp \sqrt{11}}{5}$	**A1**	**EX.**
7	$f \ -5 \ = -2$, $f \ 5 \ = 0$		The last statement holds by the Factor Theorem.

	$f\ x\ = x^3 + ax^2 + bx + c$		The question only requires the roots of the equation $f\left(\frac{1}{u}\right) = 0.$ Any coefficient of x^3 will be divided out in the end.
	$f\ 5\ = 125 + c + 25a + 5b$ $f\ {-5}\ = -125 + c + 25a - 5b$		
	$f\ 0\ + 9 = c + 9 = 4 \implies c = -5$	**M1**	
	$f\ 5\ = 120 + 25a + 5b = 0$ $f\ {-5}\ = -130 + 25a - 5b = -2$	**M1**	
	$50a + 10 = -2 \implies a = \frac{4}{25}$	**M1**	
	$b = -\frac{130 + \frac{12}{50}}{5} = -\frac{124}{5}$	**M1**	
	For convenience, let all the coefficients of the function be in $\mathbb{Z}$. $g\ x\ = k \cdot f\ x\ = k\left(x^3 + \frac{4}{25}x^2 - \frac{124}{5}x - 5\right)$ Let $k = 25$. Then we have $g\ x\ = 25f\ x\ = 25x^3 + 4x^2 - 620x - 125$ (This is **not** necessary for the candidate!)		This is possible because the question's objective is merely the roots of some equation. If $x - 5$ divides $f\ x$ then obviously it divides $g\ x$ too.
	$g\ x\ =\ x - 5\ \ \alpha x^2 + \beta x + \gamma$		
	$g\ x\ =\ x - 5\ \ 25x^2 + 129x + 25$	**M1**	Require some form of working to find $\alpha x^2 + \beta x + \gamma$, namely either long or synthetic polynomial division.

	$25x^2 + 129x + 25 = 0$		
	$x = \dfrac{-129 \pm \sqrt{14141}}{50}$ or $x = 5$	**M1**	The candidate must display working (quadratic formula or completing the square) to solve the quadratic. Else, do not award the mark.
	$u = \dfrac{1}{x}$, so $u = \dfrac{1}{5}$ or $u = \dfrac{50}{-129 \pm \sqrt{14141}}$	**A2**	Award **A1** for $u = \frac{1}{5}$ and **A1** for the other solution.
8a	$T_{r+1} = \dbinom{32}{r}\left(\dfrac{k}{20}\right)^{r}$		
	$T_1 + T_2 + T_3 + T_4 + T_5$ $= \dbinom{32}{0}\left(\dfrac{k}{20}\right)^{0} + \dbinom{32}{1}\left(\dfrac{k}{20}\right)^{1} + \dbinom{32}{2}\left(\dfrac{k}{20}\right)^{2} + \dbinom{32}{3}\left(\dfrac{k}{20}\right)^{3} + \dbinom{32}{4}\left(\dfrac{k}{20}\right)^{4}$	**M1***	
	$= 1 + \dfrac{8}{5}k + \dfrac{31}{25}k^2 + \dfrac{31}{50}k^3 + \dfrac{899}{4000}k^4$		
	$1.1\ ^{32} = \left(1 + \dfrac{2}{20}\right)^{3} 2 \Longrightarrow k = 2$	**M1** dep	**SOI** by the following **M1**. The key result is that $k = 2$.
	$1 + \dfrac{8}{5}\ 2\ + \dfrac{31}{25}\ 2^2\ + \dfrac{31}{50}\ 2^3\ + \dfrac{899}{4000}2^4$	**M1**	Candidates must clearly show evidence of substitution.
	$= 17.716$	**A1**	Require candidates to leave their answer in decimal form.

8b	*Proof.* Consider the general term of the expansion, $$T_{r+1} = \binom{n}{r} \; ax^{\;r} \left(\frac{b}{x}\right)^{n-r}$$ $$= \binom{n}{r} a^r x^r b^{n-r} \left(\frac{1}{x}\right)^{n-r}$$ $$= \binom{n}{r} a^r x^r b^{n-r} \; x^{-1} \;{}^{n-r}$$ $$= \binom{n}{r} a^r x^r b^{n-r} x^{r-n}$$ $$= \binom{n}{r} a^r x^{2r-n} b^{n-r}$$	**M1**	Award **M1** for this simplification of the general term.
	Since n is positive and even, $n = 2m$ for some positive integer m. Hence $$2r - n = 2r - 2m = 2\; r - m$$	**M1**	
	Since $0 \le r \le n$ and $m = \frac{n}{2}$, there is a case where $r = m$, implying that $r - m = 0$. Hence an independent term always exists and the proof is complete.	**A1**	
9a	$$\frac{x^3+3x^2-6x+9}{(x-1)^2(x^2+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+1}$$	**M1**	
	$$x^3+3x^2-6x+9 = A\; x-1 \;\; x^2+1 \; + B\; x^2+1 \; + \; Cx+D \;\; x-1 \;{}^2$$		
	When $x = 1$ $$1+3-6+9 = 7 = 2B \Longrightarrow B = \frac{7}{2}$$	**M1**	
	$$x^3+3x^2-6x+9 - \frac{7}{2}\; x^2+1$$		
	$$x^3+3x^2-6x+9-\frac{7}{2}\; x^2+1 \; = \frac{1}{2}\; x-1 \;\; 2x^2+x-11$$ $$= A\; x-1 \;\; x^2+1 \; + \; Cx+D \;\; x-1 \;{}^2$$		

	$\frac{1}{2}(2x^2+x-11)=A(x^2+1)+(Cx+D)(x-1)$	**M1**	Candidate is forced to expand, due to the presence of x^2+1. Award the mark for correct factorization. If the candidate immediately expands and does not divide by $x-1$ as done so earlier, verify that his expansion, and therefore the formulation of his simultaneous equations, is correct.
	$\frac{1}{2}(2x^2+x-11)=Ax^2+A+Cx^2-D-Cx+Dx$ $= \ A+C\ x^2- \ C-D\ x+ \ A-D$		
	$\begin{cases} A+C=1 \\ C-D=-\frac{1}{2} \\ A-D=-\frac{11}{2} \end{cases}$		
	$A=-2,\ C=3,\ D=\frac{7}{2}$	**M1**	
	$\frac{x^3+3x^2-6x+9}{(x-1)^2(x^2+1)}=\frac{-2}{x-1}+\frac{7}{2(x-1)^2}+\frac{6x+7}{2(x^2+1)}$ $=\frac{7}{2\ x-1\ ^2}+\frac{6x+7}{2\ x^2+1}-\frac{2}{x-1}$	**A1**	**OE**.
9b	$\int\left(\frac{7}{2\ x-1\ ^2}-\frac{2}{x-1}\right)dx=\frac{7}{2}\int\ x-1\ ^{-2}\ dx-2\log\ x-1\ +C$	**M2**	Award **M1** for the integration of *each term*, up to a maximum of **M2**. **SOI**.
	$=-\frac{7}{2\ x-1}-2\log\ x-1\ +C$	**A1**	Do not award if candidate omits the constant of integration. Require **M2**; do not award **M1A1** or **M0A1**.

10	$\frac{1}{3}(3+\sqrt{5})h = 17-\sqrt{30}$	**M1**	Correct formulation of cone equation. Allow **ECF**.
	$h = \frac{3(17-\sqrt{30})}{3+\sqrt{5}}$		
	$h = \frac{51-3\sqrt{30}}{3+\sqrt{5}}$		
	$h = \frac{51-3\sqrt{30}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$		
	$h = \frac{(51-3\sqrt{30})(3-\sqrt{5})}{4}$	**M1**	
	$h = \frac{153-9\sqrt{30}-51\sqrt{5}+3\sqrt{150}}{4}$		
	$a+b+c+d = 153+9+51+3 = 216$	**A2**	**A1** for correct values of a, b, c, d in **any order**, and **A1** for the sum. Deny **ECF** here.
11a	$h = BT + CR$		
	$BT = x \sin\,\theta$	**M1**	
	$\angle ABT = 180° - 90° - \theta = 90° - \theta$ (angles in $\triangle$)	**M1***	Do not award this mark if
	$\angle TBR = 90°$ (angles in rectangle)		

	$\angle CBR = 120° - 90° - \ 90° - \theta \ = \theta - 60°$		
	$CR = y \sin \ \theta - 60°$	**M1** dep	
	$h = x \sin \ \theta \ + y \sin \ \theta - 60°$	**A1**	
11b	OQ is horizontal, so h $= 0$		
	$x\, sin\ \theta\ \ + y\, sin\ \theta - 60°\ \ = 0$	**M1**	Correct setting of h.
	$x\, sin\ \theta\ \ + y\ \ sin\ \theta\ \ cos\ 60°\ \ - sin\ 60°\ \ cos\ \theta\ \ \ = 0$	**M1**	Usage of identity **2.4**.
	$x\, sin\ \theta\ \ + \frac{y\, sin\ \theta}{2} - \frac{y\sqrt{3}\, cos\ \theta}{2} = 0$		
	$x\, tan\ \theta\ \ + \frac{y\, tan\ \theta}{2} = \frac{y\sqrt{3}}{2}$		
	$\left(\frac{2x+y}{2}\right) tan\ \theta\ \ = \frac{y\sqrt{3}}{2}$	**M1**	
	$tan\ \theta\ \ = \frac{y\sqrt{3}}{2x+y}$	**A1**	
12a	$h = \frac{25.0}{2} = 12.5$	**B2**	Award **B2** first, before deducting one mark from this question **for every incorrect value of** $\boldsymbol{a}, \boldsymbol{h}, \boldsymbol{k}$.
	$k = 15$		

	$0 = a\ 25 - 12.5\ ^2 + 15 = a\ 12.5\ ^2 + 15$		
	$a = -\dfrac{15}{12.5^2} = -0.096$		
	$y = -0.096\ x - 12.5\ ^2 + 15$		
12b	$\int_0^{25} -0.096\ x - 12.5\ ^2 + 15\, dx$		
	$= -0.096 \int_0^{25} x - 12.5\ ^2 - 156.25\, dx$		
	$= -0.096 \int_0^{25} x^2 - 25x\ \ dx$		Permit preparation of integration by substitution.
	$= -0.096 \left[\dfrac{x^3}{3} - \dfrac{25x^2}{2}\right]_0^{25}$	**A1**	Correct integration.
	$= 250$	**M1**	
	$250 \times 500\ \ \mathrm{cm}^3 = 0.125\,\mathrm{m}^3$	**M1**	The multiplication of the area by the length, and the conversion, must be clearly seen.
	$0.125 \times \$160 = \$20$	**A1**	
13	$a > 0$	**A1***	The question requires **linear inequalities**.
	$b^2 - 4a^2 < 0$		

	$b + 2a \quad b - 2a \quad < 0$		
	$-2a < b < 2a$	**A1***	
	$a = 9, b = 1$	**A1** dep	Require all the above marks first. **Accept any values fulfilling the inequality.**

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