

XINMIN SECONDARY SCHOOL 新民中学 SEKOLAH MENENGAH XINMIN Preliminary Examination 2022

CANDIDATE NAME

CLASS

INDEX NUMBER

4048/02

25 August 2022

2 hours 30 minutes

# MATHEMATICS

Secondary 4 Express/5 Normal (Academic)

Setter: Ms Pang Hui Chin Vetter: Ms Low Yan Jin Moderator: Ms Yap Bee Leng

Candidates answer on the Question Paper

## **READ THESE INSTRUCTIONS FIRST**

Write your name, register number and class in the spaces at the top of this page. Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

### Answer all questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact,

give the answer to three significant figures. Give answers in degrees to one decimal place.

For  $\pi$ , use either your calculator value or 3.142, unless the question requires the answer in terms of  $\pi$ .

At the end of the paper, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question. The total number of marks for this paper is **100**.

Errors	Qn No.	Errors	Qn No.
Accuracy		Simplification	
Brackets		Units	
Geometry		Marks Awarded	
Presentation		Marks Penalised	

Parent's/Guardian's Signature:



Compound Interest

Total amount = 
$$P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = 
$$\pi rl$$

Surface area of a sphere =  $4\pi r^2$ 

Volume of a cone = 
$$\frac{1}{3}\pi r^2 h$$

Volume of a sphere = 
$$\frac{4}{3}\pi r^3$$

Area of triangle 
$$ABC = \frac{1}{2}ab\sin C$$

Arc length =  $r\theta$ , where  $\theta$  is in radians

Sector area = 
$$\frac{1}{2}r^2\theta$$
, where  $\theta$  is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

**Statistics** 

$$Mean = \frac{\sum fx}{\sum f}$$

Standard deviation = 
$$\sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

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1 (a) Express 
$$\frac{2x}{(2x-3)^2} - \frac{1}{2x-3}$$
 as a single fraction in its simplest form.

**(b)** It is given that 
$$V = \frac{\pi h(r^2 - p^2)}{3}$$
.

(i) Find the value of V when h = 4.5, r = 7 and p = -3.4.

Answer  $V = \dots$  [1]

(ii) Express r in terms of V, h and p.

(c) (i) Simplify 
$$\left(\frac{729}{x^6}\right)^{\frac{2}{3}}$$
.

(ii) Given that  $\frac{4x^3}{y^2} \times \frac{px^q}{y^r} = 20x^{10}$ , where *p*, *q* and *r* are constants, find the value of *p*, *q* and *r*.

Answer  $p = \dots$  $q = \dots$  $r = \dots$  [3]

- 6
- 2 The diagram shows a circle *PQRS*, with centre *O*. SOP is a straight line. When produced, the line SP and the tangent to the circle at Q meet at T. Angle  $PRQ = 35^{\circ}$ .



- (a) Giving reason(s) for each answer,
  - (i) show that angle  $SOQ = 110^{\circ}$ ,

[3]

(ii) calculate the angle *OTQ*.

[2]

- (b) Given that the radius of the circle is 6 cm and PR = 10.4 cm, find
  - (i) the length of OT,

*Answer* ...... cm [2]

(ii) the exact value of  $\cos \angle RPT$ .

- 3 (a) The point X is (12, -5) and the point Y is (-4, 7).
  - (i) Express  $\overrightarrow{XY}$  as a column vector.

(ii) The point Z is such that XZ : XY = 3 : 4 and Z is on the line XY. Find the coordinates of Z.



*Answer* ..... [1]

**(b)**  $\frac{\text{area of } \Delta AFG}{\text{area of } \Delta DFE}.$ 

4 The diagram shows a circle with centre *B* and two congruent semicircles with centres *A* and *C* inscribed in a large semicircle with centre *O*. The circle with centre *B* has a radius of (2r + 1) cm. The semicircles with centres *A* and *C* have a radius of (12 - 6r) cm.



(a) (i) Write down an expression, in terms of r, for the length of BC.

*Answer* ..... cm [1]

(ii) Show that the length of *BO* is 
$$(23-14r)$$
 cm. [1]

(b) Hence, form an equation in r and show that it reduces to  $6r^2 - 19r + 14 = 0$ . [3] Answer (c) Solve the equation  $6r^2 - 19r + 14 = 0$ .

Answer  $r = \dots$ [2]

(d) Hence, find the length of arc *XY*.

A karaoke lounge offers three different room types for singing, A, B and C.
It charges different hourly rates on a weekday and a weekend for each room type.
The matrix H shows the hourly rates of each room type during non-peak hours.

$$\mathbf{H} = \begin{pmatrix} 25 & 40 & 65 \\ 30 & 50 & 80 \end{pmatrix}$$
 weekday  
weekend

(a) (i) The lounge has 7 type A rooms, 3 type B rooms and 2 type C rooms.Represent the number of rooms in a 3×1 column matrix N.

Answer  $\mathbf{N} = \dots$  [1]

(ii) Evaluate  $\mathbf{M} = \mathbf{H}\mathbf{N}$ .

(iii) State what the elements in (ii) represent.

(b) During peak hours, the hourly rates of room types A, B and C are 140%, 150% and 120% of the respective rates during non-peak hours.
Represent the percentages in a 3×3 matrix C and hence evaluate P = HC.

Answer  $\mathbf{P} = \dots$  [2]

(c) The karaoke lounge is open from 12 pm to 3 am daily such that 8 hours each day are considered non-peak while the rest are peak hours. If the lounge is fully booked for 1 week, how much would they collect in 1 week?

*Answer* \$ ..... [4]

6 The speed of 600 vehicles travelling on an expressway in January are recorded. The cumulative frequency curve below shows the distribution of the speeds.





(a) (i) Complete the frequency table for the speed of the vehicles. [1]

Speed (v km/h)	$40 \le v < 60$	$60 \le v < 80$	$80 \le v < 100$	$100 \le v < 120$
Frequency	40			130

(ii) Calculate an estimate of the mean speed of the vehicles.

(b) Use the curve to estimate

(i) the median speed,

*Answer* ...... km/h [1]

(ii) the interquartile range of the speeds.

*Answer* ...... km/h [2]

(c) The speed limit on the road is 90 km/h.Estimate the percentage of cars that exceeded the speed limit.

Answer ...... % [2]

(d) The speeds of 600 cars travelling on the same expressway in March are recorded.

The box-and-whisker plot shows the distribution of the speeds.





Make two comments comparing the speeds of the vehicles in January and in March.

- At a clothing sale, a shop sold mystery bags containing clothing of a variety of designs at a discounted price. The sizes of the clothes in each bag are the same. There are three different sizes: small (S), medium (M) and large (L), where there are 7 size S bags, 6 size M bags and 5 size L bags.
  - (a) Each customer receives a chance to purchase at most 2 randomly selected mystery bags, without replacement. They are not entitled to choose their preferred size.

Complete the tree diagram below to show this information for the first customer.

[2]

Answer



- (b) Irene won a lucky draw that gave her a chance to be the first to purchase the mystery bags. She decided on purchasing 2 bags.Find, in its simplest form, the probability that
  - (i) both of the bags are of the same size,

(ii) both of the bags are of different sizes

Answer ..... [1]

(iii) at least one of the bags is of size M.

(c) Vicky is the second customer to make her purchase and she buys only 1 bag. Given that Irene received 1 size S and 1 size L bag, Vicky thinks that the probability of getting a size S bag is as high as getting a size M bag. Is Vicky correct? Justify your answer using probability.

8	<b>(a)</b>	Com	plete the ta	ble of valu	tes for $y =$	$=x+\frac{24}{x}-$	9.			
Give your answer correct to 1 decimal place. [1]										
	x	2	2.3	2.6	3	4	5	6	7	8
	у	5		2.8	2	1	0.8	1	1.4	2
,	(b)	On th	ne grid, dra	w the grap	wh of $y = x$	$x + \frac{24}{x} - 9$	for $2 \le x$	≤8.		[2]
Î.										

(c) By drawing a tangent, find the gradient of the curve at (3,2).

$(\mathbf{u})$ $(\mathbf{l})$ On the graph $(\mathbf{b})$ , and the nucleum $\mathbf{L}y = \mathbf{x} + 0$ for $\mathbf{L} \ge \mathbf{x} \ge 0$ .	[2]	(d) (i) On the grid in (b), draw the line $2y = -x+8$ for $2 \le x \le 8$ .
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(ii) Write down the *x*-coordinates of the points where this line intersects the curve.

*Answer* x = ..... and ..... [2]

(iii) These values of x are the solutions of the equation  $3x^2 + Ax + B = 0$ . Find the value of A and the value of B.

*Answer A* = .....







Answer ..... m [2]

(ii) angle *ABD*,

*Answer* ......° [3]

(iii) area of the ground *ABCD*.

*Answer* ...... m<sup>2</sup> [3]

(b) Mr Hafiz is driving a truck along a road. His line of sight is limited by the bottom edge of the windshield such that he is unable to see anything that is within the shaded area as shown in the diagram. This area is also known as the blind spot zone.

This area is also known as the blind spot zone.



(i) Given that the distance from the bottom edge of the windshield to the ground level, *BC*, is 200 cm and the width of his blind spot zone, *AB*, is up to 190 cm from the front of the truck, find Mr Hafiz's angle of depression before his view is blocked.

*Answer* .....° [2]

(ii) A student of height 120 cm stands at 100 cm from the front of the truck.Will the student be visible to Mr Hafiz?You must show your calculations.

Answer	 	 	 •••••	 •	

The map below (drawn to scale) shows one of the routes, route *A*, that her taxi could take to MacRitchie Reservoir Park from her home.



(a) Estimate the distance travelled via route A. Leave your answer in kilometres.

(b) Ms Koh travelled via route *B*, which did not have any ERP gantry, on one occasion which took 9 minutes for her to reach MacRitchie Reservoir Park.

The taxi first travelled at an average speed of 50 km/h before increasing to an average speed of 58 km/h for the remaining journey. The remaining journey is 2 km shorter than the first part.

Calculate the total distance travelled on route *B*, leaving your answer in kilometres.

*Answer* ...... km [3]

	Type of Vehicle						
Time	Cars/Light Motorcycles Heavy Goods/						
	Goods/Taxis		Small Buses				
07:00-08:00	\$1.00	\$0.50	\$1.50				
08:00 - 08:30	\$2.00	\$1.00	\$3.00				
08:30 - 08:35	\$2.50	\$1.25	\$3.75				
08:35 - 09:25	\$3.00	\$1.50	\$4.50				

#### Taxi Fare

<b>Booking Fee</b>	Peak hours	\$3.30		
	6 am to 9.29 am on weekdays and			
	6 pm to 11.59 pm daily			
	\$2.30			
Base fare		\$4.10		
Distance fare		\$0.50 per km		
Per minute fare		\$0.15 per minute		
Surcharge	Peak hours	+25% on top of		
	6 pm to 11.59 pm daily	metered fare*		
	Midnight to 5.59 am daily	+50% on top of		
		metered fare*		

*Note*: Prevailing ERP charges are payable on top of the total fare \*Metered fare does not include ERP charges and booking fee

(c) Ms Koh would book a taxi at 7.30 am on Tuesdays to MacRitchie Reservoir Park.

The taxi driver would always ask if she would like to travel via Route A or Route B. The average speed when traveling on route A is 55 km/h.

Information on ERP rates and taxi fare are on page 24.

Suggest a suitable route that Ms Koh should take. Justify the decision you make and show your calculations clearly.

[6]

#### **END OF PAPER**

Answer Key

1a	3	5ai	(7)
	$\frac{1}{(2-2)^2}$		
	(2x-3)		$\mathbf{N} = \begin{vmatrix} 3 \end{vmatrix}$
			(2)
1bi	176 (3sf)	5aii	(425)
			520
1bii		5aiii	The elements represent the hourly rate of
1011	$r = \pm \sqrt{\frac{3V}{2} + p^2}$	Sam	booking all rooms on a weekday and a
	$\sqrt{\pi h}$	_	weekend respectively during non-peak
1ci	81		hours.
	$x^4$		
1cii	p = 5, q = 7, r = -2	5b	$(1.4 \ 0 \ 0)$
2ai	110°		$\mathbf{C} = \begin{bmatrix} 0 & 15 & 0 \end{bmatrix}$
2aii	20°		
2bi	17.5 cm		$\begin{pmatrix} 0 & 0 & 1.2 \end{pmatrix}$
2bii	13		<b>b</b> $(35 \ 60 \ 78)$
	$-\frac{1}{15}$		$\mathbf{P} = \begin{pmatrix} 42 & 75 & 96 \end{pmatrix}$
3ai	(-16)	5c	\$55.609
cui		6ai	130: 300
		Jui	100,000
3aii	(0, 1)	6aii	$87\frac{1}{2}$
3bi	3a-3b		3
3bii	$\angle GAF = \angle DCF$ (alt. $\angle s$ , // lines)	6bi	88 km/h
	$\angle AGF = \angle CDF$ (alt. $\angle s$ , // lines)	6bii	20 km/h
	AFG = / CFD (vert onn /s)	6c	45%
	$2 \text{ in } \mathbf{G} = 2 \text{ or } \mathbf{D}$ (vert. opp. 2.3)	6 <b>d</b>	• The cars generally <u>travelled slower in</u>
	$\therefore \Delta AGF$ and $\Delta CDF$ are similar		March than in January as the median
	(AA, similarity)	-	speed in March is lesser than in
3biiia	1		January.
	2		• The speeds of the cars in January were
3biiib	1		then in March as the interquertile
	$\overline{3}$		range in January is smaller than in
4ai	13-4 <i>r</i>		March
4aii	23–14 <i>r</i>	7a	From top to bottom
<b>4</b> b	$(13-4r)^2 - (12-6r)^2 = (23-14r)^2$		Einterlaution 6 5
	$(10^{-11})^{-1}(12^{-01})^{-1}(120^{-11})^{-$		First selection: $\frac{1}{18}, \frac{1}{18}$
	$169 - 104r + 16r^{2} - (144 - 144r + 36r^{2})$		Second selection:
	$=529-644r+196r^{2}$		<u>6 6 5 7 5 5 7 6 4</u>
	$216r^2 - 684r + 504 = 0$		17'17'17'17'17'17'17'17'17
	$36(6r^2-19r+14)=0$	7bi	46
	$6r^2 - 19r + 14 = 0$ (shown)		153
<b>4</b> c	$2 \dots 1^{1}$	7bii	107
	2 (rej.)  or  1-6		153
4d	31.4 cm	7biii	29
			$\left \frac{-2}{51}\right $

	2	7	
7c	Yes, Vicky is correct as the probability of		
	getting a size S bag would be $\frac{6}{-}$ and that		
	16 16		
	of a size M bag is $\frac{6}{-1}$ too.		
	16		
<b>8</b> a	3.7		
8b	<b>B1</b> – plotted points		
	<b>B1</b> – smooth curve		
8c	M1 – drawn tangent at (3, 2)		
	A1 – accept gradient from $-1.85$ to $-1.50$		
8di	<b>B1</b> – correct gradient		
	<b>B1</b> – correct straight line		
8dii	<b>B1</b> – accept 2.55, 2.6, 2.65, 2.7, 2.75		
	B1 – accept 6 only		
8diii	A = -26		
	B = 48		
9ai	85.3 m		
9aii	31.9°		
9aiii	2120 m <sup>2</sup>		
9bi	46.5°		
9bii	<multiple are="" available="" of="" solving="" ways=""></multiple>		
	Since <i><reason></reason></i> , the student will be visible		
	to Mr Hafiz.		
10a	Accept 6.5 km to 7 km		
10b	$7.91 \text{ km} (\text{or } 7\frac{49}{-} \text{ km})$		
	$7.91 \text{ km} (01.7 \frac{1}{54} \text{ km})$		
10c	M1 – finding time for route A		
	M1 – finding additional charges of either		
	route A or B		
	M1 – finding distance charge of either		
	route A or B		
	M1 – finding duration charge of either		
	route A or B		
	A1 – accurate cost calculation for either		
	route A or B		
	A1 – Suitable conclusion with justification		
	+ accurate cost for both routes A and B		