RAFFLES INSTITUTION
YEAR 5-6 PHYSICS DEPARTMENT

Tutorial 7 GRAVITATION

Self-Check Questions

S1 What is a gravitational field?

S2 What does Newton's law of gravitation states? What kind of objects can it be applied to?

S3 What is gravitational field strength? State an expression for the gravitational field strength.

S4 What is gravitational potential? State an expression for the gravitational potential.

S5 What can you say about the total energy of a satellite that is bound to planet?

S6 What is a geostationary satellite? Why is the geostationary orbit unique?

S7 State the expressions for the escape velocity of an object from the surface Earth and the orbital velocity of the same object around the Earth? What are the differences in the two expressions?

Self-Practice Questions

$F = \frac{G m_1 m_2}{r_2} = ma$ $\quad$ $kg\,m\,s^{-2} = \frac{G\,kg\,kg}{m^2}$ [illegible] $\quad$ $m^3\,kg^{-1}\,s^{-2}$

SP1 The SI base units of the gravitational constant G is

A $m\ s^{-2}$ **B** $N\ m^{-2}\ kg^{-2}$ **C** $m^3\ kg^{-1}\ s^{-2}$ **D** $m^2\ kg^{-2}$

SP2 Outside a uniform sphere of mass M, the gravitational field strength is the same as that of a point mass M at the centre of the sphere. The Earth may be taken to be a uniform sphere of radius r. The gravitational field strength at its surface is g.

What is the gravitational field strength at a height h above the ground?

A $\dfrac{gr^2}{(r+h)^2}$ **B** $\dfrac{gr}{(r+h)}$ **C** $\dfrac{g(r-h)}{r}$ **D** $\dfrac{g(r-h)^2}{r^2}$

SP3 A planet has a mass of 5.0×10^{24} kg and a radius of 6.1×10^{6} m. The energy needed to lift a mass of 2.0 kg from its surface into outer space is

A 1.8×10^{1} J **B** 5.5×10^{7} J **C** 1.1×10^{8} J **D** 2.2×10^{8} J

SP4 X and Y are two points at respective distances R and $2R$ from the centre of the Earth, where R is greater than the radius of the Earth. The gravitational potential at X is -800 kJ kg^{-1}.

When a 1 kg mass is taken from X to Y, the work done on the mass is

A -400 kJ **B** -200 kJ **C** $+200$ kJ **D** $+400$ kJ

SP5 An Earth satellite is moved from one stable circular orbit to another stable circular orbot at a greater distance from the Earth.

Which one of the following quantities increases for the satellite as a result of the change?

A gravitational force
B centripetal acceleration
C linear speed in the orbit
D angular velocity
E gravitational potential energy

SP6 A planet of mass P moves in a circular orbit of radius R round a sun of mass S with period T.

Which one of the following correctly shows how T depends on P, R, S?

A $T \propto R^{\frac{1}{2}}$ **B** $T \propto R^{\frac{3}{2}}$ **C** $T \propto S^{\frac{1}{2}}$ **D** $T \propto P$

SP7 A body of mass m is projected from the Earth's surface. At the point of launch, the acceleration of free fall is g and the radius of the Earth is R.

To escape from the gravitational field of the Earth, the speed of the body must be at least

A $\sqrt{gR}$ **B** mgR **C** $\sqrt{2gR}$ **D** $\dfrac{mg}{2R}$

SP8 Which quantity is not necessarily the same for satellites that are in geostationary orbits around the Earth?

A angular velocity
B centripetal acceleration
C kinetic energy
D orbital period

Converted from Discussion Questions (due to time constraint)

SP9 **[J90/II/2 - modified]**
A planet of mass m orbits the Sun of mass M in a circular path of radius r.

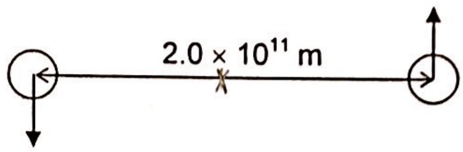
(a) Write down an expression, in terms of G, m, M and r, for the force F exerted by the Sun on the planet.

(b) Use this expression to find the angular velocity of the planet in the orbit.

(c) Deduce the expression for the time taken to complete one orbit of the Sun.

(d) The earth is 1.50×10^{11} m from the centre of the Sun and takes exactly one year to complete one orbit. The planet Jupiter takes 11.9 years to complete an orbit of the Sun. Calculate the radius of Jupiter's orbit.

SP10 The figure below shows a binary star system which consists of two stars, each of mass 4.0×10^{30} kg, separated by 2.0×10^{11} m. The stars rotate about the centre of mass of the system.



(a) Label on the above diagram, with a letter X, a point where the gravitational field strength is zero. Explain why you have chosen this point.

(b) Determine the gravitational potential at X.

(c) Calculate the force on each star due to the other star.

(d) Calculate the linear speed of each star in the system

Discussion Questions

D1 **[J96/III/2 - part]**

(a) The Earth may be considered to be a uniform sphere of radius 6370 km, spinning on its axis with a period of 24 hours. The gravitational field at the Earth's surface is identical to that of a point mass of 5.98 x 10^{24} kg at the Earth's centre. For a 1.00 kg mass situated at the Equator,

(i) calculate, using Newton's law of Gravitation, the gravitational force on the mass,
(ii) determine the force required to maintain the circular path of the mass,
(iii) deduce the reading on an accurate newton-meter (spring balance) supporting the mass.

(b) Using your answers to **(a)**, state what would be the acceleration of the mass at the Earth's surface due to

(i) the gravitational force alone,
(ii) the force as measured on the Newton-meter.

(c) A student, situated at the Equator, releases a ball from rest in a vacuum and measures its acceleration towards the Earth's surface. He then states that this acceleration is the 'acceleration due to gravity'. Comment on his statement.

D2 A communications satellite of mass 3000 kg is to be put into an equatorial orbit in which it has an angular velocity equal to that at which the Earth rotates, so that the satellite remains above the same point on the Earth's surface. Assume that the mass of the Earth is 5.98×10^{24} kg and its radius is 6.37×10^{6} m.

(a) What is the angular velocity of the Earth's rotation about its axis? Give your answer in rad s^{-1}.

(b) Find the altitude of the satellite's orbit.

(c) List the characteristics of a geostationary satellite.

D3 **(a)** A small satellite is in a stable circular orbit of radius 7000 km around a planet of mass 5.7×10^{24} kg and radius 6500 km. Calculate

(i) the orbital speed of the satellite,

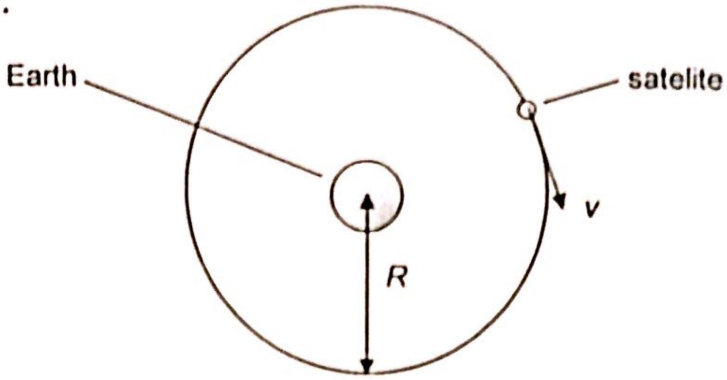
(ii) the escape velocity from the surface of the planet.

(b) By what factor would the escape velocity be reduced if the linear dimensions of the planet were 10^{3} smaller, its mean density remaining unchanged?

(c) In the light of your answer explain why many small planets do not have gaseous atmosphere.

D4 [N04/II/3]

A satellite orbits the Earth in a circular path, as illustrated below.



Both the Earth and the satellite may be considered to be point masses with their masses concentrated at their centres. The satellite has speed v and the radius of its orbit about the Earth is R.

(a) (i) Show that the speed v is given by the expression

$$v^2 = \frac{GM}{R},$$

where M is the mass of the Earth and G is the gravitational constant.

(ii) The mass of the satellite is m. Determine the expression for the kinetic energy E_k of the satellite in terms of G, M, m and R.

(b) (i) State an expression, in terms of G, M, m and R for the gravitational potential energy E_p of the satellite.

(ii) Hence, show that the total energy E_t of the satellite is given by

$$E_t = -\frac{GMm}{2R}.$$

(c) As the satellite orbits the Earth, it gradually loses energy because of air resistance.

(i) State whether the total energy E_t becomes more or less negative.

(ii) Hence, state and explain the effect of this change on

1. the radius of the orbit,
2. the speed of the satellite.

D5 **(a)** Calculate the Earth's gravitational field strength at a height of 0.12×10^6 m above the Earth's surface. Assume mass and radius of Earth are 5.98×10^{24} kg and 6.37×10^6 m, respectively.

(b) Explain briefly why an astronaut in a satellite orbiting the Earth at this altitude may be described as weightless.

(c) The value of the gravitational potential ϕ at a point in the Earth's field is given by the equation

$$\phi = -\frac{GM}{r},$$

where M is the mass of the Earth and r is the distance of the point from the centre of the Earth (r is greater than the radius of the Earth). Explain

(i) what is meant by the term gravitational potential,

(ii) why the potential has a negative value.

(d) Use the expression given in **(c)** to calculate the gain in the potential energy of a satellite of mass 3000 kg between its launch and when it is at a height of 0.12×10^6 m above the Earth's surface.

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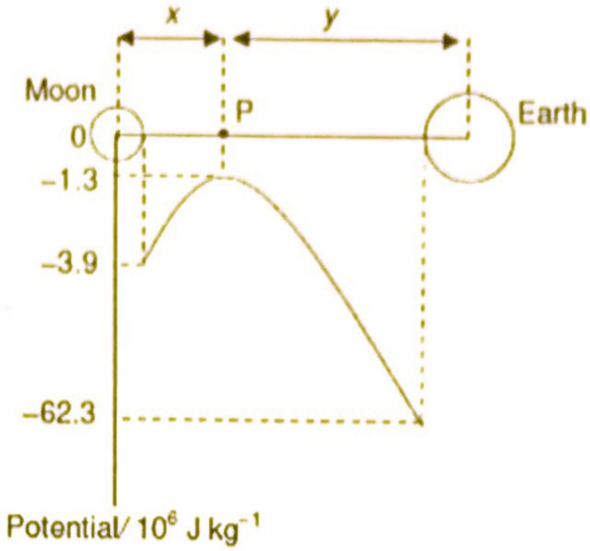
D6 [J87/II/8]

A point mass m is at a distance r from the centre of the Earth. **Write down an expression**, in terms of m, r, the Earth's mass m_E and the gravitational constant G, for the gravitational potential energy U of the mass (consider only the values of r greater than the Earth's radius).



Certain meteorites (tektites) found on the Earth have a composition identical with that of lunar granite. It is thought that they may be debris from a volcanic eruption on the Moon. The figure above, which is not drawn to scale, shows how the gravitational potential between the surface of the Moon and the surface of the Earth varies along the line of centres. At the point P, the gravitational potential is a maximum.

Given: mass of the Earth = 6.0×10^{24} kg
mass of the Moon = 7.4×10^{22} kg

(a) By considering the separate contributions of the Earth and the Moon to the gravitational potential, explain qualitatively why the graph has a maximum and why the curve is asymmetrical.

(b) State how the resultant gravitational force on the tektite at any point between the Moon and the Earth could be deduced from the graph.

(c) When a tektite is at P, the gravitational forces on it due to the Moon and the Earth are F_M and F_E respectively. State the relation which applies between F_M and F_E. Hence find the value of x/y, where x and y are the distances of P from the centre of the Moon and the centre of the Earth respectively.

(d) If a tektite is to reach the Earth, it must be projected from a volcano on the Moon with a certain minimum speed v_o. Making use of appropriate values form the graph, find this speed. Explain your reasoning.

(e) Discuss very briefly whether a tektite will reach the Earth's surface with a speed less than, equal to or greater than the speed of projection. (Neglect atmospheric resistance.)

increase in potential: 2.6 from P, ~~[illegible]~~ no WD required as grav force will act
$\downarrow KE = \uparrow PE$

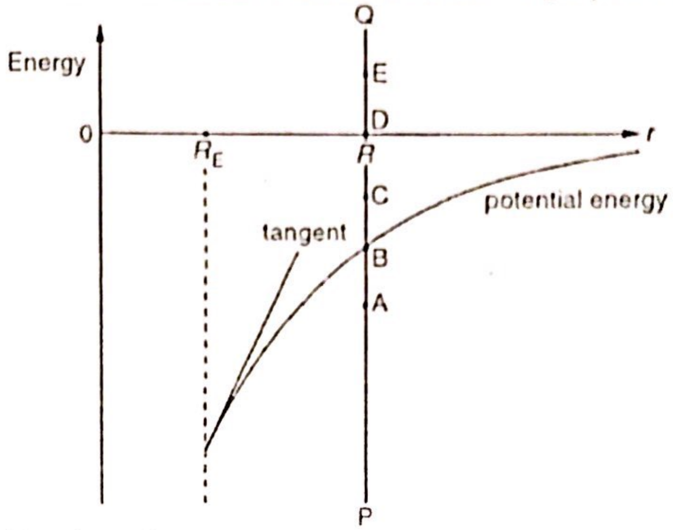
$\frac{1}{2}mv^2 = 2.6 \times 10^6 (m)$

$v = \sqrt{5.2 \times 10^6} = 2.3 \times 10^3$

D7 **[J85/II/9]**

The curve below shows the way in which the gravitational potential energy of a body of mass m in the field of the Earth depends on r, the distance from the centre of the Earth, for values of r greater than the Earth's radius R_E.

What does the gradient of the tangent to the curve at $r = R_E$ represent?



The body referred to above is a rocket which is projected vertically upwards from the Earth. At a certain distance R from the centre of the Earth, the total energy of the rocket (its gravitational potential energy plus kinetic energy) may be represented by a point on the line PQ. Five points A, B, C, D, E have been marked on this line.

Which point (or points) could represent the total energy of the rocket

(a) if it were momentarily at rest at the top of its trajectory at $r = R$,

(b) if it were falling towards the Earth at $r = R$,

(c) if it were moving away from Earth at $r = R$, with sufficient energy to reach an infinite distance?

In each case, explain briefly how you arrive at your answer.

Challenging Questions

C1 A particle of mass m is located inside a uniform solid sphere of radius R and mass M. If the sphere is at a distance r from the centre of the sphere, show that the gravitational potential energy of the system is

$$U = \frac{GMmr^2}{2R^3} - \frac{3GmM}{2R}.$$

How much work is done by the gravitational force in bringing the particle from the surface of the sphere to its centre?

[ANS: $\frac{GMm}{2R}$]

Tutorial 7 Gravitational Field Suggested Solutions

Self - Check Questions

S1 A gravitational field is a region of space in which a mass placed in that region experiences a gravitational force.

S2 Newton's law of gravitation states that two point masses attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

It can be applied to point masses or uniform spherical objects (where their separation is taken to be the distance between their centres of mass).

S3 The gravitational field strength at a point in space is defined as the gravitational force experienced per unit mass at that point.

$$g = \frac{F}{m} = \frac{GM}{r^2}$$

S4 The gravitational potential at a point in a gravitational field is defined as the work done per unit mass by an external force in bringing a small test mass from infinity to that point.

$$\varphi = \frac{U}{m} = -\frac{GM}{r}$$

S5 The total energy of a satellite bound to a planet is equal to the sum of its kinetic energy and gravitational potential energy. The total energy is negative (since a total energy ≥ 0 means that the object will be able to escape to infinity).

S6 A geostationary satellite is a satellite that orbits directly above the equator (sharing the same axis of rotation as the Earth) in the same direction as Earth's rotation. Its orbital period is the same as the period of the Earth's rotation (24 hrs). The orbit radius is unique since it must follow the same period as the Earth's rotation and it is also independent of the mass of the satellite.

S7

$$v_{escape} = \sqrt{\frac{2GM}{R}}, \qquad v_{orbital} = \sqrt{\frac{GM}{r}}$$

Escape velocity is derived from the principle of conservation of energy, whereas the orbital velocity is derived from Newton's second law of motion.

The radius in the escape velocity refers to the radius of the Earth, whereas the radius in the orbital velocity refers to the radius of the orbit.

There is a factor 2 in the escape velocity.

Self - Practice Questions

SP1 C

$$F = \frac{GMm}{r^2}$$

$$G = \frac{Fr^2}{Mm}$$

$$\text{units of G} = \frac{\text{kg m s}^{-2}\ \text{m}^2}{\text{kg}^2} = \text{m}^3\ \text{kg}^{-1}\ \text{s}^{-2}$$

SP2 A

$$g = \frac{GM}{r^2} \Rightarrow GM = gr^2$$

$$g_{r+h} = \frac{GM}{(r+h)^2} = \frac{gr^2}{(r+h)^2}$$

SP3 C

$$U_f = 0$$

$$U_i = -\frac{GMm}{r}$$

Energy needed = gain in gravitational potential energy

$$= U_f - U_i$$

$$= 0 - \left(-\frac{GMm}{r}\right)$$

$$= \frac{GMm}{r}$$

$$= \frac{6.67\times10^{-11}(5.0\times10^{24})(2.0)}{6.1\times10^{6}}$$

$$= 1.1\times10^{8}\ \text{J}$$

SP4 D

$$\phi_X = -\frac{GM}{R} = -800\ \text{kJ kg}^{-1} \Rightarrow U_X = m\phi_X = -800\ \text{kJ}$$

$$\phi_Y = -\frac{GM}{2R} = -400\ \text{kJ kg}^{-1} \Rightarrow U_Y = m\phi_Y = -400\ \text{kJ}$$

Work done $= \Delta U = U_f - U_i = U_Y - U_X = -400 - (-800) = 400$ kJ

SP5 E

$$U = -\frac{GMm}{r}$$

When $r\uparrow$, $U\uparrow$ (ie less negative).

A: $F = \dfrac{GMm}{r^2} \Rightarrow$ when $r\uparrow$, $F\downarrow$.

B: $a_c = \dfrac{F}{m} = \dfrac{GM}{r^2} \Rightarrow$ when $r\uparrow$, $a_c\downarrow$.

C: $v_{orbital} = \sqrt{\dfrac{GM}{r}} \Rightarrow$ when $r\uparrow$, $v_{orbital}\downarrow$.

D: $\omega = \dfrac{v_{orbital}}{r} = \sqrt{\dfrac{GM}{r^3}} \Rightarrow$ when $r\uparrow$, $\omega\downarrow$.

SP6 **B**

Since planet orbits the sun,

$$\frac{GM_S M_P}{R^2} = M_P R\omega^2$$

$$\frac{GSP}{R^2} = PR\left(\frac{2\pi}{T}\right)^2$$

$$T^2 \propto \frac{R^3}{S}$$

$$\therefore T \propto R^{3/2} \text{ or } T \propto S^{-1/2}$$

SP7 **C**

Minimum energy to escape Earth's attraction: $E_{Earth} = E_{infinity}$

$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = 0$$

$$v^2 = \frac{2GM}{R}$$

$$= 2\frac{GM}{R^2}R \quad \text{, since } g = \frac{GM}{R^2}$$

$$= 2gR$$

$$v = \sqrt{2gR}$$

SP8 **C**

For geostationary satellites, the following are the same:

- Magnitude of centripetal acceleration
- Linear speed, angular speed
- Radius, height
- Period, frequency

As force, kinetic energy and potential energy depends on the mass of the satellite, they may not be the same.

SP9 (a) $F = \frac{GMm}{r^2}$

(b)

$$F_g = F_c$$

$$\frac{GMm}{r^2} = mr\omega^2$$

$$\omega = \sqrt{\frac{GM}{r^3}}$$

(c)

$$\omega = \sqrt{\frac{GM}{r^3}}$$

$$\frac{2\pi}{T} = \sqrt{\frac{GM}{r^3}}$$

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

(d)

$$\frac{T_J^2}{T_E^2} = \frac{r_J^3}{r_E^3}$$

$$r_J = \left(\frac{T_J}{T_E}\right)^{2/3} r_E$$

$$= \left(\frac{11.9}{1}\right)^{2/3} \left(1.50 \times 10^{11}\right)$$

$$= 7.82 \text{ x } 10^{11} \text{ m}$$

SP10 **(a)** X is at the mid-point between the centres of the two stars. At this point, the gravitational fields due to the stars have the same magnitude due to equal mass and equal distance from the centre of the stars. However, they act in opposite directions and hence the resultant gravitational field strength is zero.

(b)

$$\phi_X = \left(-\frac{GM}{r}\right) + \left(-\frac{GM}{r}\right)$$

$$= 2\left(-\frac{\left(6.67 \times 10^{-11}\right)\left(4.0 \times 10^{30}\right)}{1.0 \times 10^{11}}\right)$$

$$= -5.34 \times 10^{9} \text{ J kg}^{-1}$$

(c)

$$F = \frac{GM_1M_2}{r^2}$$

$$= \frac{\left(6.67 \times 10^{-11}\right)\left(4.0 \times 10^{30}\right)^2}{\left(2.0 \times 10^{11}\right)^2}$$

$$= 2.67 \times 10^{28} \text{ N}$$

(d) Note that the centre of rotation is the centre of mass of the binary-star system.

$$F = \frac{mv^2}{r}$$

$$2.668 \times 10^{28} = \frac{\left(4.0 \times 10^{30}\right)v^2}{1.0 \times 10^{11}}$$

$$v = 2.58 \times 10^{4} \text{ m s}^{-1}$$

<u>Answers to Self-Practice Questions</u>

SP1 C **SP2** A **SP3** C **SP4** D
SP5 E **SP6** B **SP7** C **SP8** C
SP9(d) 7.82×10^{11} m
SP10(b) -5.34×10^{9} J kg^{-1} **(c)** 2.67×10^{28} N **(d)** 2.58×10^{4} m s^{-1}

<u>Answers to Discussion Questions</u>

D1(a) ~~9.38~~ 9.83 N, 0.0337 N, 9.80 N **(b)** 9.83 m s^{-2}, 9.80 m s^{-2}
D2(a) 7.27×10^{-5} rad s^{-1} **(b)** 3.59×10^{7} m
D3(a) 7.37×10^{3} m s^{-1} , 1.08×10^{4} m s^{-1}
D5(a) ~~9.44~~ 9.47 N kg^{-1} **(d)** 3.47×10^{9} J
D7(a) B **(b)** C **(c)** D, E