Section B: Statistics [60 marks]

6 A bus runs at regular intervals. The delay time, X, is the number of minutes that the bus arrived after its scheduled arrival time at the interchange. It is given that X follows a normal distribution, and that a bus arrives earlier than the scheduled arrival time at the interchange with a probability of 0.06 and at least 15 minutes late with a probability of 0.029.

(i) Find the mean and standard deviation of X. [4]

(ii) Sketch this distribution for delay times between –8 and 8 minutes. [2]

7 A and B are events such that $\mathrm{P}(A) = \frac{1}{5}$, $\mathrm{P}(A \cup B) = \frac{9}{20}$, $\mathrm{P}(A \mid B) = \frac{1}{6}$ and $\mathrm{P}(A \cap B) = k$, where $0 < k < 1$.

(i) Find the value of k. [3]

(ii) Find $\mathrm{P}(A' \cap B)$. [2]

(iii) Explain whether events A' and B are independent. [1]

8 A factory makes lamps. The probability that a lamp is defective is 0.05. A random sample of 30 lamps is tested.

(i) Find the expected number of defective lamps in the sample. [1]

(ii) Find the probability that there is at least one defective lamp in the sample. [2]

(iii) Given that there is at least one defective lamp in the sample, find the probability that there are at most two defective lamps in the sample. [3]

9 Nine prosecutors were asked the number of years, x (years), of experience each of them had as a prosecutor, and the percentage, y (%), of their cases that ended in guilty pleas. The data obtained was recorded below:

x	10	12	8	1	3	2	7	15	5
y	93	90	82	72	76	70	84	95	83

(i) Draw a scatter diagram for the above data. [2]

(ii) Find the value of r, the product moment correlation coefficient, and comment on its value in the context of the question. [2]

(iii) Find the equation of the regression line of y on x, giving your answer in the form $y = mx + c$, with the values of m and c correct to 3 significant figures. Interpret the value of m in this context. [2]

(iv) Use the equation of your regression line to calculate an estimate for the percentage of cases ending in guilty pleas for a prosecutor with 20 years of experience. Comment on the reliability of your estimate. [3]

10 In this question you should state the parameters of any distributions that you use.

A fast-food chain sells fries in both small and large packets. The mass of fries (in grams) in a small packet follows the normal distribution $\mathrm{N}(80, 2^2)$ and the mass of fries (in grams) in a large packet follows the normal distribution $\mathrm{N}(150, 3^2)$. The distributions are independent.

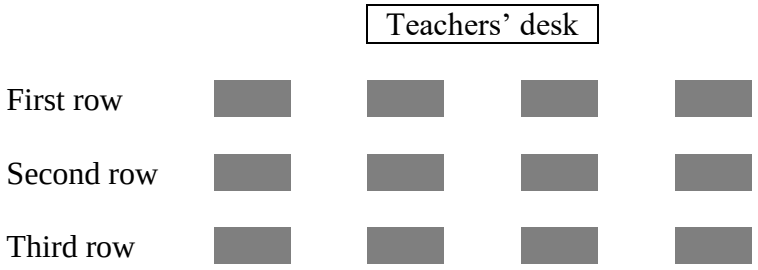
(i) Two small packets of fries are randomly chosen. Find the probability that the mass of fries in each packet is less than 77 grams. [2]

(ii) Find the probability that the total mass of fries in 4 randomly chosen small packets of fries is more than the total mass of fries in 2 randomly chosen large packets of fries by at least 15 grams. [4]

86% of the mass of fries is potato while the remaining 14% of the mass is vegetable oil.

(iii) There is a probability of more than 0.7 that the mass of potato used to make the fries in a randomly chosen large packet is less than twice the mass of potato used to make the fries in a randomly chosen small packet by more than k grams. Find the largest possible integer value of k. [4]

11 A class has 12 pupils, of whom seven are boys and five are girls. Among them, two of the boys and one of the girls play sports. The seating arrangement in the classroom is shown in the diagram below.



(i) In how many ways can the pupils be seated in the classroom so that the boys do not occupy the seats in the first row? [2]

The pupils sit at random in the classroom on the first day at school. Find the probability that

(ii) the three pupils who play sports are seated at the corners of the classroom, [2]

(iii) the three pupils who play sports are seated next to one another in the third row. [2]

Five of the pupils in the class are selected at random to form a team to represent the class at the inter-class games. Find the probability that

(iv) the team consists of at least two pupils who play sports, [2]

(v) the team consists of all boys, or at least two pupils who play sports, or both. [3]

[Turn over

12 Children and adolescents with ages ranging from 5 to 17 years of age are recommended to engage in at least 60 minutes of moderate-to-vigorous intensity physical activity per day.

An online survey is conducted on a random sample of 60 primary school children to investigate whether the mean amount of time that primary school children spend per day on moderate-to-vigorous intensity physical activities meets the minimum recommended time. The time spent per day on moderate-to-vigorous intensity physical activities, x (in minutes), of the 60 children are summarised by

$$\sum x = 3290, \qquad \sum x^2 = 201100.$$

(i) Find unbiased estimates of the population mean and variance. [3]

(ii) Explain whether a 1-tail test or 2-tail test should be carried out. [1]

(iii) Carry out the test at the 5% level of significance and give your conclusion in context. [4]

An expert thinks that the mean amount of time spent per day on moderate-to-vigorous intensity physical activities for secondary school students is not 60 minutes. He conducts a survey on a random sample of 60 secondary school students. The mean for this sample is k minutes and the population standard deviation is 15 minutes. A test at the 5% level of significance supports his claim.

(iv) Find the range of possible values of k. [4]

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