Anglo - Chinese School
(Independent)

THE BEST IS YET TO BE

FINAL EXAMINATION 2016
YEAR THREE EXPRESS
ADDITIONAL MATHEMATICS
PAPER 2

Wednesday **12 October 2016** **1 hour 30 minutes**

Additional Materials: Answer Paper(7 Sheets)
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your index number on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 60.

This question paper consists of 4 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}ab\sin C$$

Answer all the questions.

1 Prove that $(\cot x - \cos ecx)^2 \equiv \dfrac{1-\cos x}{1+\cos x}$ [4]

2 Solve $3^{x+1} = 5 - 2(3^{-x})$. [4]

3 Given that the curve $x^2 - 4xy + 3 = 0$ cuts the line $3y = 2x + 1$ at A and B, find the coordinates of A and B. [5]

4 A circle, C_1, has equation $x^2 + y^2 - 4x + 10y - 20 = 0$

(i) Find the coordinates of the centre and the radius of C_1. [3]

(ii) If circle C_1 is reflected on the line $y = -1$, find the equation of the reflected circle. [2]

5 **(a)** Find the range of values of x which satisfy the inequalities $3 - 2x \le 1$ and $2x^2 - 5x < -2$. [4]

(b) Find the range of values of k for which the line $y = kx - 1$ does not intersect the curve $y^2 = 2x - 3$. [4]

6 It is given that $f(x) = \left|3 - \dfrac{x}{2}\right| - 1$.

(i) Solve $f(x) = 0$. [2]

(ii) Sketch the graph of $f(x)$, indicating the intercept(s) and vertex of the graph. [3]

(iii) State the range of values of m for which the line $y = mx$ cuts the graph of $f(x)$ at only one point and that $m > 0$. [1]

(iv) State the value of c if there are infinitely many solutions for $\dfrac{1}{2}x + c = \left|3 - \dfrac{x}{2}\right| - 1$. [2]

7 **Answer the whole of this question on a piece of graph paper.**

The variables x and y are known to be related by the equation $y = \ln(px^2 + q) - 1$, where p and q are constants. The following table shows some experimental pairs of values of x and y.

x	1	2	3	4
y	–0.09	0.72	1.36	1.86

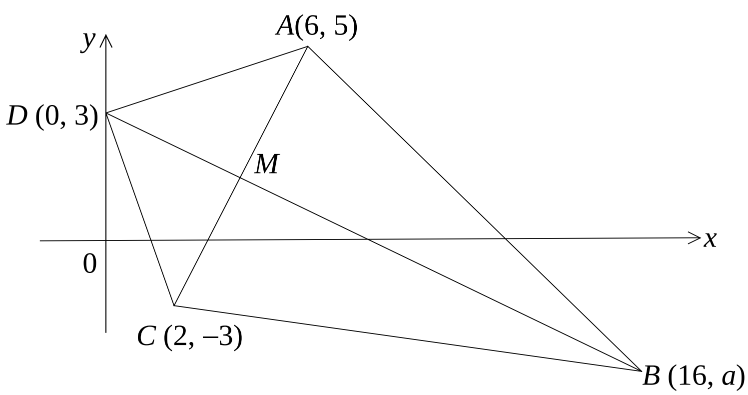
(i) By plotting e^{y+1} against x^2, draw a straight line graph and use the graph to estimate the value of p and of q, [6]

(ii) Find the possible values of x when $y = \ln 5$. [2]

8 **(a)** The roots of the equation $2x^2 + mx - 5 = 0$ are α and β. If $\alpha^2 + \beta^2 = \frac{101}{4}$, find the possible values of m. [4]

(b) The roots of an equation $x^2 + 12 = 8x$ are α and β. Obtain an equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$. [5]

9 **The solution to this question by accurate drawing will not be accepted.**



The diagram shows a kite $ABCD$ whose diagonals meet at M. The coordinates of A, B, C and D are (6, 5), (16, a), (2, –3) and (0, 3) respectively where a is a constant. Find the

(i) coordinates of M, [1]

(ii) equation of BD, [3]

(iii) value of a, [1]

(iv) area of the kite $ABCD$, [2]

(v) ratio of $DM : DB$ [2]

END OF PAPER 2

Solutions

1. LHS $= (\cot x - \operatorname{cosec} x)^2$

$$= \cot^2 x - 2\cot x \operatorname{cosec} x + \operatorname{cosec}^2 x$$
$$= \frac{\cos^2 x}{\sin^2 x} - \frac{2\cos x}{\sin^2 x} + \frac{1}{\sin^2 x}$$
$$= \frac{(1-\cos x)^2}{\sin^2 x}$$
$$= \frac{(1-\cos x)^2}{1-\cos^2 x}$$
$$= \frac{(1-\cos x)^2}{(1-\cos x)(1+\cos x)}$$
$$= \frac{1-\cos x}{1+\cos x} \text{ [Proved]}$$

2. $3^{x+1} = 5 - 2(3^{-x})$

$$3.3^{2x} = 5.3^x - 2$$
$$3.3^{2x} - 5.3^x + 2 = 0$$
$$(3.3^x - 2)(3^x - 1) = 0$$
$$3^x = \frac{2}{3} \quad \text{or} \quad 3^x = 1$$
$$x = -0.369 \qquad x = 0$$

3. $3y = 2x + 1$

$$y = \frac{2x+1}{3}$$
$$x^2 - 4x\left(\frac{2x+1}{3}\right) + 3 = 0$$
$$3x^2 - 8x^2 - 4x + 9 = 0$$
$$5x^2 + 4x - 9 = 0$$
$$(5x+9)(x-1) = 0$$
$$x = -\frac{9}{5} \quad \text{or} \quad x = 1$$
$$y = -\frac{13}{15} \qquad y = 1$$

4(a) $x^2 + y^2 - 4x + 10y - 20 = 0$

$$(x-2)^2 + (y+5)^2 = 49$$

Centre = $(2,-5)$
Radius = 7

(b) New centre = $(2,3)$

Equation: $(x-2)^2+(y-3)^2=49$

5(a) $3-2x\le 1$

$x\ge 1$

$2x^2-5x<-2$

$(2x-1)(x-2)<0$

$\frac{1}{2}<x<2$

Ans: $1\le x<2$

5(b) $(kx-1)^2=2x-3$

$k^2x^2-2kx+1=2x-3$

$k^2x^2+(-2k-2)x+4=0$

$D<0$

$(-2k-2)^2-4(4)(k^2)<0$

$3k^2-2k-1>0$

$(3k+1)(k-1)>0$

$k<-\frac{1}{3}$ or $k>1$

6(i) $\left|3-\frac{x}{2}\right|=1$

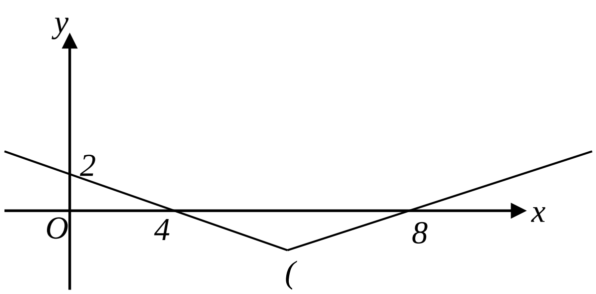
$3-\frac{x}{2}=1$

$x=4$

Or $3-\frac{x}{2}=-1$

$x=8$

(ii)



Min point (6, -1) B1
y-intercept B1
Shape B1

(ii) $m\ge\frac{1}{2}$

(iii) $c = -4$

8(a) $2x^2 + mx - 5 = 0$

$$\alpha + \beta = -\frac{m}{2}$$

$$\alpha\beta = -\frac{5}{2}$$

$$\alpha^2 + \beta^2 = \frac{m^2}{4} - 2(-\frac{5}{2}) = \frac{m^2}{4} + 5$$

$$\frac{m^2}{4} + 5 = \frac{101}{4}$$

$$m^2 = 81$$

$$m = \pm 9$$

(b) $\alpha + \beta = 8$

$$\alpha\beta = 12$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2 \beta^2}$$

$$= \frac{8^2 - 2(12)}{12^2}$$

$$= \frac{5}{18}$$

$$\frac{1}{\alpha^2} . \frac{1}{\beta^2} = \frac{1}{\alpha^2 \beta^2}$$

$$= \frac{1}{144}$$

$$x^2 - \frac{5}{18}x + \frac{1}{144} = 0$$

$$144x^2 - 40x + 1 = 0$$

9(i) $M = \left(\frac{2+6}{2}, \frac{5-3}{2}\right) = (4,1)$

(ii) gradient of AC = $\frac{5+3}{6-2} = 2$

gradient of BD $= -\frac{1}{2}$

Equation of BD: $y = -\frac{1}{2}x + 3$

(iii) when $x = 16$, $a = -0.5(16) + 3 = -5$

(iv) Area of kite $= 2 \times \frac{1}{2} \begin{vmatrix} 0 & 16 & 6 & 0 \\ 3 & -5 & 5 & 3 \end{vmatrix}$

$$= |80 + 18 - (48 - 30)|$$

$$= 80$$

(v) By similar figure,

$$\frac{3-1}{3-(-5)} = \frac{1}{4}$$