INDEX NUMBER

Anglo - Chinese School

(Independent)

THE BEST IS YET TO BE

FINAL EXAMINATION 2020

YEAR THREE EXPRESS

ADDITIONAL MATHEMATICS

PAPER 1

Monday **5 October 2020** **1 hour 30 minutes**

Candidates answer on the Question Paper.

No additional materials are required.

READ THESE INSTRUCTIONS FIRST

Write your index number in the space at the top of this page.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers in the spaces provided under each question.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 60.

For Examiner's Use
60

This document consists of **13** printed pages and **1** blank page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Answer all the questions.

1 Solve the following pair of simultaneous equations [4]

$$4x^2 + 3xy + y^2 = 1$$

$$x + y = 1$$

[Turn Over

2 Given that $\tan\theta = \frac{12}{5}$ and that θ is acute, find the exact value of

(i) $\cos(-\theta)$, [1]

(ii) $\cos(90° - \theta)$, [1]

(iii) $\tan(180° - \theta)$. [1]

3 **(a)** The graph of $y = \log_a(kx - 1)$ passes through the points with coordinates $(1, 0)$ and $(5, 2)$.

(i) Determine the value of each of the constants a and k. [4]

(ii) Write down the range of values of x such that y is defined. [1]

(b) Sketch the graph of $y = \log_4 x$. [2]

4 It is given that $f(x) = 4x^3 - 16x^2 + 21x - 9$.

(a) Find the quotient when $f(x)$ is divided by $x^2 + 1$. [2]

(b) Prove that $x - 1$ is a factor of $f(x)$. [1]

(c) Hence, factorise $f(x)$ completely. [3]

(d) Express $\dfrac{x}{f(x)}$ in partial fractions. [5]

5 The equation of a graph is $y = 2\sin 2x + 1$ for $0 \le x \le \pi$.

(i) State the period and amplitude of y. [2]

(ii) Solve $y = 0$ for $0 \le x \le \pi$, giving your answer in exact form. [3]

(iii) Sketch the graph of $y = 2\sin 2x + 1$ for $0 \le x \le \pi$. [3]

(iv) By drawing a suitable straight line on the same axis in **(iii)**, find the number of solutions to the equation $2\sin 2x = 1$. [3]

6 **(i)** Given that $3\lg(xy) = 2 + 2\lg x - \lg y$, express x in terms of y. [4]

(ii) Solve the equation $\log_4(x+2) - 4\log_{16}(x-1) = 1$. [4]

7 $f(x)$ is a cubic polynomial such that $f(x) = (x+1)(x-m)(x-3m)$, where m is an integer. It is given that $f(x)$ has a remainder of 10 when divided by $(x-1)$.

(i) Find the value of m. [3]

(ii) With the value of m found in **(i)**, write down the expression for $f(x)$ in descending powers of x. [2]

(iii) Hence, solve the equation $(y+1)^3 - 7(y+1)^2 + 4y + 16 = 0$. [2]

8 **(a)** A circle C_1 has an equation given by $x^2 + y^2 - 2x - 6y - 8 = 0$.

(i) Find the centre and radius of circle C_1. [2]

(ii) Given that the equation of the tangent to C_1 at point P is $y = x - 4$, find the coordinates of P. [4]

(b) A circle C_2 has a radius of 6 units and its centre is at $(2, 4)$. The lowest point on circle C_2 is T. Another circle C_3 has its highest point and lowest point at the centre of C_2 and T respectively. Find the equation of C_3. [3]

End of Paper 1

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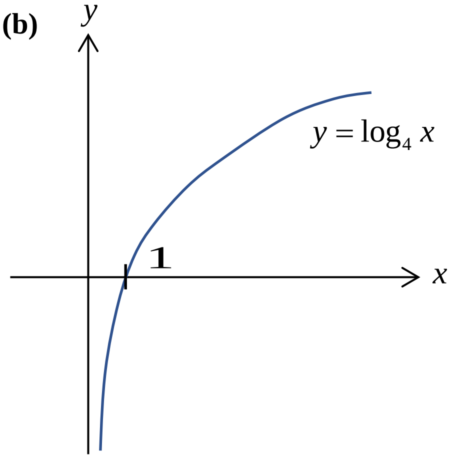
Answers

1. $x = 0,\ y = 1$ or $x = -\frac{1}{2},\ y = \frac{3}{2}$

2. **(i)** $\frac{5}{13}$ **(ii)** $\frac{12}{13}$ **(iii)** $-\frac{12}{5}$

3. **(a)(i)** $a = 3$ (rej $a = -3$), $k = 2$ **(a)(ii)** $x > \frac{1}{2}$

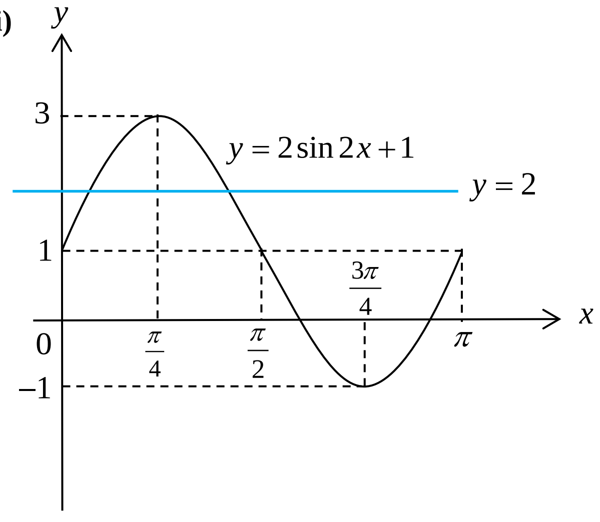
(b)



4. **(a)** $4x - 16$ **(c)** $f(x) = (x-1)(2x-3)^2$ **(d)** $\frac{x}{(x-1)(2x-3)^2} = \frac{1}{x-1} - \frac{2}{2x-3} + \frac{3}{(2x-3)^2}$

5. **(i)** period $= \pi$, amplitude $= 2$ **(ii)** $x = \frac{7\pi}{12}, \frac{11\pi}{12}$

(iii)



(iv) 2 solutions

6. **(i)** $x = \frac{100}{y^4}$ or $x = 10^{2-\lg y^4}$ or $x = 10^{2-4\lg y}$ **(ii)** $x = 2$

7. **(i)** $m = 2$ **(ii)** $f(x) = x^3 - 7x^2 + 4x + 12$ **(iii)** $y = -2,\ y = 1,\ y = 5$

8. **(a)(i)** $C(1, 3)$, radius $= 3\sqrt{2}$ units **(ii)** $P(4, 0)$ **(b)** $(x-2)^2 + (y-1)^2 = 3^2$