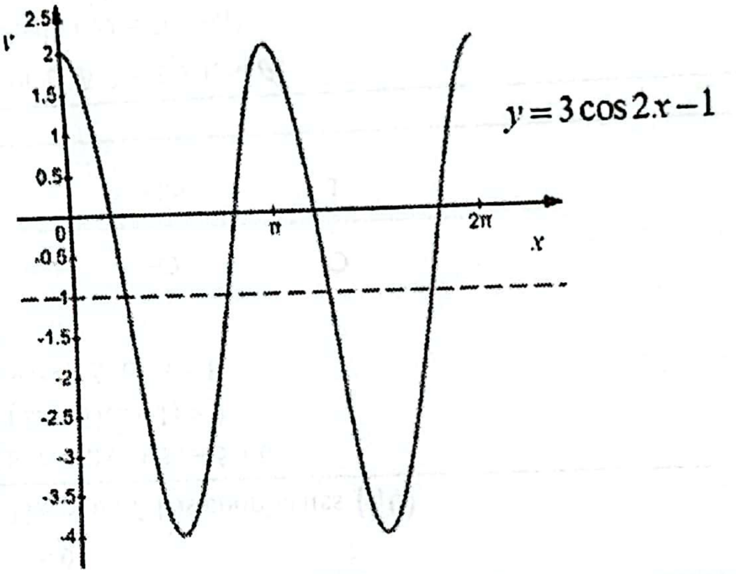
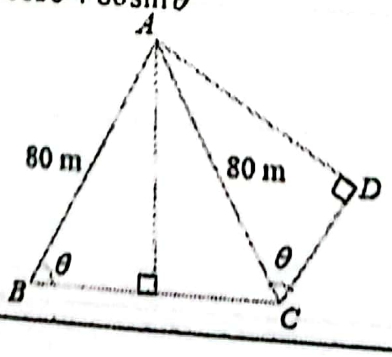
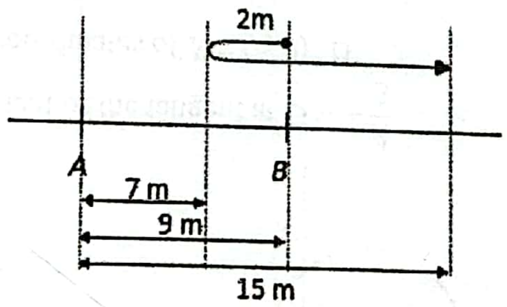
2022 PRELIM 4E5N AM P2 Marking Scheme

Q	Solution	Remarks
1a	Sub $x=40$, $y=-\frac{1}{10}(40)^2+120(40)-5000$ $y=-360$ Based on the model, it is estimated that the company will make a monthly loss of $360 if they price their product at $40.	B1: argument made based on substitution and correct y value.
1b	$-\frac{1}{10}x^2+120x-5000$ $=-\frac{1}{10}(x^2-1200x)-5000$ $=-\frac{1}{10}(x^2-1200x+600^2)-600^2\times\left(-\frac{1}{10}\right)-5000$ $=-\frac{1}{10}(x-600)^2+31000$	M1: factor coefficient of x^2 M1: Add and subtract $\left(\frac{b}{2}\right)^2$ term A1: correct vertex form
1c	The company should price their product at $600 to earn a monthly profit of $31 000.	B1: both answer required
2a	$\angle ABD=\angle BCD$ ($\angle$s in alt. segment) $\angle ADB=\angle BDC$ (DB is the angle bisector of angle ADC) By AA property, ΔABD is similar to ΔBCD.	M1: Alt seg theorem M1 A1
2b	$\angle ADB=\angle ABD$ (tangents from ext. pt.) $\therefore \angle BDC=\angle BCD$ (using (a)) So $BC=BD$.	M1 A1
3	Check that $f(3)=3^3-3^2-5(3)-3=0$, So $x-3$ is a factor of $f(x)$. Using long division or comparing coefficient method, $f(x)=(x+1)^2(x-3)$	M1: deduce one factor *must show working M1: long div or comparing coeff method A1
3a	Solving $(x+1)^2(x-3)=0$ $x=-1$ or $x=3$	M1: Apply zero pdt property A1: allow e.c.f
3b	Solving $(x+1)^2(x-3)=(x+1)(x-3)$ $(x+1)(x-3)(x+1-1)=0$ $x(x+1)(x-3)=0$ $x=0$ or $x=-1$ or $x=3$	B1 (all complete solutions)

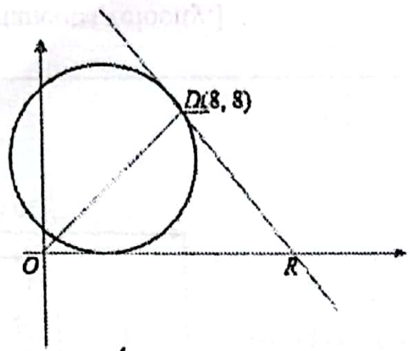
4a	$2\cos^2 x - 4\sin^2 x$ $= 1 + \cos 2x - 2(1 - \cos 2x)$ $= 3\cos 2x - 1$	M1: Use double angle for cosine A1
4b	Amplitude $= 3$ units Period $= \pi$ radians	B1 B1 (accept $180°$)
4c		G1: correct shape G1: correct principal axis G1: correct amp and no. of cycles
5a	Sub $y = 6$, $6 = \dfrac{6}{(x-2)^2}$ $(x-2)^2 = 1$ $x = 2 \pm 1$ $x = 1$ or 3 $\therefore A(1,6)$ and $B(3,6)$ Sub $x = 0$, $y = \dfrac{6}{(0-2)^2} = 1.5$. By symmetry, the $x-$coordinate of D is 4. $\therefore C(0,1.5)$ and $D(4,1.5)$	M1: Substitution A1: coord of A and B M1: Substitution A1: coord of C and D
5b	Required area $= 2\int_0^1 \dfrac{6}{(x-2)^2}\,dx + (6\times 2)$ $= 2\left[\dfrac{-6}{x-2}\right]_0^1 + (6\times 2)$ $= 2\times 3 + 12$ $= 18$ units^2	M1: form correct definite integral M1: deduce shaded part using area of rectangle M1: correct integration for $\dfrac{6}{(x-2)^2}$ A1

6a	Method 1: By comparing coefficients, (i) of x^3: $A=2$ (ii) of x^2: $4+2B+1=-1 \Rightarrow B=-3$ (iii) of constant: $-6+C=2 \Rightarrow C=8$ Method 2: By substitution, Sub $x=-2$, $-16-4+26+2=C$ $\therefore C=8$ Sub $x=0$, $2=2B+8$ $\therefore B=-3$ Sub $x=1$, $2-1-13+2=(A+1)(-2)(3)+8$ $\therefore A=2$ *Accept also a combination of substitution and comparing coefficient method.	M1 A1A1A1: for each correct coefficient
6b	Let $f(x)=3x^3-x+k$, $f(-1)=3$ $3=-3+1+k$ $k=5$	M1: apply remainder theorem A1
6c	Let $g(x)=x^3+(p+1)x^2-p^2$ and $h(x)=x^3+px^2-8x-16$ $g(2)=0$ $8+4p+4-p^2=0$ $p^2-4p-12=0$ $(p-6)(p+2)=0$ $p=-2$ or $p=6$ But $h(2)\neq 0$, I.e $8+4p-16-16\neq 0$ $p\neq 6$ $\therefore p=-2$	 M1: apply factor theorem A1: deduce 2 correct values of p A1: reject $p=6$ using given condition

7a	Let $y=2x^2+kx-5=0$, $k^2-4(2)(-5)$ $=k^2+40$, since $k^2\geq 0$ for all real values of k ≥ 40 >0 Since the discriminant >0 for all real values of k, the above equation has two real and distinct roots, i.e curve will intersect the x-axis at two real and distinct points.	M1: correct discriminant formula M1: use $k^2\geq 0$ A1
7bi	Sub $y=mx-m$ into $y=2x^2+3x-5$, $2x^2+3x-5=mx-m$ $2x^2+(3-m)x+(m-5)=0$ $(3-m)^2-4(2)(m-5)=0$ $9-6m+m^2-8m+40=0$ $m^2-14m+49=0$ $(m-7)^2=0$ $m=7$	M1: Substitution and make $ax^2+bx+c=0$ M1: Discriminant $=0$ A1
7bii	Sub $m=7$ into $2x^2+3x-5=mx-m$, Solve for x, $2x^2-4x+2=0$ $x^2-2x+1=0$ $(x-1)^2=0$ $x=1$ Sub $x=1$ into $y=7x-7$, $y=0$ The point P has coordinates $(1,0)$	M1: Substitution back into the eqn to solve for x A1
7c	Solve $2x^2+3x-5>0$ $(2x+5)(x-1)>0$ $x<-2.5$ or $x>1$ 	M1: factorization A1 A1
8a	Length $BC=2(80\cos\theta)$ Length $CD=80\cos\theta$ Length $AD=80\sin\theta$ $\therefore L=80+80+160\cos\theta+80\cos\theta+80\sin\theta$ $L=160+240\cos\theta+80\sin\theta$ 	M1: Use trigo ratio to find length BC or CD A1

8b	$R=80\sqrt{3^2+1^2}=80\sqrt{10}$ $\alpha=\tan^{-1}\left(\frac{1}{3}\right)=18.435°$ $L=160+80\sqrt{10}\cos(\theta-18.435°)$	M1 M1 A1
8c	Max $L=160+80\sqrt{10}$ or 413 m (3 s.f)	B1
8d	$160+80\sqrt{10}\cos(\theta-18.435°)=360$ $\cos(\theta-18.435°)=\frac{200}{80\sqrt{10}}$ $\theta-18.435°=37.761°$ $\theta=56.2°$	M1: apply arccosine to solve for compound angle A1
9a	When $t=0$, $AB=9$ m	B1
9b	$v=\frac{ds}{dt}=4t-4$ When $v=0 \Rightarrow \quad t=1$ s When $t=1 \Rightarrow \quad s=7$ m	M1 M1 A1
9c	When $t=3 \Rightarrow \quad s=15$ m Total distance traveled $=(9-7)+(15-7)=10$ m 	M1: Compute dist from A at $t=3$ A1
9d	When $t=3$, $v=8$ [This is to find the instantaneous velocity.] Hence from $a=t-8 \Rightarrow \quad v=\int a\,dt=\frac{1}{2}t^2-8t+C$ When $t=3$, $v=8 \Rightarrow \quad C=\frac{55}{2}$ $\Rightarrow \quad v=\frac{1}{2}t^2-8t+\frac{55}{2}$ When $v=0$, then $t^2-16t+55=0$ $\Rightarrow \quad t=5$ or $t=11$ (NA)	M1: integration to get v M1: substitution to find C A1

10a	$80\pi = \frac{2}{3}\pi(2r)^3 + \pi r^2 h$ $80 = \frac{16}{3}r^3 + r^2 h$ $h = \frac{80}{r^2} - \frac{16r}{3}$	M1: Set up equation A1
10b	$A = 3\pi(2r)^2 + 2\pi rh$ $= 3\pi(2r)^2 + 2\pi r\left(\frac{80}{r^2} - \frac{16r}{3}\right)$ $= 12\pi r^2 + \frac{160\pi}{r} - \frac{32\pi r^2}{3}$ $= \frac{160\pi}{r} + \frac{4\pi r^2}{3}$	M1: S.A of figure A1
10c	When A is stationary, $\frac{dA}{dr} = 0$. $-\frac{160\pi}{r^2} + \frac{8\pi r}{3} = 0$ $\frac{8r}{3} = \frac{160}{r^2}$ $r^3 = 60$ $r = \sqrt[3]{60}$ or 3.91 (3sf)	M1: differentiated $\frac{dA}{dr}$ correctly M1: solving cubic eqn A1
10d	$\frac{d^2A}{dr^2} = \frac{320\pi}{r^3} + \frac{8\pi}{3}$ $\left.\frac{d^2A}{dr^2}\right\|_{r^3=60} = \frac{320\pi}{60} + \frac{8\pi}{3} = 8\pi > 0$ $r = 3.91$ gives a minimum value of A. A figure with the minimum total surface area will require the least amount of paint to paint and thus most cost-efficient.	M1: apply second derivative test A1: conclude $\frac{d^2A}{dr^2} > 0 \Rightarrow$ min A B1: Justification using above results
11a	Let $(a, a+1)$ be the coordinates of the centre P. $AP = PB$ $(a-1)^2 + a^2 = (9-a)^2 + (5-a-1)^2$ $2a^2 - 2a + 1 = 81 - 18a + a^2 + 16 - 8a + a^2$ $24a = 96$ $a = 4$. Coordinates of $P = (4, 5)$. Radius $= AP$ $= \sqrt{(4-1)^2 + (5-1)^2}$ $= 5$ units Equation of the circle is $(x-4)^2 + (y-5)^2 = 25$.	M1: Use $y = x + 1$ condition for centre of circle M1: Use radius of circle to derive $AP = PB$ M1: Solve eqn to get coord of centre P M1: use Pythagoras theorem to find radius of circle A1: correct radius A1: correct eqn form

11b	On the x-axis, $y = 0$. When $y = 0$, $(x-4)^2 + 25 = 25$ $(x-4)^2 = 0$ There is only one solution $x = 4$. As the circle only intersect the x-axis at one point (4, 0), the x-axis is a tangent to the circle.	M1: Sub $y = 0$ A1 (accept use of D = 0 to show one pt of intersection)
11c	gradient of PD $= \frac{8-5}{8-4}$ $= \frac{3}{4}$  Gradient of the tangent at $D = -\frac{4}{3}$ Let coordinates of $R = (r, 0)$ $\frac{0-8}{r-8} = -\frac{4}{3}$ $24 = 4r - 32$ $r = 14$ Therefore, $R(14, 0)$. Area of triangle $ODR = \frac{1}{2}(14)(8) = 56$ units^2	M1: find m_{PD}. M1: grad of tangent at D M1: set up eqn to solve for the coordinates of R A1