NATIONAL JUNIOR COLLEGE
SENIOR HIGH 2 PRELIMINARY EXAMINATION
Higher 2

MATHEMATICS 9740/01

Paper 1

27 August 2015

3 hours

Additional Materials: Answer Paper
List of Formulae (MF15)
Cover Sheet

READ THESE INSTRUCTIONS FIRST

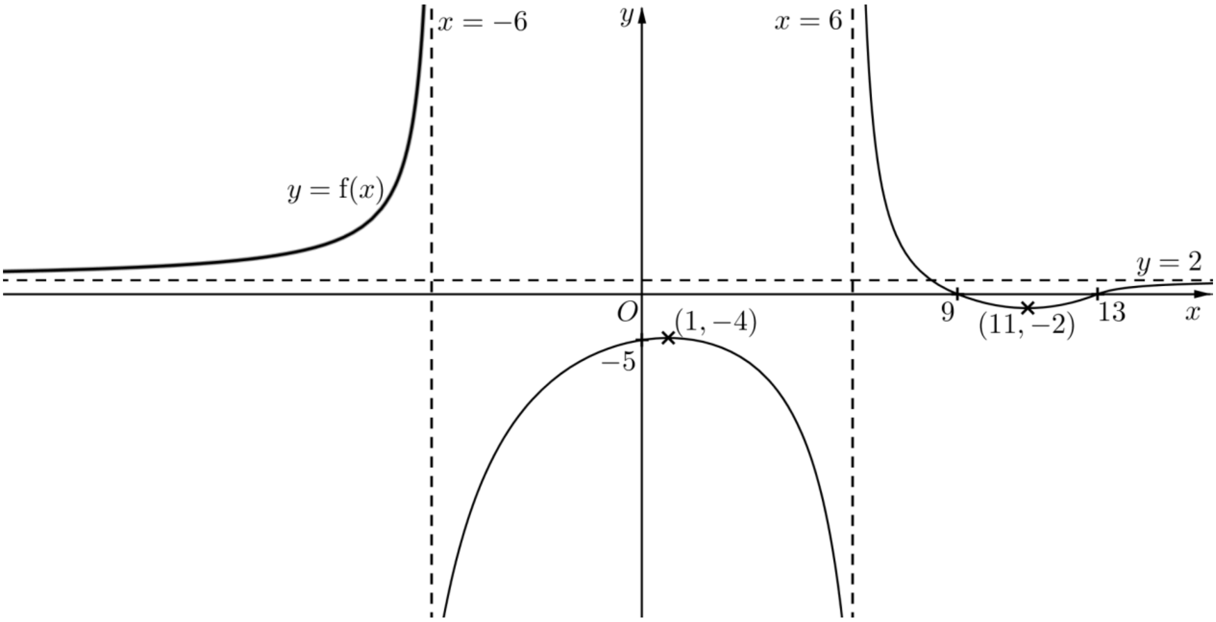
Write your name, registration number, subject tutorial group, on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in the brackets [ ] at the end of each question or part question.

1 Prove by the method of mathematical induction that $\displaystyle\sum_{n=0}^{N}\frac{3n+2}{(n+1)!3^{n+1}}=1-\frac{1}{(N+1)!3^{N+1}}$ for $N \geq 0$. [5]

2



On separate diagrams, sketch the following curves, labelling clearly any asymptotes, axial intercepts and turning points.

(i) $y=\dfrac{1}{\mathrm{f}(x)}$ [3]

(ii) $y=\mathrm{f}'(x)$ [3]

3 **(i)** Use the Maclaurin series for $\cos 2x$ to find the Maclaurin series for $\mathrm{f}(x)=\dfrac{\cos 2x}{1-x^2}$, up to and including the term in x^4. [4]

(ii) By substituting $x=\dfrac{1}{3}$ in your answer in part **(i)**, find an approximation for $\cos\left(\dfrac{2}{3}\right)$, giving your answer as an exact fraction in its simplest form. [2]

4 It is given that two vectors **a** and **b** satisfy the equation $\mathbf{a}\cdot\mathbf{b}=0$.

(i) What can be said about the vectors **a** and **b**? [2]

Assume that **a** is a non-zero vector.

(ii) Find, in terms of the vectors **a** and/or **b**, the projection vector of (**a** – **b**) onto **a**, simplifying your answer as far as possible. Show your working clearly. [3]

(iii) Show that $|\mathbf{a}\times\mathbf{b}|=|\mathbf{a}||\mathbf{b}|$. [2]

[Turn over

5 **(i)** Prove by the method of differences that $\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+3)} = \frac{1}{3} - \frac{n+1}{(2n+1)(2n+3)}$. [3]

(ii) Hence, find

(a) the exact value of $\sum_{r=5}^{\infty} \frac{1}{(2r-1)(2r+3)}$, [3]

(b) $\sum_{r=0}^{n} \frac{1}{(2r+1)(2r+5)}$ in terms of n. [2]

6 **(a)** On the same diagram, sketch the graphs of $y = 1 + \frac{1}{x+1}$ and $y = 2^x$, labelling the equations of the asymptote(s), if any. [2]

Hence, solve the inequality $1 + \frac{1}{x+1} < 2^x$. [2]

(b) Kandy, Landy and Mandy participated in a triathlon and their average speeds (in km/h) for each stage are summarised in the table below:

	Average speed (in km/h)		
	Kandy	Landy	Mandy
Swimming	3	3	2
Cycling	26	28	30
Running	12	9	7

The total time (in hours) that Kandy, Landy and Mandy took to complete the triathlon is 11.7, 12.4 and 13.9 respectively. Find the distances (in kilometres) for the swimming, cycling and running stage. [4]

7 It is given that the curve C has equation

$$y = \frac{(x-1)(x-2)}{x-3},\ x \in \mathbf{R},\ x \neq 3.$$

(a) Use an algebraic method to find exactly the set of values that y can take. [4]

(b) State an equation of the curve that is obtained after translating C in the positive y-direction by $\frac{2}{3}$ units. Hence or otherwise, find the volume of revolution when the region bounded by C and the line $y = -\frac{2}{3}$ is rotated completely about the line $y = -\frac{2}{3}$. Give your answer correct to 3 decimal places. [4]

8 The function f is defined by

$$\mathrm{f} : x \mapsto \cos^2 x, \text{ for } x \in \mathbf{R},\ 0 \le x \le k.$$

(i) If $k = \dfrac{2\pi}{3}$, find the range of f. [2]

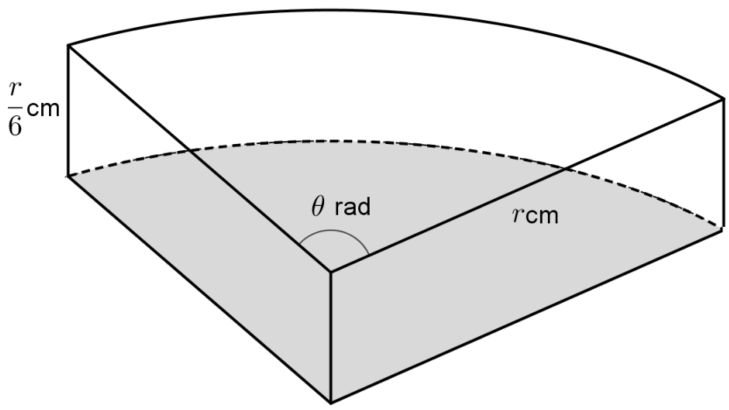
(ii) If it is given instead that f^{-1} exists, state the largest possible value of k exactly. [1]

Assume now that k has the value found in part **(ii).**

(iii) Sketch the graphs of $y = \mathrm{f}(x)$, $y = \mathrm{f}^{-1}(x)$, and $y = x$ on the same diagram, labelling clearly the coordinates of any points of intersection with the axes.
[You do not have to label the coordinates of any points of intersection between the graphs.] [3]

(iv) Without using a calculator, find the exact area bounded by the curve $y = \mathrm{f}^{-1}(x)$, the x-axis and the y-axis. [3]

9



The diagram shows a right container with the top face removed. Its base is a circular sector of radius r cm, with angle θ radians at the centre, where $0 < \theta < \pi$. The height of the container is $\dfrac{r}{6}$ cm. The thickness of its wall and base can be assumed to be negligible.

(i) It is given that the total *exterior* surface area of this container is a fixed value A cm^2, and the volume is a maximum. Use differentiation to show that $\theta = 1$. [6]

(ii) It is given instead that the volume of the container is 72 cm^3 and its total exterior surface area is 108 cm^2. Find the value of r and justify why this is the only possible value. [4]

[Turn over

10 The population (in thousands) of fish present in a lake at time t years is denoted by x. It is found that the growth rate of x is proportional to $(200 - 2t - x)$.

It is given that the initial population of the fish in the lake is 8000 and the population grows at a rate of 16000 per year initially. Show that the growth rate of x at time t years can be modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{200 - 2t - x}{12}. \qquad [2]$$

Find x in terms of t by using the substitution $u = 2t + x$. Deduce, to the nearest number of years, the time taken for the fish to die out in this lake. [6]

It is given that the solution curve that describes that population size of the fish at time t years intersects the graph of $x = 200 - 2t$ at the point (t_1, x_1). Describe, in context, what t_1 and x_1 represent. [2]

11 The curve C has equation $4y^2 + 8y - x^2 = 0$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [2]

(ii) Find the equation of the normal at the point $\left(-\frac{3}{2}, \frac{1}{4}\right)$ on C. [3]

(iii) Show that the equation for C can be expressed in the form $\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$, where a, b, h, and k are real constants to be determined. Sketch the curve C, labelling clearly the stationary point(s) and the equation(s) of the asymptote(s), if any. [5]

12 **Do not use a graphing calculator for this question.**

(a) Let $z_1 = \sqrt{2} - \left(\sqrt{2}\right)\mathrm{i}$ and $z_2 = 1 + \left(\sqrt{3}\right)\mathrm{i}$.

(i) Find $z_3 = -\frac{(z_2)^2}{z_1^*}$ exactly in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \le \pi$. [4]

(ii) On a single Argand diagram, sketch the points representing the complex numbers z_1, z_2 and z_3. Label each point and show clearly any geometrical relationships. [3]

(iii) Explain whether there is a fixed complex number k such that z_1, z_2 and z_3 are all roots of the equation $z^3 = k$. [2]

(b) Given that $w = \frac{1}{\mathrm{e}^{\mathrm{i}4\theta} - 1}$, show that $\mathrm{Re}(w) = -\frac{1}{2}$ and find $\mathrm{Im}(w)$ in terms of θ. [4]

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