Name : _____

METHODIST GIRLS' SCHOOL
Founded in 1887

PRELIMINARY EXAMINATION 2022
Secondary 4

Tuesday	**MATHEMATICS**	**4048/01**
2 August 2022	**Paper 1**	2 hours

Candidates answer on the Question Paper.

INSTRUCTIONS TO CANDIDATES

Write your class, name and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For p, use either your calculator value or 3.142, unless the question requires the answer in terms of p.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80**.**

80
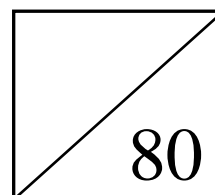
Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle} = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1. Arrange the following numbers from the smallest to the largest.

    $$\frac{2\pi}{3} \quad , \quad -4 \quad , \quad 2.\dot{3} \quad , \quad 227\% \quad , \quad -\sqrt{17}$$

    Answer ……………………………..………………………. [1]

2. The marked price of a washing machine in a shop is $m.
    During the Great Singapore Sale, it was sold at a discount of *d* %.

    (a) Express the selling price as a single fraction in terms of *m* and *d*.

    Answer $ …..……...…………… [1]

    (b) The shopkeeper made a profit of 25% from the sale of the washing machine.
    Express the cost price as a single fraction in terms of *m* and *d*.

    Answer $ …..………..………… [1]

3. Factorise completely $4px^2 - 3k + 12kx^2 - p$.

    Answer …………………..……………………… [3]

4. Solve $\dfrac{x-4}{4} - \dfrac{2x-1}{6} = 1$.

Answer $x = $ ……………………… [2]

5. **(a)** Given that $p = 2^4 \times 3^2 \times 11$ and $q = 2 \times 3^2 \times 5 \times 11$.
Find the greatest integer that divides p and q exactly.

Answer …………………………… [1]

(b) A number has exactly nine factors. Two of the factors are 12 and 18.
List all the factors of the number.

Answer ………………………………………………………………… [2]

6. $\varepsilon = \{\text{all real numbers}\}$

$A = \{\text{all prime numbers}\}$

$B = \{\text{all rational numbers}\}$

$C = \{\text{all integers}\}$

$D = \{\text{all negative numbers}\}$

$E = \{1, 2, 3, 4, 5, 6, 7\}$

(a) List all the elements contained in the set $A \cap E$.

Answer ……………………………………… [1]

(b) Explain why $C \cap B' = \phi$.

Answer ……………………………………………………………………………………………

………………………………………………………………………………………………………..

………………………………………………………………………………………………………..

…………………………...

…………………………………………………………………………………………………… [1]

(c) Which of the following statement(s) is/are correct?

Statement 1: $C \subset B$

Statement 2: $A \cup C = A$

Statement 3: $A \cap D = \phi$

Statement 4: $C \cap D = \phi$

Answer Statement(s) number ……………….…………….is/are correct [2]
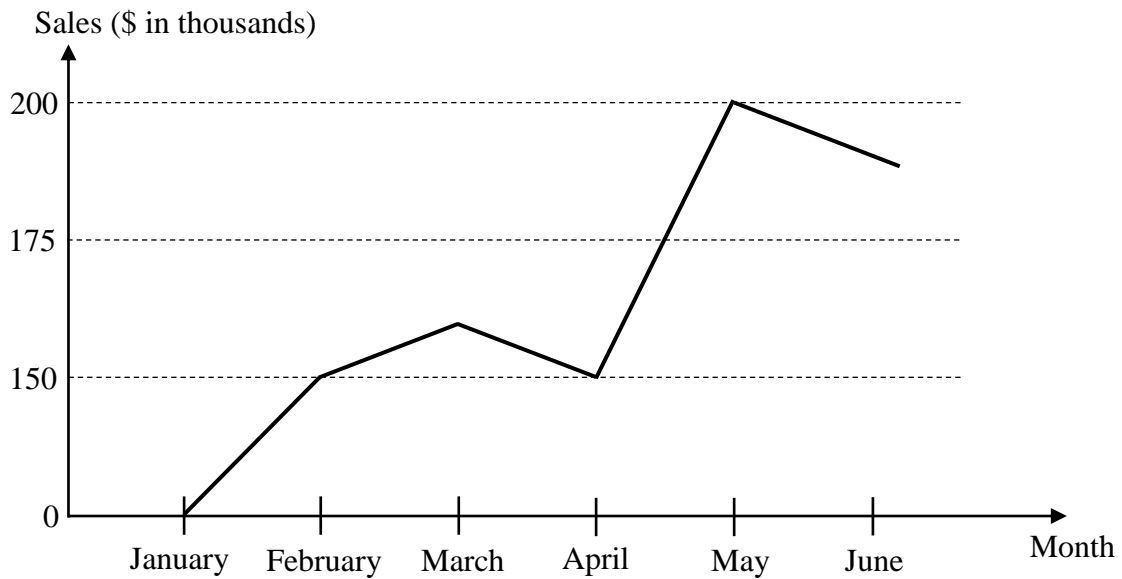
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7. Given that $\sqrt{\dfrac{h^2k-3}{5w}} = 2h$, express h in terms of k and w.

Answer ………………………………… [3]

8. Given that n is a positive integer, explain whether $(7n+3)^2 - 4^2$ is divisible by 7. Show your working clearly.

Answer [2]

9. The chart below shows the monthly sales of an electrical company from January to June in 2022.

(a) State one feature of the graph that may be misleading.

Answer …………………………………………………………………………………

……………………………………………………………………………………… [1]

(b) Explain how this may lead to a misinterpretation of the graph.

Answer …………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

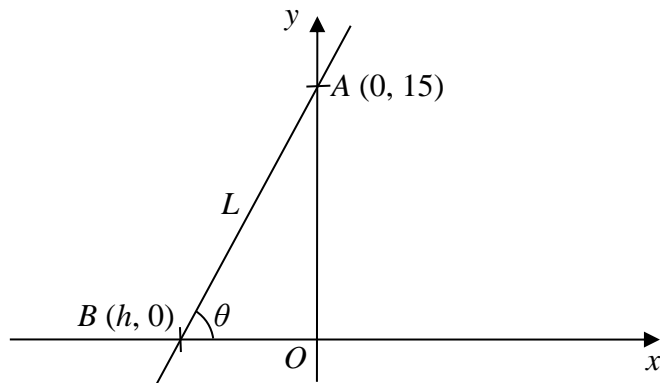……………………………………………………………………………………………

……………………………………………………………………………………… [1]

10. In the diagram, the line L cuts the y-axis at A (0, 15) and the x-axis at B (h, 0).

Given that $\tan \theta = \dfrac{5}{2}$, where θ is the angle made by the line L and the horizontal x-axis, find

(a) the value of h,

Answer h = …………………… [1]

(b) the equation of the line L.

Answer …………….…………… [1]

(c) The coordinates of point C is (−6 , 21). Calculate the length of AC.

Answer …………….…… units [2]

(d) *ABCD* is a trapezium. Find the coordinates of a possible point D.

Answer (……… , ……..) [1]

11. (a) Solve the inequality $-5 \leq \frac{1}{2}(3+5x) < 7+x$.

Answer ………….…………… [3]

(b) Illustrate the solutions of the inequality on a number line clearly.

Answer [1]

→ x

(c) Find the smallest integer value of x, which satisfies the inequality.

Answer ………….…………… [1]

12. Given that $2^{2n-3} = 2^{2022} - 2^{2021}$, find the value of n.

Answer $n =$ …..…………… [2]

13.

	Vietnam	Singapore
Population	9.8×10^7	5.4×10^6

Use the information from the table to answer the following.

(a) How many more people live in Vietnam than in Singapore?
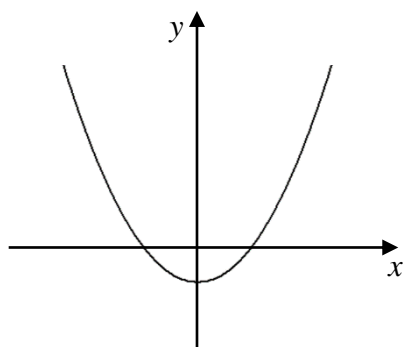Give your answer in standard form, to a sensible degree of accuracy.

Answer .... [1]

(b) The land area of Vietnam is 331 690 km².
Calculate the average number of people per square kilometre living in Vietnam.

Answer .... people/km² [1]

14. Select a possible equation from the box to represent each of the sketch graphs below.

$xy = 2$	$y = x^3 - 2$	$y = 2 - x^3$
$y = 2 - x^2$	$y = x^2 - 2$	$x^2 y = 2$

(a)

Answer …………………………….. [1]

(b)
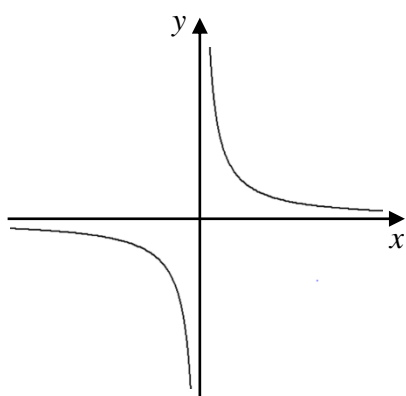
Answer …………………………….. [1]

(c)
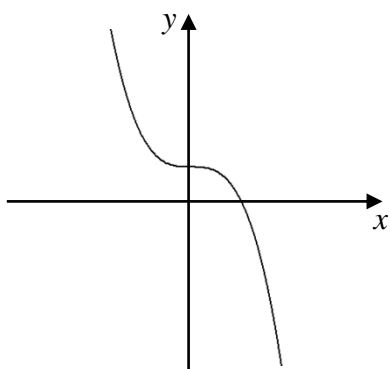
Answer …………………………….. [1]

15. The diagram shows a regular octagon and a regular pentagon.
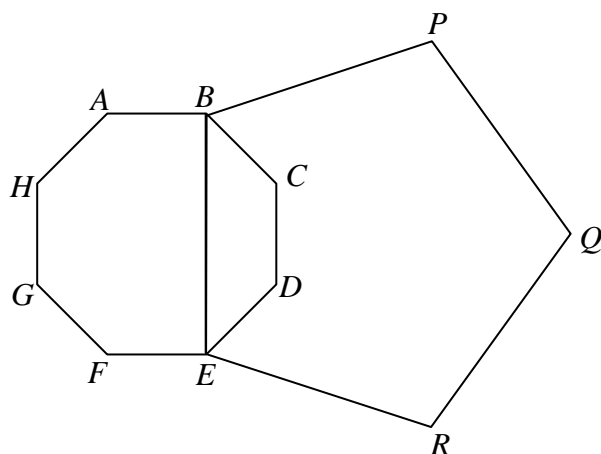
(a) **(i)** Find $\angle PBE$.

Answer $\angle PBE$ = ……….………….. [1]

(ii) Find $\angle PBC$.

Answer $\angle PBC$ = ……….………….. [2]

(b) Is *BE* parallel to *CD*? Explain your answer with working clearly.

Answer [1]

16. The heights of 20 students were measured.
The results are shown in the stem-and-leaf diagram below.

```
14 | 6 6 7 8
15 | 1 2 3 5 5 6
16 | 0 1 1 1 4 6 7 7 9
17 |
18 |
19 |
20 | 5
```

Key 14 | 6 means 146 cm

(a) Find the median height.

Answer …………………… cm [1]

(b) Which is a more appropriate measure of central tendency, median or mean, to represent this distribution? Explain the reason clearly.

Answer ………………… is more appropriate because ………………………………..

……………………………………………………………………………………………

……………………………………………………………………………………………… [1]

17. The diagram shows a parallelogram *ABCD*.
APS, *BRS*, *CRQ* and *DPQ* are straight lines that bisect angles *A*, *B*, *C* and *D* respectively.
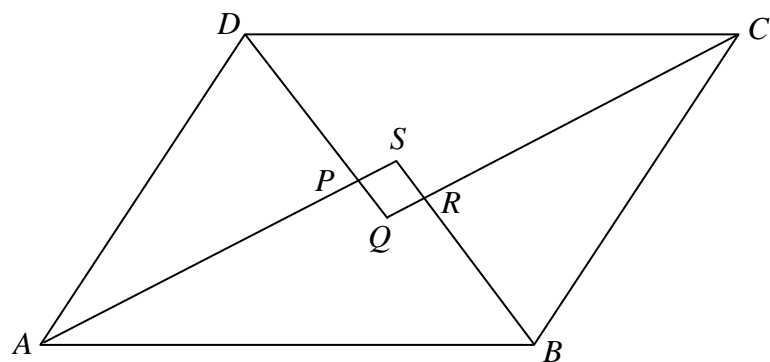
Prove that Δ*PAD* and Δ*RCB* are congruent.

Answer [3]

18. The cash price of a motorcycle is $18 000.

Mr Lim made a down payment of 40% on the cash price. He took a loan of the balance amount from a bank that charged a simple interest of r % per annum. He paid 30 equal monthly instalments to the bank.

Mr Lim paid a total of $20 025 for the motorcycle.

(a) Find the value of r.

Answer $r = $ …………………….. [2]

(b) Calculate each monthly instalment.

Answer $ ....………..…..……… [2]

_____

19. The terms T_1, T_2, T_3, T_4 of a sequence are given as follows:

$$T_1 = \frac{1}{3} = \frac{1}{1 \times 3} = \frac{1}{2 \times 1} - \frac{1}{2 \times 3}$$

$$T_2 = \frac{1}{8} = \frac{1}{2 \times 4} = \frac{1}{2 \times 2} - \frac{1}{2 \times 4}$$

$$T_3 = \frac{1}{15} = \frac{1}{3 \times 5} = \frac{1}{2 \times 3} - \frac{1}{2 \times 5}$$

$$T_4 = \frac{1}{24} = \frac{1}{4 \times 6} = \frac{1}{2 \times 4} - \frac{1}{2 \times 6}$$

(a) **(i)** Write down the next term, T_5, in this sequence $\dfrac{1}{3}$, $\dfrac{1}{8}$, $\dfrac{1}{15}$, $\dfrac{1}{24}$, ...

Answer T_5 = ...………………… [1]

(ii) Write down the nth term of this sequence, T_n, in terms of n.

Answer T_n = ...………………… [1]

(b) Find the exact value of $T_1 + T_3 + T_5 + \ldots + T_{99}$.

Answer ……...………………… [2]

20. The braking distance, d, of a car is directly proportional to the square of its speed, v.
When the speed is p metres per second, the braking distance is 8 metres.

(a) When the speed is increased by 200%,

(i) write down an expression, in terms of p, for the new speed of the car,

Answer …………………… m/s [1]

(ii) hence, find the braking distance.

Answer ………………… m [2]

(b) Find the percentage increase in the braking distance.

Answer ……………… % [2]

21. The diagram shows a circle with centre *O*. *ABCDE* is a straight line. *C* is the mid-point of *BE*. It is given that $OD = 8$ cm, $BC = 10$ cm, $\angle ODB = 90°$.
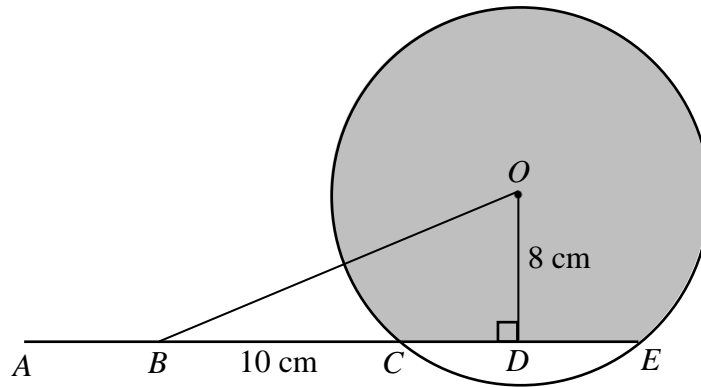
(a) Find the exact value of cos $\angle OBA$.

Answer ...………………… [2]

(b) **(i)** Show that $\angle DOE = 0.5586$ radian, correct to 4 significant figures.

Answer [2]

(ii) Hence, find the area of the shaded region.

Answer ..………………… cm^2 [3]

22. The diagram shows the speed-time graph of a motorcycle.
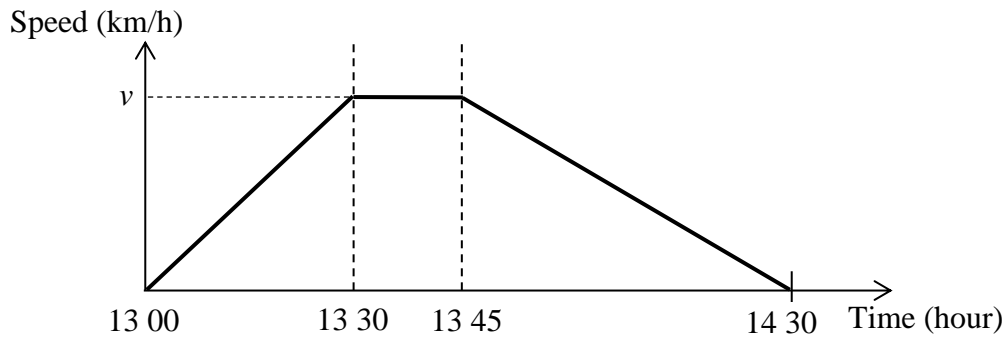
(a) Given that the deceleration of the motorcycle at 14 00 hour is 100 km/h^2, calculate the maximum speed, v, in km/h.

Answer v = ... km/h [2]

(b) Calculate the total distance travelled by the motorcycle from 13 00 hour to 14 30 hour.

Answer .... km [2]

(c) On the grid in the answer space, sketch the distance-time graph of the motorcycle for the same journey from 1300 to 1430.

Answer [2]

Distance (km)

0

13 00   13 30   13 45   14 30   Time (hour)
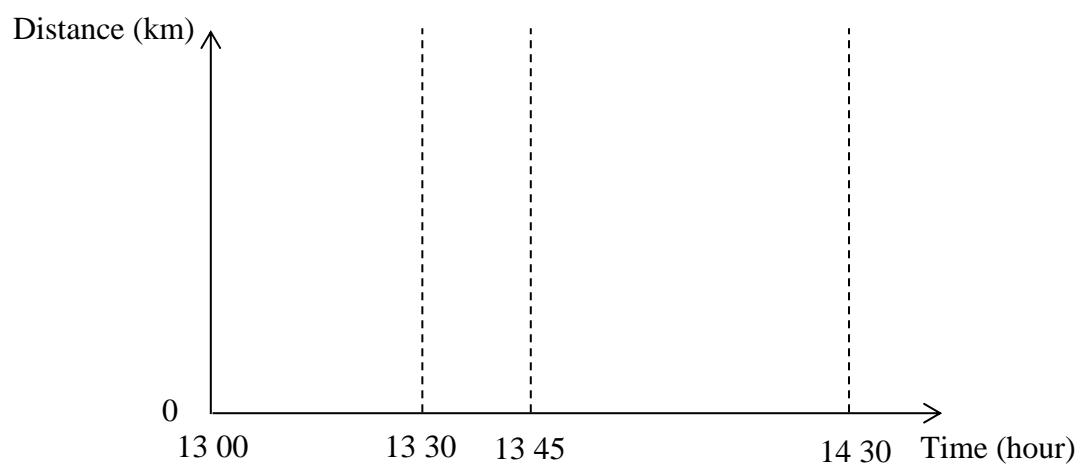
23. The diagram is a scale drawing of a triangular park, *XYZ*.

Scale 1 : 5000

North

X

Y

Z
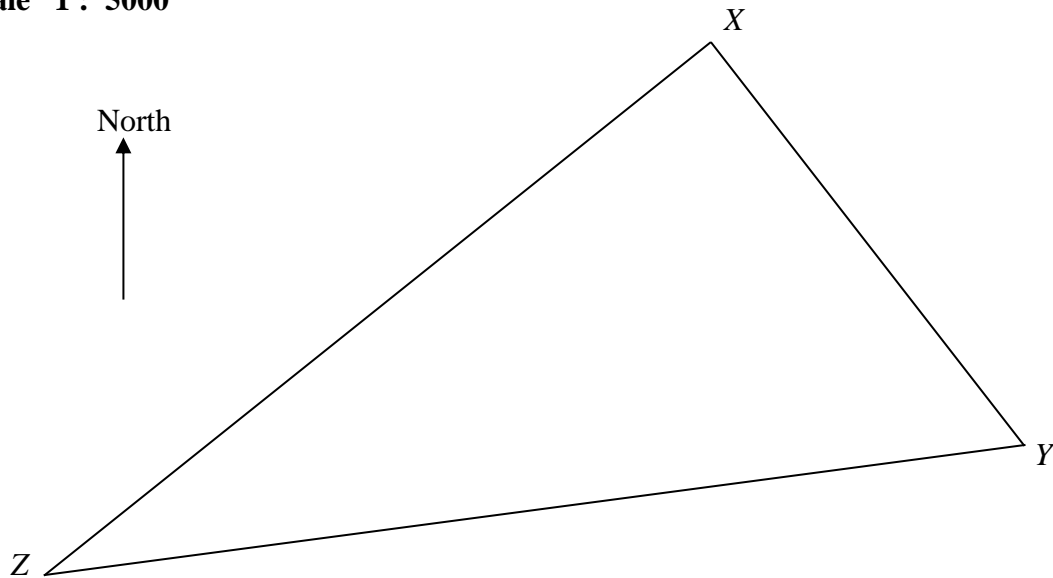
(a) Measure and write down the bearing of *Y* from *Z*.

Answer .... [1]

(b) On the diagram, using compasses, construct

    (i) the bisector of angle *ZXY*,   [1]

    (ii) the bisector of angle *XYZ*.   [1]

(c) A circular playground is to be built inside the triangular park *XYZ*.

    (i) Using compasses, construct a circle touching the three sides of triangle *XYZ*.   [1]

    (ii) Find the greatest possible actual area, in m^2, of the circular playground, correct to the nearest m^2.

Answer .... m^2 [2]

End of paper

Answer Key:

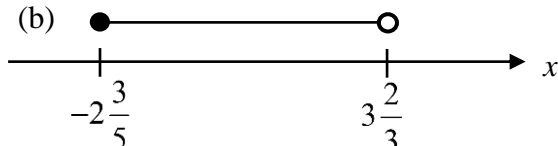|	
1. $-\sqrt{17}$, -4 , $\dfrac{2\pi}{3}$, 227% , $2.\dot{3}$	9(a) The scale / intervals on the vertical axis is not consistent. (b) It may mislead the reader to think that the <mark>increase in sales</mark> from April to May (\$50 000) appears to be twice of that from January to February (\$150 000). Actually the amount of increase is only one-third. Or It may mislead the reader to think that the <mark>amount of sales</mark> in May (\$200 000) appears to be three time of that in April (\$150 000). Actually it is only $1\dfrac{1}{3}$ times.
2(a) $\dfrac{m\,(100-d)}{100}$ (b) $\dfrac{m\,(100-d)}{125}$	
3. $(p+3k)\,(2x+1)\,(2x-1)$	10(a) $h=-6$ (b) $y = \dfrac{5}{2}x + 15$ (c) 8.49 units
4. $x=-22$	
5(a) 198 (b) 1 , 2 , 3 , 4 , 6 , 9 , 12 , 18 , 36	(d) $(0\,,\,20)$ Any value of $y>15$
6(a) $\{2\ ,\ 3\ ,\ 5\ ,\ 7\}$ (b) $B^{'}$ are irrational numbers which are non-terminating and non-recurring decimal numbers that cannot be expressed as a fraction like the integers in C. Therefore, $C\cap B^{'} = \phi$. (c) Statements 1 and 3	11(a) $-2\dfrac{3}{5} \le x < 3\dfrac{2}{3}$ (b) (c) -2
7. $h = \pm\sqrt{\dfrac{3}{k-20w}}$	12. $n=1012$
8. $(7n+3)^2 - 4^2$ $= 7\,(7n^2+6n-1)$ or $7\,(n+1)\,(7n-1)$ Since 7 is a factor, $(7n+3)^2 - 4^2$ is divisible by 7.	13(a) 9.3×10^7 (2sf) (b) 295 people/km^2 (3sf)

14(a) $y = x^2 - 2$ (b) $xy = 2$

  (c) $y = 2 - x^3$

15(ai) 108°   (aii) 63°

  (b) Since $\angle CBE + \angle BCD$
    $= 45° + 135°$
    $= 180°$
    By property of interior angles, BE is parallel to CD.

16(a) 158 cm

  (b) Median… there is an extreme height of 215 cm which will give a value of the mean height that skews the representation of the central tendency for this distribution.

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17. $AD = BC$ (opp. sides of parallelogram)

$\angle DAB = \angle DCB$ (opp. angles of parallelogram)
$\angle DAP = \angle BCR$ (since APS bisects $\angle A$, CRQ bisects $\angle C$)

$\angle ADC = \angle ABC$ (opp. angles of parallelogram)
$\angle ADP = \angle CBR$ (since DRQ bisects $\angle D$, BRS bisects $\angle B$)

$\therefore \triangle PAD \equiv \triangle RCB$ (AAS)

18(a) $r = 7.5$ (b) $427.50

19(ai) $T_5 = \dfrac{1}{35}$

  (aii) $T_n = \dfrac{1}{n^2 + 2n}$ or $\dfrac{1}{n(n+2)}$

  (b) $\dfrac{50}{101}$

20(ai) $3p$ (aii) 72   (b) 800%

21(a) $-\dfrac{15}{17}$

  (bi) $\tan \angle DOE = \dfrac{5}{8}$

    $\angle DOE = \tan^{-1}\left(\dfrac{5}{8}\right)$

          $= 0.5586$ rad (4sf)   (shown)

  (bii) 270 cm^2

22(a) $v = 75$ km/h

  (b) 65.625 km

  (c)

Distance (km)

23(a) 083°   (Accept 082° - 084°)

  (bi, ii), (ci)

  (cii)
Area of circle on drawing
$= \pi(2.4)^2$ cm^2 (Accept radius 2.3 – 2.5 cm)

Actual area of circle
$= 45239$ m^2 (to nearest m^2)

(Accept 41548 – 49087 m^2 based on the radius on the drawing)