# HUA YI SECONDARY SCHOOL PRELIMINARY EXAM 2024

# 4-G3 / 5-G2

NAME

CLASS

INDEX NUMBER

## ADDITIONAL MATHEMATICS 4049/01

## PAPER 1

**22 September 2024**

**2 hour 15 minutes**

Candidates answer on the Question Paper

No Additional Materials is required.

### READ THESE INSTRUCTIONS FIRST

Write your Name, Class, and Index Number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue, or correction fluid.

Answer **all** the questions.
The number of marks is given in the brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer. Omission of essential working will result in loss of marks.
The total number of marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate. If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Gives answers in degrees to one decimal place. For π, use either your calculator value or 3.142.

This document consists of **21** printed pages and **1** blank page.



Setter: Ms Lee Hui Ling

**[Turn Over**

*Mathematical Formulae*

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial expansion*

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

*Formulae for $\Delta ABC$*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

**1** A cuboid has a base area of $(7+4\sqrt{5})\text{cm}^2$ and a volume of $(16+18\sqrt{5})\text{cm}^3$. Find, without using a calculator, the height of the cuboid, in cm, in the form $(a+b\sqrt{5})$, where $a$ and $b$ are integers.

[3]

**2** The curve $5x - xy = 20$ and the line $x - 2y - 3 = 0$ intersects at the points $A$ and $B$.
Find the $y$-coordinate of $A$ and of $B$. [3]

**3** **(a)** Express $12x - 13 - 3x^2$ in the form $a(x+b)^2 + c$ and hence state the coordinates of the turning point of the curve $y = 12x - 13 - 3x^2$. [4]

**(b)** State the range of $k$ such that $y = k$ will intersect the curve $y = 12x - 13 - 3x^2$. [1]

**4** Integrate $\frac{4}{5x+1} - \frac{6}{x^3}$ with respect to $x$. [2]

**5** Express $\frac{7x^2-17x+1}{(x^2+1)(2-3x)}$ in partial fractions. [5]

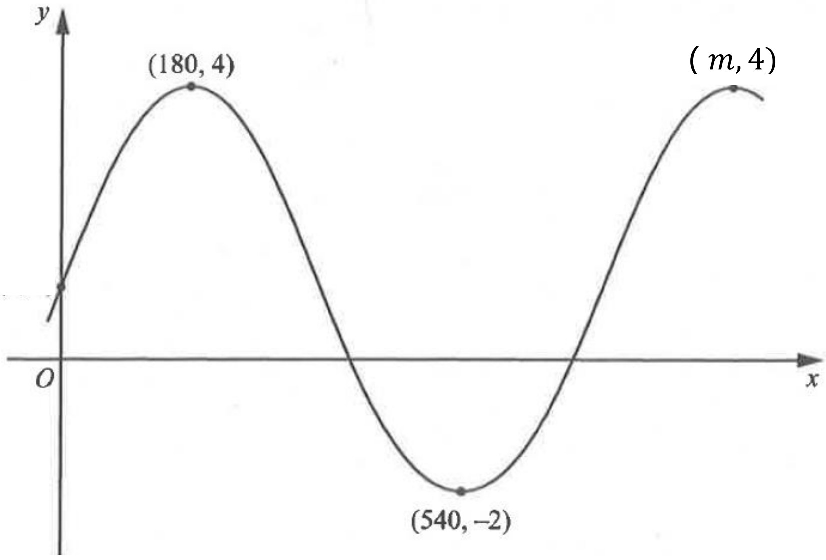
**6** Given that $x^5 + ax^3 + bx^2 - 3 \equiv (x^2 - 1)\mathrm{Q}(x) - x - 2$, where $\mathrm{Q}(x)$ is a polynomial,

**(a)** state the degree of $\mathrm{Q}(x)$, [1]

**(b)** show that $a = -2$ and $b = 1$, [3]

**(c)** find the polynomial $\mathrm{Q}(x)$. [3]

**7**



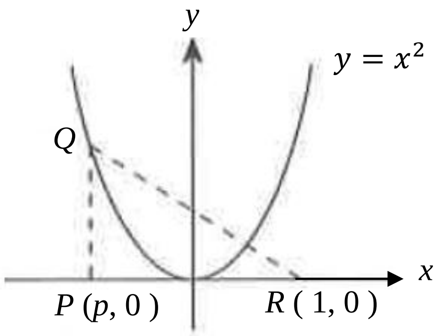

The sketch above shows part of the graph of $y = a\sin\left(\frac{x}{b}\right) + c$, where $x$ is in degrees.

**(a)** Explain why $c = 1$. [2]

**(b)** State the value of $b$. [1]

**(c)** Find the value of $m$ and explain how you get it. [2]

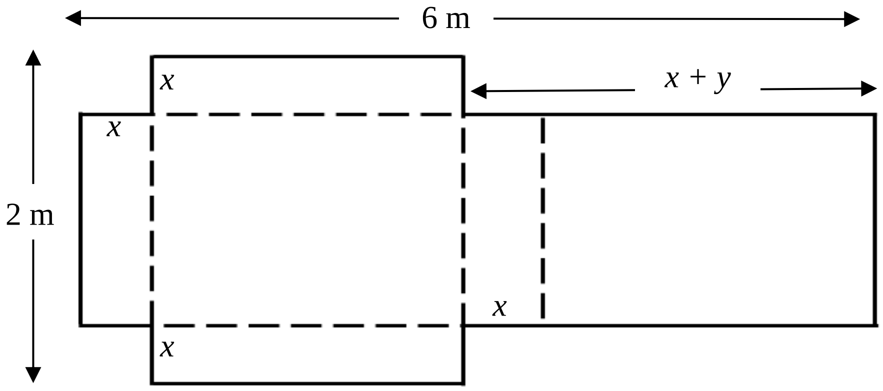
**8** The figure shows the curve $y = x^2$ and the point $R(1,0)$. The variable point $P(p, 0)$ moves along the $x$ axis and $PQ$ is vertical. It is given that $p$ decreases at the rate of 1.2 units per second.




**(a)** Show that the area of the triangle $PQR$, $A$ units², is $A = \frac{1}{2}p^2 - \frac{1}{2}p^3$. [2]

**(b)** Find the rate at which $A$ is increasing at the instant when $p = -7$. [4]

**9** From a rectangular piece of metal of width 2m and length 6m, two squares of side $x$ m and two rectangles of sides $x$ m and ( $x + y$) m are removed as shown. The metal is then folded about the dotted lines to give a closed box with height $x$ m.

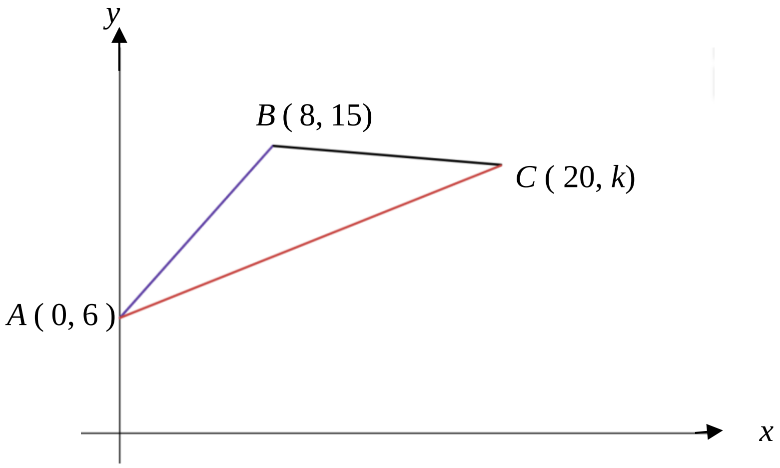



**(a)** Show that the volume of the box, $V$ m$^3$, is given by $V = 2x^3 - 8x^2 + 6x$. [3]

**(b)** Given that $x$ can vary, find the stationary value of $V$. [4]

**(c)** Show that this value of $V$ is a maximum. [2]

**10** The diagram shows a triangle $ABC$ with vertices at $A\,(\,0,\,6\,)$ , $B\,(\,8,\,15\,)$ and $C\,(\,20,\,k\,)$.




**(a)** Given that $AB = BC$, find the value of $k$. [4]

**(b)** $D$ is a point on the $x$ axis such that $ABCD$ is a kite. Find the coordinates of $D$. [4]

**(c)** Hence find the area of the kite $ABCD$. [2]

**11 (a)** Prove the identity $\cot\theta + \frac{\sin\theta}{1+\cos\theta} = \text{cosec}\,\theta$. [3]

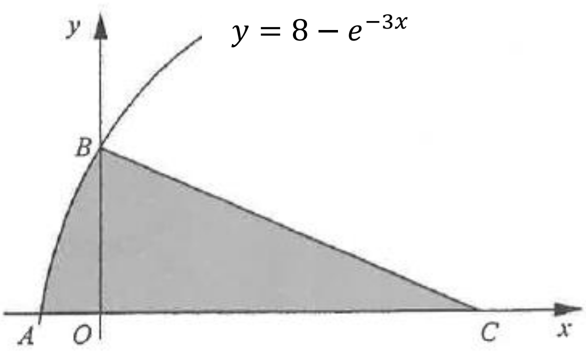
**(b)** Hence solve the equation $\cot 3\theta + \frac{\sin 3\theta}{1+\cos 3\theta} = -2$ for $-90° \leq \theta \leq 90°$. [4]

**12 (a)** Solve $6^{2x-1} = 4^x \times 5$. [4]

**(b)** Given that $\log_x 3 = p$, express $\log_3 \frac{27}{x}$ in terms of $p$. [2]

**(c)** Solve the equation $\lg x - 1 = \lg(x - 1)$ [3]

**13** The diagram shows part of the curve $y = 8 - e^{-3x}$ which crosses the axes at $A$ and at $B$.

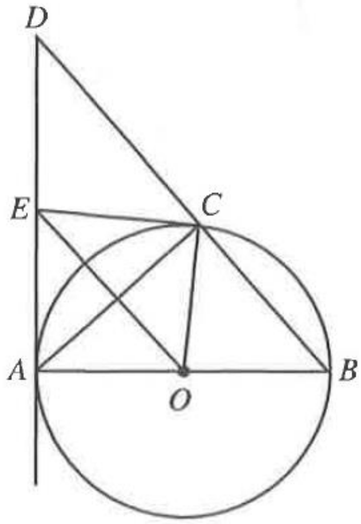



**(a)** Show that the $x$ coordinate of $A$ is $-\ln 2$ . [3]

**(b)** The normal to the curve at $B$ meets the $x$ axis at $C$. Find the coordinates of $C$. [4]

**(c)** Find the area of the shaded region. [4]

**14**




The diagram shows a circle, centre $O$, with diameter $AB$. The point $C$ lies on the circle. The tangent to the circle at $A$ meets $BC$ extended at $D$. The tangent to the circle at $C$ meets the line $AD$ at $E$.

**(a)** Prove that triangles $EOA$ and $EOC$ are congruent. [3]

**(b)** Prove that triangles *ADC* and *BAC* are similar. [3]

**(c)** Hence prove that $AC^2 = CD \times BC$. [2]

**End of Paper**

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