MERIDIAN JUNIOR COLLEGE
JC2 Preliminary Examination
Higher 2

H2 Mathematics 9740/01

Paper 1 **15 September 2015**

3 Hours

Additional Materials: Writing paper
List of Formulae (MF 15)

READ THESE INSTRUCTIONS FIRST

Write your name and civics group on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

1 In the quadrilateral *OABC* where *O* is the origin, the position vectors of the points *A*, *B* and *C* are **a**, **b** and **c** respectively. Given that *AC* is perpendicular to *OB* and *OB* bisects the angle *AOC*, express $|\mathbf{a}|$ in the form of $k|\mathbf{c}|$, where *k* is a constant to be determined. [4]

2 It is given that

$$\mathrm{f}(x) = \begin{cases} x\mathrm{e}^{x} & \text{for } 0 < x \le 1, \\ \mathrm{e}(2-x) & \text{for } 1 < x \le 2, \end{cases}$$

and that $\mathrm{f}(x) = \mathrm{f}(x+2)$ for all real values of *x*.

(i) Sketch the graph of $y = \mathrm{f}(x)$ for $-2 < x \le 4$. [3]

(ii) Find the exact value of $\int_{-1}^{3} \mathrm{f}(x)\,\mathrm{d}x$. [4]

3 Consider the curve $y = \dfrac{x^2-49}{x^2-4}$.

(i) Show, by differentiation, that there is only one stationary point. [3]

(ii) Sketch the graph of $y = \dfrac{x^2-49}{x^2-4}$, stating the equations of any asymptotes, coordinates of any turning points and the coordinates of the points where the curve crosses the axes. [3]

(iii) Hence find the range of values of *p* such that $\dfrac{x^2-49}{x^2-4} = \sqrt{p^2-x^2}$ has no real roots. [2]

4 **(a)** Given that $\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$, find $\sum_{r=1}^{n}\left(n^r + (2r+2)(2r-2)\right)$. (There is no need to express your answer as a single algebraic fraction.) [3]

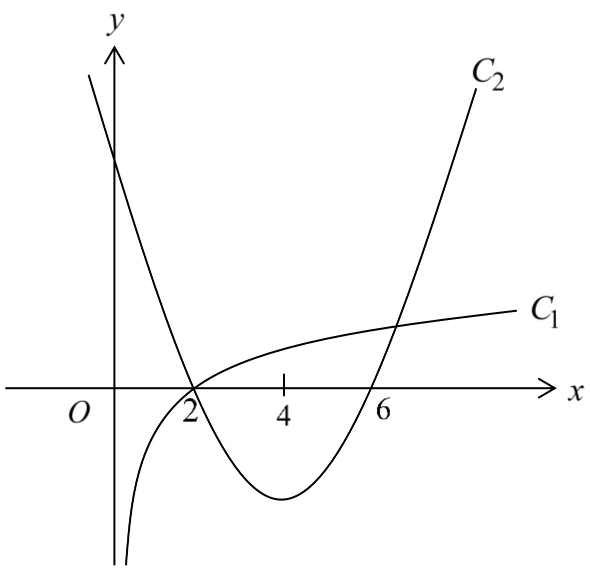
(b) Using the formula for $\cos P - \cos Q$, show that

$$\cos\left[(n-1)\theta\right] - \cos\left[(n+1)\theta\right] = 2\sin n\theta \sin\theta .$$ [2]

Hence show that

$$\sum_{r=2}^{n} \sin r\theta \sin\theta = \cos\frac{\theta}{2}\left[\cos\frac{3\theta}{2} - \cos\frac{(2n+1)\theta}{2}\right].$$ [4]

5



The diagram shows the curve C_1 with parametric equations $x = t^2 + t,\ y = t - \frac{1}{t}$ where $t > 0$ and curve C_2 with equation $y = (x-4)^2 - 4$. C_1 crosses the positive x-axis at $x = 2$. C_2 crosses the positive x-axis at $x = 2$ and $x = 6$ and has a minimum turning point at $x = 4$.

(i) The region R is bounded by C_1 and C_2 from $x = 2$ to $x = 6$. Using a non-calculator method, find the exact area of R. [6]

(ii) Find the numerical value of the volume of revolution when the region bounded by C_2, the lines $y = -3$ and $y = 4$ is rotated completely about the y-axis. [3]

6 Matthew embarks on a skipping regime, 6 days a week to lose weight. In week 1, he skips 50 times per day. In week 2, he skips 60 times per day. On each subsequent week, the number of skips per day is 10 more than on the previous week.

(i) Show that the number of skips completed by Matthew in week 8 is 720. [1]

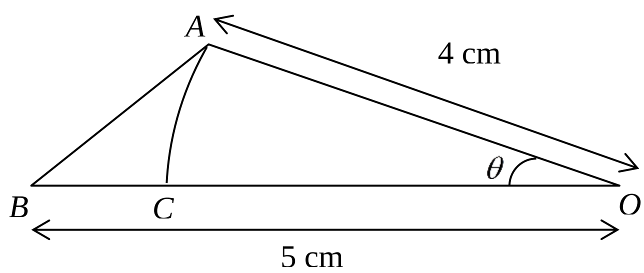
(ii) After n weeks, Matthew found that he exceeded 5000 skips in total. Express this information as an inequality in n and hence find the least value of n. [4]

As a result of the skipping, Matthew starts to lose weight. He measures his initial weight and records his weight at the end of each week and notices that his weights follow a geometric progression. At the end of week 32, Matthew's weight is 83 kg.

(iii) Given that he lost 10% of his initial weight at the end of week 25, find Matthew's initial weight. [3]

7 **(a)** Given that $\mathrm{f}(x)=\mathrm{e}^{\sin ax}$, where a is a non-zero real constant, find $\mathrm{f}(0)$, $\mathrm{f}'(0)$ and $\mathrm{f}''(0)$. Hence write down the first three non-zero terms in the Maclaurin series of $\mathrm{f}(x)$. Give the coefficients in terms of a. [5]

(b) [It is given that arc length of a circular sector with radius r and angle θ radians is $r\theta$.]



The diagram shows a triangle OAB with $OA=4$ cm, $OB=5$ cm and $\angle AOB=\theta$ radians. Given that OA and OC are radii of a circle with centre O and θ is a sufficiently small angle, show that the perimeter of ABC can be approximated by $a+b\theta+c\theta^2$ for constants a, b and c to be determined. [5]

8 A cup of hot liquid is placed in a room where the temperature is a constant $25°\text{C}$. As the liquid cools down, the rate of decrease of its temperature $\theta°\text{C}$ after time t minutes is proportional to the temperature difference $(\theta - 25)°\text{C}$. Initially the temperature of the liquid is $75°\text{C}$.

(i) Find θ in terms of t and sketch this solution curve. [7]

(ii) After 10 minutes, the temperature of the liquid was recorded to be $35°\text{C}$. Find the time it takes for the liquid to cool from $75°\text{C}$ to $30°\text{C}$, giving your answer to the nearest minute. [3]

9 The function f is defined as follows.

$$\mathrm{f} : x \mapsto x + 1 + \frac{1}{x-2} \quad \text{for} \quad x \in \mathbb{R},\ x \neq 2.$$

(i) Sketch the graph of $y = \mathrm{f}(x)$. [3]

(ii) If the domain of f is further restricted to $x \geq k$, state with a reason the least value of k for which the function f^{-1} exists. [2]

The function g is defined as follows.

$$\mathrm{g} : x \mapsto x + \frac{1}{x} \quad \text{for} \quad x \in \mathbb{R},\ x \neq 0.$$

(iii) Describe fully a sequence of transformations which would transform the curve $y = \mathrm{f}(x)$ onto $y = \mathrm{g}(x)$. [2]

A function h is said to be odd if $\mathrm{h}(-x) = -\mathrm{h}(x)$ for all x in the domain of h.

(iv) Show that g is odd. [1]

(v) Find $\sum_{x=-m}^{-1} \mathrm{g}(x) + \sum_{x=2}^{m+1} \mathrm{g}(x)$, where m is a positive integer. [2]

10 **(a)** On the same diagram, sketch the loci

(i) $|z-3-6\mathrm{i}|=4$,

(ii) $\arg(z-3-2\mathrm{i})=\dfrac{\pi}{6}$. [4]

The complex number w is represented by the point of intersection of the loci in parts **(i)** and **(ii)**. Find w in the form x + iy, leaving the values of x and y in the exact non-trigonometrical form. [3]

(b) Sketch the locus of z such that $2 \le |z| < 4$ and the argument of z follows an arithmetic progression where the first term is $-\dfrac{3\pi}{4}$ radians with common difference $\dfrac{\pi}{2}$ radians. Find the exact minimum value of $|z+3\mathrm{i}|$. [4]

11 A curve C has parametric equations

$$x=2\cos t, \qquad y=\sin t, \qquad \text{for } 0 \le t < 2\pi.$$

Show that the equations of the tangent and normal to C at the point P with parameter θ are $(\cos\theta)x+(2\sin\theta)y=2$ and $(2\sin\theta)x-(\cos\theta)y=3\sin\theta\cos\theta$ respectively. [5]

(i) Show algebraically that the tangent to C at the point P does not cut the curve C again. [3]

(ii) The normal to C at the point P cuts the x-axis and y-axis at points A and B respectively. By finding the mid-point of AB, determine a cartesian equation of the locus of the mid-point of AB as θ varies. Hence describe the locus formed. [6]

END OF PAPER