2014 H1 A Level Mathematics Solution

Q1.

$$\int_1^6 \frac{1}{\sqrt{(1+4x)}}\,dx$$

$$= \int_1^6 (1+4x)^{-\frac{1}{2}}\,dx$$

$$= \left[\frac{(1+4x)^{\frac{1}{2}}}{2}\right]_1^6$$

$$= \frac{1}{2}\left[5-\sqrt{5}\right]$$

Q2.

(i) $\frac{\mathrm{d}}{\mathrm{d}x}\ln\left(x^2+4\right) = \frac{2x}{x^2+4}$

(ii) Gradient of C = K

$$\Rightarrow \frac{2x}{x^2+4} = k$$

$$\Rightarrow 2x = k(x^2+4)$$

$$\Rightarrow kx^2 - 2x + 4k = 0 \text{ (shown)}$$

$$kx^2 - 2x + 4k = 0$$

(iii) For equal roots,

$$b^2 - 4ac = 0$$

$$4 - 4k\left(4k\right) = 0$$

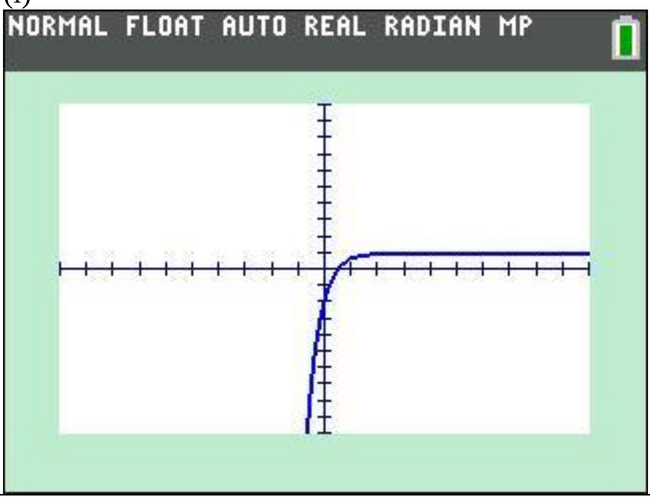
$$16k^2 = 4$$

$$k^2 = \frac{1}{4}$$

$$k = \frac{1}{2} \text{ or } k = -\frac{1}{2}$$

Q3.

(i)



NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl

X	Y₁
-2	-147.4
-1.5	-53.6
-1	-19.09
-.5	-6.389
0	-1.718
.5	0
1	.63212
1.5	.86466
2	.95021
2.5	.98168
3	.99326

X=3

At x axis intercept	At y axis intercept
$1-e^{1-2x}=0$ $e^{1-2x}=1$ $1-2x=0$ $x=\frac{1}{2}$	$y=1-e^{1-2(0)}$ $y=1-e$

The coordinates of points of intersection with the axes are (0 , 1- e) and ($\frac{1}{2}$, 0).

Asymptote : $y=1$

(ii) When x = 1, y = $1-\frac{1}{e}$

$$\frac{\mathrm{d}y}{\mathrm{d}x}=2e^{1-2x}$$

At x=1

$$\frac{\mathrm{d}y}{\mathrm{d}x}=2e^{-1}=\frac{2}{e}$$

Equation of Tangent to C at x = 1

$$y-\left(1-\frac{1}{e}\right)=\frac{2}{e}\left[x-(1)\right]$$

$$y=\frac{2}{e}x+1-\frac{3}{e}$$

$$m=\frac{2}{e}\quad c=1-\frac{3}{e}$$

Q4. (i) By Pythagoras's Theorem,

$$(y-x)^2+(2x)^2=\left(2\sqrt{65}\right)^2$$

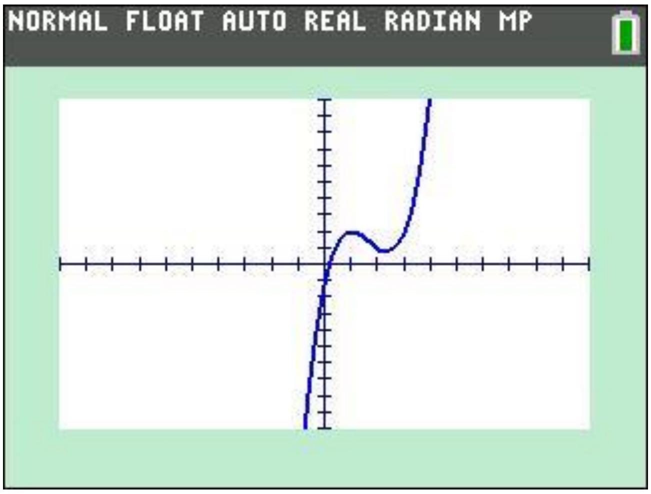
$$y^2-2xy+x^2+4x^2=260$$

$$5x^2+y^2-2xy=260 \text{ (shown)}$$

(ii) Since the perimeter of the rectangle $ABCD$ is 60 cm,

	$2y+6x=60$ $y+3x=30$ $y=30-3x$ Using $5x^2+y^2-2xy=260$ $5x^2+(30-3x)^2-2x(30-3x)=260$ $x=4,\ y=18$ $x=8, y=6$
Q5.	(i) $y=x^3+kx^2+7x+c$ $\frac{\mathrm{d}y}{\mathrm{d}x}=3x^2+2kx+7$ For stationary point, $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ $3x^2+2kx+7=0$ For $x=1$, $3+2k+7=0$ $k=-5$ (shown) Given that A has coordinates (1 , 2), $y=x^3-5x^2+7x+c$ $2=1-5+7+c$ $c=-1$ $y=x^3-5x^2+7x-1$ (ii) $\frac{\mathrm{d}y}{\mathrm{d}x}=3x^2-10x+7=0$ $x=1$ or $x=2\frac{1}{3}$ For $x=2\frac{1}{3}$, $y=\left(2\frac{1}{3}\right)^3-5\left(2\frac{1}{3}\right)^2+7\left(2\frac{1}{3}\right)-1$ $=\frac{22}{27}$ Co-ordinates of $B\left(2\frac{1}{3},\frac{22}{27}\right)$

(iii)



Using GC, co-ordinates of point that crosses the x-axis (0.161 , 0)

(iv)

Exact Area of Region =

$$\int_1^2 x^3 - 5x^2 + 7x - 1 \, dx$$

$$= \left[\frac{x^4}{4} - \frac{5x^3}{3} + \frac{7x^2}{2} - x \right]_1^2$$

$$= \left(4 - \frac{10}{3} + 14 - 2 \right) - \left(\frac{1}{4} - \frac{5}{3} + \frac{7}{2} - 1 \right)$$

$$= 1\frac{7}{12} \text{ units}^2$$

Q6. Let X be the random variable of the heights of the girls in the school.

$$X \sim N\left(142.2, 6^2\right)$$

(i) $\mathrm{P}\left(X < 146\right) = 0.73674 = 0.737$ (3 s.f)

(ii) $\mathrm{P}\left(142.2 - 5 < X < 142.2 + 5\right)$

$= \mathrm{P}\left(137.2 < X < 147.2\right)$

$= 0.595$(3 s.f)

Q7. **[This part is not in syllabus]**

(i) Systematic Sample

To draw a sample of 100 households from a population of 5000 households, we first assign a consecutive number for each household starting from 1.

$k = 50$ ($\approx \frac{5000}{100}$).

Taking the 1st number randomly from 1 to 50, say 8, we select every 50th household i.e 8th, 58th, 108th until all 100 households are selected.

(ii)

To obtain a sample of 100 households, we draw random samples from the age-groups with sample size in the same proportion as the size of each age-group:

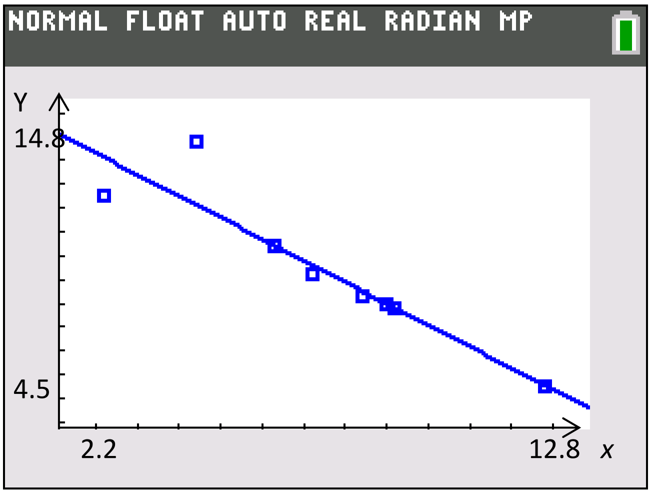
	Supermarket	Online
Under 25 years	10	20
25-60 years	18	32
Over 60 years	16	4

(iii)

Stratified Sampling will be preferred over systematic sampling due to the following reasons:

1. When there are clear strata present, this method can provide more accurate estimates
   than simple random sampling and is therefore more likely to give a good representative sample of the population.
2. Each of the strata can be treated separately, and so the sampling is convenient and more accurate.

8(i) **[This part is not in syllabus for 2020]**



(ii) $r = -0.92582$

The value of r shows a strong negative linear correlation between the number of hours spent travelling to and from work, and the number of hours spent watching television.

(iii) $y = -0.9021x + 16.15$

(iv) When $x = 13.2$, $y = 4.2423 \approx 4.24$

The estimate is unreliable as the value of 13.2 hours is out of the given data range of the number of hours of television watched by a person, i.e. the estimate is an extrapolation.

9(i) Let X be r.v "Number of cakes containing fruit, out of 6 cakes"

$$X \sim B(6, \frac{2.4}{6}) = B(6, 0.4)$$

(a) $P(X = 0) = 0.46656 \approx 0.467$

(b) $P(X \le 2) = 0.54432 \approx 0.544$

(ii)	Let Y be r.v "Number of packs containing at most two cakes containing fruit, out of 8 packs" $Y \sim B(8, 0.54432)$ $\text{P}(Y \geq 4) = 1 - \text{P}(Y \leq 3)$ $= 0.72867 \approx 0.729$
(iii)	Let A be r.v "Number of packs containing at most two cakes containing fruit, out of 150 packs" $A \sim B(150, 0.54432)$ Since n is large, and $np = 81.648 > 5$ $nq = 68.352 > 5$ $A \sim N(150 \times 0.54432, 150 \times 0.54432 \times 0.45568)$ $A \sim N(68.352, 31.147)$ $\text{P}(A > 75) \xrightarrow{c.c} \text{P}(A > 75.5)$ $= 0.10013$ ≈ 0.100
10(i) (ii) (iii)	Let X be r.v "Length of leaves from beech trees" $H_0 : \mu = 7$ $H_1 : \mu < 7$ Since n is large, $\overline{X} \sim N(7, \frac{4.4}{50})$ approximately by C.L.T. p-value $= 0.045946 < 0.05$ We reject H_0 and conclude that there is sufficient evidence to show that the population mean length of leaves from beech trees in this forest is less than 7cm. Unbiased estimate of the population mean $= \frac{310.4}{50} = 6.208 \approx 6.21$ Unbiased estimate of the population variance $= \frac{1}{49}\left(2209.2 - \frac{310.4^2}{50}\right) = 5.7599$ $H_0 : \mu = 7$ $H_1 : \mu \neq 7$ p-value $= 0.019623$ To reject H_0, p-value $< \alpha\%$ $0.019623 < \alpha\%$ $\alpha > 1.96$
11(i)	$\text{P}(L) = \frac{48+10+20+12}{48+10+20+12+130+15+55+x} = \frac{90}{290+x}$ $\text{P}(G) = \frac{10+55+20+15}{48+10+20+12+130+15+55+x} = \frac{100}{290+x}$ L and G are independent $\Rightarrow \text{P}(L \cap G) = \text{P}(L) \times \text{P}(G)$ $\frac{30}{290+x} = \frac{90}{290+x} \times \frac{100}{290+x}$ $\frac{30}{290+x} = \frac{9000}{(290+x)^2}$

	$1=\dfrac{300}{290+x}$ $x=10(shown)$
(ii)	$\text{P}(L\cup T)=\dfrac{48+10+20+12+15+130}{300}=\dfrac{47}{60}$
(iii)	$\text{P}(T\cap G')=\dfrac{12+130}{300}=\dfrac{71}{150}$
(iv) (v)	$\text{P}(L\mid G)=\text{P}(L)=\dfrac{90}{300}=\dfrac{3}{10}$ P(1 student owns exactly two items) $=\dfrac{10+15+12}{300}=\dfrac{37}{300}$ P(2 students owns exactly two items) $=\dfrac{37}{300}\times\dfrac{36}{299}=\dfrac{111}{7475}$
12(i) (ii) (iii)	$A\sim N(50,\sigma^2)$ $\text{P}(A>75)=0.0189$ $\text{P}(Z>\dfrac{75-50}{\sigma})=0.0189$ $\text{P}(Z\le\dfrac{25}{\sigma})=0.9811$ $\dfrac{25}{\sigma}=2.0770$ $\sigma=12.037$ $\sigma^2=144.877\approx145(shown)$ $B\sim N(75,64)$ $B_1+...+B_7\sim N(75\times7,64\times7)=N\left(525,448\right)$ $P(B_1+...+B_7<500)=0.11877\approx0.119$ Let $X=B_1+...+B_7-2\left(A_1+...+A_5\right)$ $E(X)=7E(B)-2[5E(A)]=25$ $Var(X)=7Var(B)+4[5Var(A)]=448+2900=3348$ $X\sim N(25,3348)$ $P(X>0)=0.66715\approx0.667$