JURONG PIONEER JUNIOR COLLEGE
9749 H2 PHYSICS /8867 H1 PHYSICS

MEASUREMENT

Content

- **Physical quantities and SI units**
- **Scalars and vectors**
- **Errors and uncertainties**

Learning Outcomes

Candidates should be able to:

(a) recall the following base quantities and their SI units: mass (kg), length (m), time (s), current (A), temperature (K), amount of substance (mol).

(b) express derived units as products or quotients of the base units and use the named units listed in ‘Summary of Key Quantities, Symbols and Units’ as appropriate.

(c) use SI base units to check the homogeneity of physical equations.

(d) show an understanding of and use the conventions for labelling graph axes and table columns as set out in the ASE publication *Signs, Symbols and Systematics (The ASE Companion to 16-19 Science, 2000)*.

(e) use the following prefixes and their symbols to indicate decimal sub-multiples or multiples of both base and derived units: pico (p), nano (n), micro (μ), milli (m), centi (c), deci (d), kilo (k), mega (M), giga (G), tera (T).

(f) make reasonable estimates of physical quantities included within the syllabus.

(g) distinguish between scalar and vector quantities, and give examples of each.

(h) add and subtract coplanar vectors.

(i) represent a vector as two perpendicular components.

(j) show an understanding of the distinction between systematic errors (including zero errors) and random errors.

(k) show an understanding of the distinction between precision and accuracy.

(l) assess the uncertainty in a derived quantity by addition of actual, fractional, percentage uncertainties or by numerical substitution (a rigorous statistical treatment is not required).

Introduction

- Physics aims to understand the natural world around us and to represent the various phenomenon via mathematical relationships. Scientific experiments are then designed to test out the validity of the mathematical relationships.

- Such experiments involve the measurements of various physical quantities. The reliability of the measurements is important so as to correctly verify the concepts and theories. The precisions of any experimental results should also be quoted to an appropriate numbers of significant figures to reflect on the order or accuracy.

- In this lecture, we will learn to use different physical quantities, their SI units and how to work with the errors and uncertainties incurred when taking measurements. We will also be learning the classification of physical quantities as scalars or vectors, the addition and subtraction of vectors and the resolving of a vector into its components.

1 SI units

(a) *Candidates should be able to recall the following base quantities and their units: mass (kg), length (m), time (s), current (A), temperature (K), amount of substance (mol).*

1.1 Physical quantities

- Physical quantities are quantities that can be measured.
- A physical quantity consists of a numerical value and a unit.
- For example, the height of a man is about 1.70 m.

numerical value

h = 1.70 m

physical quantity

unit

In experiments, instruments are used for the measurement and recording of various physical quantities. Some examples are:

Physical quantity	**Instrument**
Mass, weight	Spring balance, lever balance
Length	Ruler, vernier callipers, micrometer screw gauge
Time	Stopwatch, clock, cathode ray oscilloscope
Temperature	Thermometer
Angle	Protractor
Electric current	Ammeter
Potential difference	Voltmeter

- In 1960, the international scientific community adopted a number of conventions about physical quantities and their units. The *Système Internationale d'Unités* (International System of Units) is based on seven base quantities and their corresponding units, called *base units*.

1.2 Base quantities and base units

- Base quantities are physical quantities that are fundamental and are not defined in terms of other physical quantities.

base quantity	**usual symbol for base quantity**	**SI base unit**	**symbol for base unit**
Mass	m	kilogram	kg
Length	l	metre	m
Time	t	second	s
Electric current	I	ampere	A
Thermodynamic Temperature	T	kelvin	K
Amount of substance	n	mole	mol
Luminous intensity*	L	candela	cd

*not in syllabus

- There are other physical quantities such as velocity and pressure that need to be measured. Such physical quantities are called derived quantities.

(b) *Candidates should be able to express derived units as products or quotients of the base units and use the named units listed in 'Summary of Key Quantities, Symbols and Units' as appropriate.*

1.3 Derived quantities and derived units

- Derived quantities are physical quantities that are defined in terms of base quantities according to a defining equation.

  For example, velocity is a derived quantity and it has the defining equation $v = \dfrac{\text{change in displacement}}{\text{time taken}}$ ..

- Units of derived quantities are called derived units and are expressed as products or quotients of base units.

- Derived units can be obtained from the defining equation as follows:

  The defining equation for velocity is $v = \dfrac{\text{change in displacement}}{\text{time taken}}$ ..

  Hence, the unit of velocity is m s^{-1} (metre per second).

  [Note the space break between the m and s^{-1}].

- In determining derived units, it is important to differentiate between symbols used for the physical quantities and the corresponding symbols for units. A summary of the usual symbols and units for different physical quantities can be found on pages 34 – 35 of the 9749 H2 Physics syllabus document (www.seab.gov.sg).

Derived quantity	Defining equation	Derived unit	Usual unit
acceleration	$\dfrac{\text{change in velocity}}{\text{time}} = \dfrac{v-u}{t}$	$\dfrac{\text{m s}^{-1}}{\text{s}} = \text{m s}^{-2}$	-
force	$\dfrac{\text{change in momentum}}{\text{time}}$ $= \dfrac{m(v-u)}{t}$	$\dfrac{\text{kg m s}^{-1}}{\text{s}} = \text{kg m s}^{-2}$	newton (N)
pressure	$\dfrac{\text{force}}{\text{area}} = \dfrac{F}{A}$	$\dfrac{\text{kg m s}^{-2}}{\text{m}^2}$ $= \text{kg m}^{-1}\text{ s}^{-2}$	pascal (Pa)
work	force $\times$ displacement in the direction of the force $= Fs$	$\text{kg m s}^{-2}\text{ m}$ $= \text{kg m}^2\text{ s}^{-2}$	joule (J)
power	$\dfrac{\text{work}}{\text{time}} = \dfrac{E}{t}$	$\dfrac{\text{kg m}^2\text{ s}^{-2}}{\text{s}}$ $= \text{kg m}^2\text{ s}^{-3}$	watt (W)
potential difference	$\dfrac{\text{work}}{\text{charge}} = \dfrac{E}{Q}$	$\dfrac{\text{kg m}^2\text{ s}^{-2}}{\text{A s}}$ $= \text{kg m}^2\text{ s}^{-3}\text{ A}^{-1}$	volt (V)

1.4 Dimensionless quantities and dimensionless constants

- Dimensionless quantities are physical quantities that have no units. Some examples are refractive index and relative molecular mass.

- All real numbers and some mathematical constants like π have no units. They are called dimensionless constants.

- Note: Some physical quantities that are constants have units. Some examples are:

    - acceleration of free fall, $g = 9.81 \text{ m s}^{-2}$,
    - elementary charge, $e = 1.60 \times 10^{-19}\text{ C}$

(c) Candidates should be able to use SI base units to check the homogeneity of physical equations.

1.5 Homogeneity of equations

- An equation is called homogeneous or dimensionally consistent if every term on both sides of the equation has the same units. That is, units on the LHS = units on the RHS.

- For example, for an equation W = X + Y, the units of the physical quantities W, X and Y must be the same.

- Base units can be used to check the homogeneity of physical equations.

Example 1

Check whether the following equations are homogeneous and state whether they are physically correct.
(a) $v = u + at$
(b) $v = u + 2at$
(c) $s = ut + at$

where s is displacement, v is final velocity, u is initial velocity and a is acceleration.

Solution:

(a) $v = u + at$
Unit of v = m s^{-1} (unit of velocity)

Unit of u = m s^{-1} (unit of velocity)

Unit of at = m s^{-2} × s = m s^{-1}

Hence equation is homogeneous. This is physically correct.

(b) $v = u + 2at$
Unit of v = m s^{-1}

Unit of u = m s^{-1}

Unit of 2at = m s^{-2} × s = m s^{-1}

Hence equation is homogeneous. But it is not physically correct.

(c) $s = ut + at$
Unit of s = m
Unit of ut = m s^{-1} × s = m
Unit of at = m s^{-2} × s = m s^{-1}

Equation is not homogeneous and thus definitely physically incorrect.

- An equation that is physically correct must be dimensionally consistent or homogeneous.

- However, note that an equation that is dimensionally consistent or homogeneous may NOT be physically correct.

Example 2

In the ideal gas law, $pV = nRT$ where n is the number of moles of gas, p is pressure, V is gas volume and T is the thermodynamic temperature. What are the possible units of R?

Solution:

$$pV = nRT$$

$$R = \frac{pV}{nT}$$

$$\text{Units of } R = \frac{\left(\text{kg m}^{-1}\text{ s}^{-2}\right)\text{m}^3}{(\text{mol})\text{K}} = \text{kg m}^2\text{ s}^{-2}\text{ mol}^{-1}\text{ K}^{-1}$$

Example 3

Bernoulli's equation, which applies to fluid flow, states that

$$p + h\rho g + \frac{1}{2}\rho v^2 = k$$

where p is a pressure, h a height, ρ a density, g an acceleration, v a velocity and k a constant. Show that the equation is dimensionally consistent and state an SI unit for k.

Solution:

(a) Since $p = \frac{F}{A}$,

$$\begin{aligned} \text{units of } p &= \frac{\text{units of } F}{\text{units of } A} \\ &= \frac{\text{kg m s}^{-2}}{\text{m}^2} \\ &= \text{kg m}^{-1}\text{ s}^{-2} \end{aligned}$$

$$\begin{aligned} \text{units of } h\rho g &= \text{units of } h \times \text{units of } \rho \times \text{units of } g \\ &= \text{m kg m}^{-3}\text{ m s}^{-2} \\ &= \text{kg m}^{-1}\text{ s}^{-2} \end{aligned}$$

$$\begin{aligned} \text{units of } \frac{1}{2}\rho v^2 &= \text{units of } \rho \times \text{units of } v^2 \\ &= \text{kg m}^{-3}\left(\text{m s}^{-1}\right)^2 \\ &= \text{kg m}^{-1}\text{ s}^{-2} \end{aligned}$$

Since the terms separated by the addition signs have the same SI base units, the equation is dimensionally consistent. Hence, the SI base units of k should be $\text{kg m}^{-1}\text{ s}^{-2}$.

(e) *Candidates should be able to use the following prefixes and their symbols to indicate decimal sub-multiples or multiples of both base and derived units: pico (p), nano (n), micro (μ), milli (m), centi (c), deci (d), kilo (k), mega (M), giga (G), tera (T).*

1.6 Standard form

- A way of writing very small or very large quantities.

- Standard form expresses a number as $N \times 10^n$ where n is an integer, either positive or negative, and N is any number such that $1 \leq N < 10$ or $-10 > N < -1$.

- For example, the sun's diameter is 1,392,000,000 m and an atom's average diameter is 0.000 000 000 3 m. Thus,

  Sun's diameter = 1.392×10^9 m = 1.392 Gm

  Average atom's diameter = 3×10^{-10} m = 0.3 nm

1.7 Prefixes

- The following prefixes and their symbols can be used to indicate decimal sub-multiples or multiples of both base and derived units:

		Multiplying factor	Prefix	Symbol
Decimal sub-multiples	0.000 000 000 001	10^{-12}	pico	p
	0. 000 000 001	10^{-9}	nano	n
	0. 000 001	10^{-6}	micro	μ
	0. 001	10^{-3}	milli	m
	0. 01	10^{-2}	centi	c
	0. 1	10^{-1}	deci	d
Decimal multiples	1 000	10^{3}	kilo	k
	1 000 000	10^{6}	mega	M
	1 000 000 000	10^{9}	giga	G
	1 000 000 000 000	10^{12}	terra	T

E.g. **0.00000123 J** can be expressed as **1.23 μJ.**

(d) *Candidates should be able to show* an understanding of and use the conventions for labelling graph axes and table columns as set out in the ASE publication *Signs, Symbols and Systematics (The ASE Companion to 16-19 Science, 2000)*.

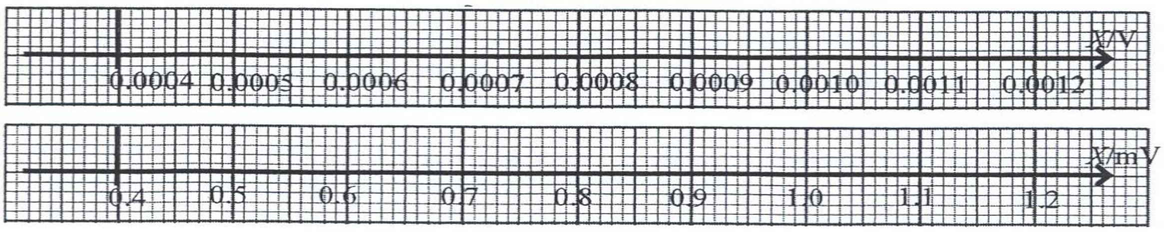
1.8 Conventions used for labelling table columns and graph axes

- The table column headings and the axes of the graph should show the physical quantity and the unit, separated by a solidus (slash). No units should be shown beside the values in the body of the table.

- In the table below, columns 2 and 3 are both correctly tabulated.

T / K	$\frac{1}{T}$ / K^{-1}	$\frac{1}{T}$ / 10^{-3} K^{-1}
273	0.00366	3.66
283	0.00353	3.53
293	0.00341	3.41

- However column 3 is preferred as it gives a better representation.
- Although the labelling of the two axes shown below are correct, the latter gives a better presentation.



(f) *Candidates should be able to make reasonable estimates of physical quantities included within the syllabus.*

1.9 Estimates Order of magnitude of physical quantities

- The ability to estimate the value of a physical quantity and to compute an approximate answer is a necessary skill in problem solving. The estimated answer can serve as a check on the calculation.

- In making an estimation or approximation of the physical quantities, it is usually necessary to make some assumptions.

Example 4

What is a reasonable estimate for the amount of energy needed to boil a flask of water? The energy needed is given by $Q = mc\Delta\theta$ where c is the specific heat capacity of water and is approximately 4.2 x 10^3 J kg^{-1} K^{-1}.

<u>Solution:</u>

Estimated room temperature = 30 °C

Thus, $\Delta\theta = 70$ K.

What about the mass of water?
Estimated volume of normal flask = 1.5 litres.
Thus, the mass of water is about 1.5 kg.

The estimated amount of energy needed $Q = mc\Delta\theta = (1.5)\left(4.2\times10^3\right)70 = 4.4\times10^5$ J

- Some estimation are only to the ‘power of ten’ of the number that describes that physical quantity. This is referred to as an *order of magnitude* of the physical quantity.

Physical quantity	Estimate / Order of Magnitude
Diameter of a nucleus	10^{-15} m
Diameter of an atom	10^{-10} m
Weight of a man	10^{3} N
Weight of a car	10^{4} N

1.10 Units Conversion

- Sometime physical quantities are given in units other than S.I. Units. For calculation in Physics, the values have to be converted to S.I. Units.
  - 1 cm^3 = _______________ m^3
  - 60 km h^{-1} = _______________ m s^{-1}
  - 1 g cm^{-3} = _______________ kg m^{-3}

(j) Candidates should be able to show an understanding of the distinction between systematic errors (including zero errors) and random errors.

2 Errors and uncertainties

- When an experimenter conducts an experiment and uses the instruments to take readings, the experimenter has to depend on his own skills to obtain as accurate a reading as possible.
- However, each instrument also has a limit of accuracy within which the experimenter is working with.
- Thus, physical quantities cannot be measured exactly with any instrument.
- Errors are uncertainties in measurements. They may arise due to:

  (a) the measuring instruments used,

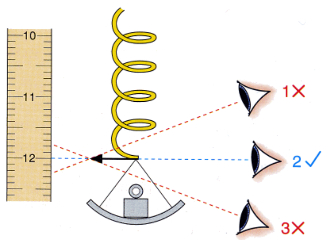
  (b) the techniques or skills of the experimenter,

  (c) the surrounding physical conditions,

  (d) the experimental design or method used.
- These errors caused measurements to deviate from their actual or true values.

2.1 Systematic and Random Errors

- There are two types of errors; systematic and random.
- The table below shows the comparison between a systematic and a random error.

Systematic error	Random error
Definition: Systematic errors result in readings taken being either always consistently over or under its true or actual value.	Definition: Random errors result in a scatter of readings about a mean value, such that the readings have an equal chance of being over or under its true or actual value.
The error is thus systematic if: (a) repeated measurements under the same conditions yield the same error in magnitude and sign. (b) its value changes in a predictable manner.	The error is thus random if: (a) repeated measurements results in errors with different magnitudes and signs.
Reducing Error: Systematic errors can be reduced by using good experimental techniques, replacing faulty equipment or via calibration curves. Systematic errors cannot be eliminated by taking an average of several readings.	Reducing Error: Random errors can be reduced by taking an average of several readings. This is as random errors tend to cancel each other out when taking average as both positive and negative errors are equally likely to occur.
Examples Zero errors on instruments. For example, when the gap of a micrometer screw gauge is closed, its reading is not zero but +0.01 mm. The zero error thus caused the measurement to be larger than the actual value. If the zero error is −0.01 mm, then the measurement will be smaller than the actual value. To reduce the error, the zero error should be subtracted from the measured value to obtain the actual value.	Examples Random parallax error when reading a scale. For example, the line of sight of the experimenter is not perpendicular to the scale. The randomness in the error occurs because the angle between the line of sight and the scale is not consistent throughout the experiment. This results in readings that are larger or smaller than the actual values.  The error may be reduced by having the scale as close as possible to the object to be measured, and viewing the scale with the line of sight perpendicular to the scale.

Reaction time of the experimenter. For example, in a time measurement, there may be a consistent delay between the experimenter observing an event and starting a stopwatch. The error may be reduced by starting and stopping the stopwatch to the same stimulus.	Random error due to irregularity of a physical quantity. For example, the diameter of a rod is not uniform and this causes random error in the measurement of its diameter. The error may be reduced by measuring the diameter at several positions along the length of the rod, and then averaging.

(k) *Candidates should be able to show an understanding of the distinction between precision and accuracy.*

2.2 Precision and Accuracy

2.2.1 Accuracy

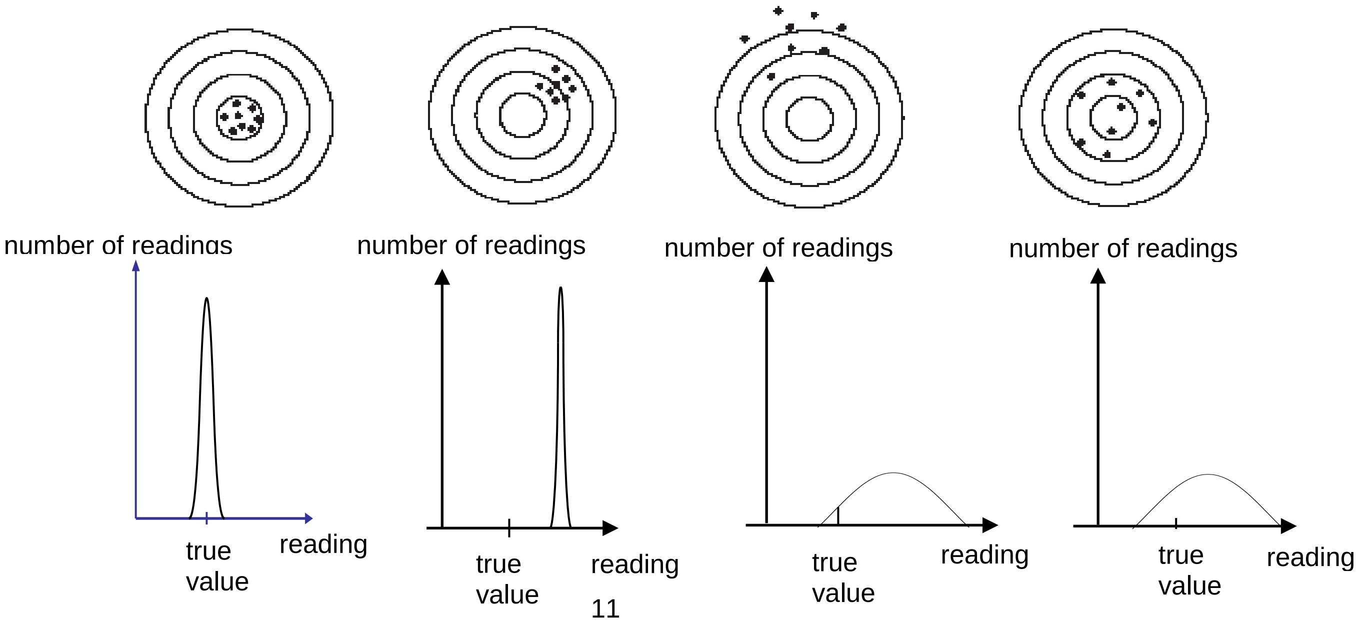
- Accuracy is the degree of closeness the measurements are to the true value. Accurate measurements will have readings that are close to the actual or true value of the physical quantity.

- Accuracy depends on the instrument used, the skills of the experimenter and the techniques involved.

2.2.2 Precision

- Precision is the degree of agreement of repeated measurements of the same quantity. Precise measurements will have a small spread of readings, resulting in small random errors. Imprecise measurements will have a large spread of readings and thus large random errors.

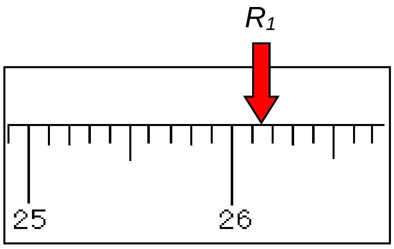
- On the other hand, precision of a measuring instrument refers to the extent or limit of sensitivity of the physical quantity being measured. For example, the precision of a metre rule is up to 0.1 cm, while the precision of a micrometer screw gauge is up to 0.001 cm. Hence, a micrometer screw gauge is a more precise instrument compared to a metre rule.



2.3 Uncertainty

2.3.1 Absolute or Actual uncertainty

- In any instrument, there is always a limit of accuracy within which the experimenter can attain. For example, what is the reading of R_1 below?



Reading, $R_1 \approx$ 26.13 cm or 26.14 cm?

- The last digit is an **estimate**. Thus, all readings will have a certain 'uncertainty'.

- The uncertainty is dependent on the instruments used and the experimenter can only read as accurately as what the instrument can measure.

- The actual uncertainty in the scale reading or pointer reading of an instrument is generally taken as half the smallest scale division or graduation of the instrument.

- For the above reading, R_1 = (26.15 $\pm$ 0.05) cm.

- Absolute or actual uncertainty of R_1, written as ΔR_1 is = 0.05 cm.

- Generally, any reading or measurements of a physical quantity A can be recorded as below:

$$A = a \pm \Delta a$$

where a is the reading from the instrument and Δa is the absolute uncertainty.

2.3.2 Fractional and percentage uncertainties

- For a reading $A = a \pm \Delta a$,
- The fractional uncertainty in A is $\dfrac{\Delta a}{a}$.
- The percentage uncertainty in A is $\dfrac{\Delta a}{a} \times 100\,\%$.

- For the above reading of R_1 = (26.15 $\pm$ 0.05) cm
  - absolute uncertainty in R_1, $\Delta R_1 = 0.05\ \text{cm}$
  - fractional uncertainty in R_1, $\dfrac{\Delta R_1}{R_1} = \dfrac{0.05}{26.15}$
    $= 0.002$
  - percentage uncertainty in R_1, $\dfrac{\Delta R_1}{R_1} \times 100\% = 0.2\%$

- In deciding which instrument to use when measuring a physical quantity, low fractional or percentage uncertainty is required. For examples, vernier calipers and micrometer screw gauge are used for small readings.

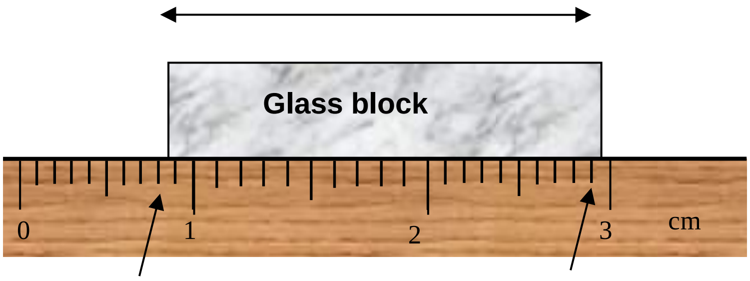
(l) *Candidates should be able to assess the uncertainty in a derived quantity by addition of actual, fractional or percentage uncertainties or by numerical substitution (a rigorous statistical treatment is not required).*

2.3.3 Uncertainties in derived quantities

- The final result of an experiment is seldom obtained by a single measurement. It is usually calculated from an expression containing different measured quantities.

- Consider the measurement of the length of a glass block using a ruler.

$L = (2.1 \pm 0.1)$ cm



$R_1 = (0.85 \pm 0.05)$ cm $\qquad R_2 = (2.95 \pm 0.05)$ cm

- The measurement of the length, L, is obtained from two readings, R_1 and R_2.
  - $R_1 = (0.85 \pm 0.05)$ cm
  - $R_2 = (2.95 \pm 0.05)$ cm
  - $L = (R_2 - R_1) =$ 2.10 cm

- What about the uncertainty of the length measurement?

- Consider from first principle.
  - $L_{max} = 3.00 - 0.80 = 2.20$ cm
  - $L_{min} = 2.90 - 0.90 = 2.00$ cm

  Hence, L range from 2.20 cm to 2.00 cm.

  Thus, we can write the length as follows:

  $L = l \pm \Delta l = 2.1 \pm 0.1$ cm

  The uncertainty of L is $\Delta l = 0.1$ cm

  It is also obtained as follows: $\Delta l = \Delta R_1 + \Delta R_2 = 0.05 + 0.05 = 0.1$ cm

- The table below shows how the uncertainties of the derived quantities A, B, C, D, E, and F are calculated. The measured quantities x and y have uncertainties Δx and Δy respectively, while k, m and n are all dimensionless constants.

Operation	Equation	Uncertainty
Addition	$A = mx + ny$	Absolute uncertainty, $\Delta A = m\Delta x + n\Delta y$
Subtraction	$B = mx - ny$	Absolute uncertainty, $\Delta B = m\Delta x + n\Delta y$
Multiplication	$C = kxy$	Fractional uncertainty, $\frac{\Delta C}{C} = \frac{\Delta x}{x} + \frac{\Delta y}{y}$
Division	$D = k\frac{x}{y}$	Fractional uncertainty, $\frac{\Delta D}{D} = \frac{\Delta x}{x} + \frac{\Delta y}{y}$
Product with powers	$E = kx^m y^n$	Fractional uncertainty, $\frac{\Delta E}{E} = m\frac{\Delta x}{x} + n\frac{\Delta y}{y}$
Quotient with powers	$F = k\frac{x^m}{y^n}$	Fractional uncertainty, $\frac{\Delta F}{F} = m\frac{\Delta x}{x} + n\frac{\Delta y}{y}$

2.3.4 Significant figures

- The number of significant figures in a result is simply the number of figures that are known with some degree of reliability.

- Thus, it will be sufficient to give uncertainties to one significant figure (s.f.).

- The measured quantities are then rounded off to the same decimal place (d.p.) as the uncertainties.

> Note:
>
> 1. All uncertainties are rounded off to 1 significant figure.
>
> 2. The derived quantity is then rounded off to the same decimal place as its uncertainty.

- Thus, in measuring the length of the glass block, L = 2.1 ± 0.1 cm.

  - L = 2.10 ± 0.10 cm is wrong because uncertainty written has 2 significant figure. Uncertainty can only has 1 s.f.

  - L = 2.10 ± 0.1 cm is wrong because the length should be expressed to 1 dp, same number of dp as its uncertainty.

Example 5

A rectangle has a length l = (34.3 ± 0.1) cm and breadth b = (21.8 ± 0.1) cm. Calculate the perimeter P and area A of the rectangle and express their values with their absolute uncertainties.

Solution:

$P = 2l + 2b = 2 \times (34.3 + 21.8) = 112.2$ cm

$\Delta P = 2\Delta l + 2\Delta b = 2 \times (0.1 + 0.1) = 0.4$ cm (to 1 s.f.)

$\therefore P = (112.2 \pm 0.4)$ cm (to 1 dp, similar to dp of uncertainty)

$A = lb = 34.3 \times 21.8 = 747.74$ cm^2

$$\frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta b}{b} = \frac{0.1}{34.3} + \frac{0.1}{21.8} = 0.00750$$

$\Delta A = 0.00750 \times A = 0.00750 \times 747.74 = 5.61$ cm^2 = 6 cm^2 (to 1 s.f.)

$\therefore A = (748 \pm 6)$ cm^2

Example 6

In a simple pendulum experiment to determine the acceleration due to gravity g, the equation used is $T = 2\pi\sqrt{\frac{l}{g}}$, where T is the period and l is the length of the pendulum. The values for T and l are (2.16 ± 0.01) s and (1.150 ± 0.005) m respectively. Determine the value of g with its uncertainty.

Solution:

Rearranging the equation, we have $g = \frac{4\pi^2 l}{T^2}$.

$$g = \frac{4\pi^2 \times 1.150}{2.16^2}$$

$$g = 9.7308 \text{ m s}^{-2}$$

Fractional uncertainty in g, $\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$, and we have

$$\frac{\Delta g}{9.7308} = \frac{0.005}{1.150} + 2\left(\frac{0.01}{2.16}\right)$$

$$\Delta g = 0.1 \text{ m s}^{-2} \quad \text{(1 s.f.)}$$

Therefore, the value of g is (9.7 ± 0.1) m s^{-2}.

Example 7

A metal cube of side *L* has a mass of *M*.

(a) If $M = (0.065 \pm 0.001)$ kg and $L = (0.200 \pm 0.001)$ m, determine its density ρ and express ρ with its associated uncertainty.

(b) What are the possible errors in the measurements and how can they be reduced?

(c) Suggest possible improvements to the determination of the density.

<u>Solution:</u>

(a) Density, $\rho = \dfrac{M}{L^3} = \dfrac{0.065}{0.200^3} = 8.125 \text{ kg m}^{-3}$

Fractional error $= \left(\dfrac{\Delta M}{M}\right) + 3\left(\dfrac{\Delta L}{L}\right)$

= (0.001 / 0.065) + 3(0.001 / 0.200)

= 0.0304

Absolute error, $\Delta\rho = 0.0304 \times 8.125$

= 0.247

= 0.2 kg m^{-3} (to 1 s.f.)

Therefore, density, $\rho = (8.1 \pm 0.2)$ kg m^{-3}

(b) Random error: parallax error in length reading
non-uniformity of the length of the sides of the cube

Systematic error: zero error of weighing balance
zero error of metre rule if zero mark of ruler is not clear

(c) Random error: use a marker/pointer when measuring length
take repeat readings and determine the average length

Systematic error: take into account zero error

Improvements: use vernier calipers to measure length.

Example 8

The equation connecting object distance *u*, image distance *v* and focal length *f* for a lens is $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$. A student measures values of *u* and *v* with their associated uncertainties, which are $u=(50\pm3)$ mm and $v=(200\pm5)$ mm. Determine the value of *f* with its uncertainty.

<u>Solution:</u>

$$\frac{1}{f}=\frac{1}{50}+\frac{1}{200}=40\text{ mm}$$

To determine f_{max},

$$\frac{1}{f_{max}}=\frac{1}{u_{max}}+\frac{1}{v_{max}}$$

$$\frac{1}{f_{max}}=\frac{1}{53}+\frac{1}{205}$$

$$f_{max}\approx 42.112\text{ mm}$$

To determine f_{min},

$$\frac{1}{f_{min}}=\frac{1}{u_{min}}+\frac{1}{v_{min}}$$

$$\frac{1}{f_{min}}=\frac{1}{47}+\frac{1}{195}$$

$$f_{max}\approx 37.872\text{ mm}$$

Therefore, the actual uncertainty in *f* is

$$\Delta f=\frac{f_{max}-f_{min}}{2}$$

$$\Delta f=\frac{42.112-37.872}{2}$$

$$\Delta f=2\text{ mm}\qquad\text{(1 s.f.)}$$

Therefore, the value of *f* is (40 ± 2) mm.

3 Scalars and vectors

(g) Candidates should be able to distinguish between scalar and vector quantities, and give examples of each.

3.1 Scalars and vectors

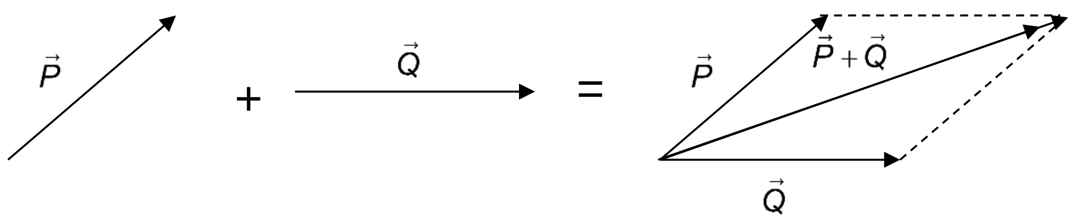
- Physical quantities can be classified as scalars or vectors.
- A scalar quantity has a magnitude only, while a vector quantity has both a magnitude and a direction. For example, the speed of a car is 12 m s^{-1}, but the velocity of the car will be 12 m s^{-1} in the direction horizontally towards the right.
- Some examples of scalar quantities are mass, distance, time, speed, volume, temperature and energy.
- Some examples of vector quantities are displacement, velocity, acceleration, weight, force and momentum

(h) Candidates should be able to add and subtract coplanar vectors.

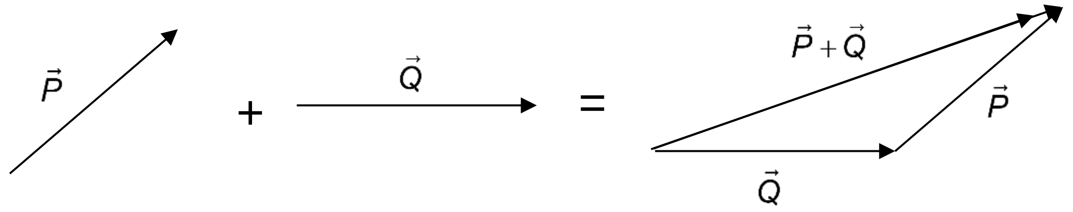
3.2 Vector addition and subtraction

- A vector is represented by a line drawn in a particular direction. The length of the line represents the magnitude of the vector while the direction of the arrow represents the direction of the vector.
- Coplanar vectors are vectors that lie in the same plane.
- Two vectors $\vec{P}$ and $\vec{Q}$ can be added together using either the parallelogram rule or triangle rule of vector addition.

3.2.1 Parallelogram rule of vector addition

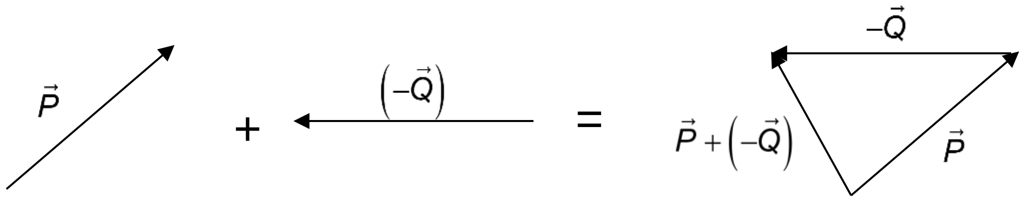


3.2.2 Triangle rule of vector addition



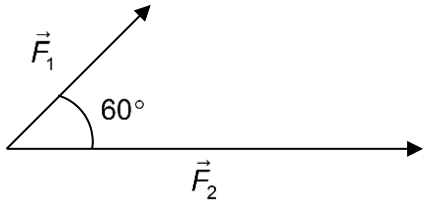
3.2.3 Vector subtraction

- Likewise, two vectors $\vec{P}$ and $\vec{Q}$ can be subtracted to give a resultant vector. This can be done by writing the subtraction as an addition of vectors instead.
- Thus, $\vec{P}-\vec{Q}=\vec{P}+\left(-\vec{Q}\right)$.



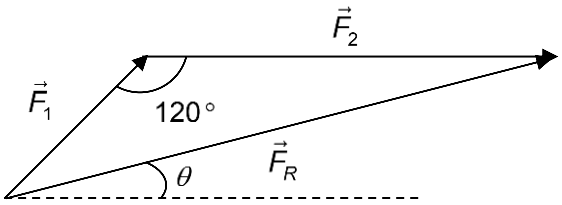
Example 9

Forces $\vec{F}_1$ and $\vec{F}_2$ of magnitude 10.0 N and 20.0 N respectively act at the same point on an object. Given that the angle between the two forces is 60°, determine the magnitude and direction of the resultant force acting on the object.



Solution:

Using the triangle rule of vector addition, $\vec{F}_R=\vec{F}_1+\vec{F}_2$.



Using the cosine rule,

$$F_R{}^2 = F_1{}^2 + F_2{}^2 - 2F_1F_2\cos 120^\circ$$

$$F_R{}^2 = 10.0^2 + 20.0^2 - \left(2\times 10.0\times 20.0\cos 120^\circ\right)$$

$$F_R = 26.5 \text{ N}$$

Using the sine rule,

$$\frac{\sin\theta}{F_1}=\frac{\sin 120^\circ}{F_R}$$

$$\frac{\sin\theta}{10.0}=\frac{\sin 120^\circ}{26.5}$$

$$\theta = 19.1^\circ$$

Therefore, the resultant force is 26.5 N at an angle of 19.1° above the horizontal, as shown in the diagram.

Example 10

A car changes its velocity from 30 m s^{-1} due East to 25 m s^{-1} due South.

Calculate (a) the change in speed, and

(b) the change in velocity of the car.

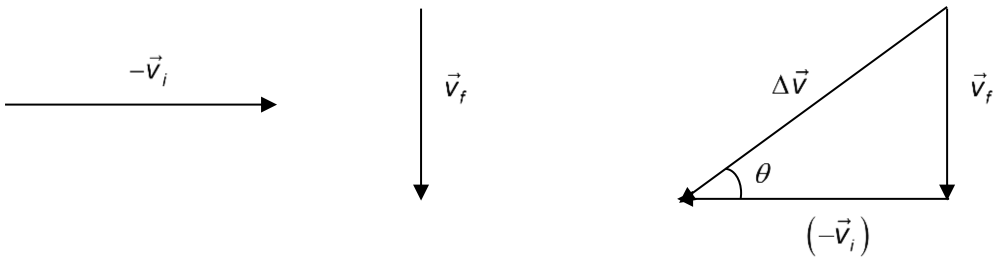
Solution:

(a) Since speed is a scalar quantity, it has a magnitude only. Hence, the change in speed is $25 - 30 = -5\ \text{m s}^{-1}$.

(b) Change in velocity, $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$

$$\Delta\vec{v} = \vec{v}_f + \left(-\vec{v}_i\right)$$

Drawing a vector diagram, we have



Using the Pythagoras' Theorem,

$$\Delta v^2 = v_i^2 + v_f^2$$

$$\Delta v^2 = 30^2 + 25^2$$

$$\Delta v = 39\ \text{m s}^{-1}$$

$$\tan\theta = \frac{v_f}{v_i}$$

$$\tan\theta = \frac{25}{30}$$

$$\theta = 40°$$

Therefore, the change in velocity is 39 m s^{-1} at an angle of 40^{o} below the horizontal, as shown in the diagram.

(i) Candidates should be able to represent a vector as two perpendicular components.

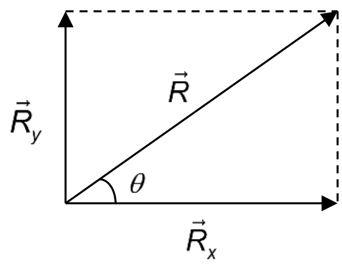
3.3 Components of vectors

- A vector can be split up into two parts which are called the components of the vector. The components are usually chosen to be along two mutually perpendicular directions.

- The process of finding the components is known as resolving the vector into its components.

- For example, the horizontal and vertical components of $\vec{R}$ are:

  $\vec{R}_x = \vec{R}\cos\theta$ and $\vec{R}_y = \vec{R}\sin\theta$ respectively, as shown below.

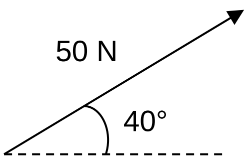


- Note that the magnitude of $\vec{R}$ can be obtained from its components using the Pythagoras' Theorem, where $R^2 = R_x{}^2 + R_y{}^2$, and the angle θ can be calculated using $\tan\theta = \dfrac{R_y}{R_x}$.

- The resolution of vectors into their components provides an alternative way of adding and subtracting vectors, and is useful in determining the resultant vector for more than two vectors.

Example 11

Calculate the horizontal and vertical components of a force of 50 N, acting at an angle of 40° above the horizontal.



Solution:

Resolving the force into its components,

$F_x = 50\cos 40°$

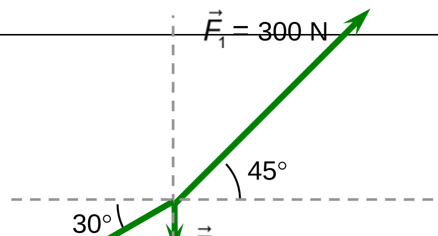
$F_x = 38$ N

$F_y = 50\sin 40°$

$F_y = 32$ N

Therefore, the horizontal component of the force is 38 N, while the vertical component of the force is 32 N.

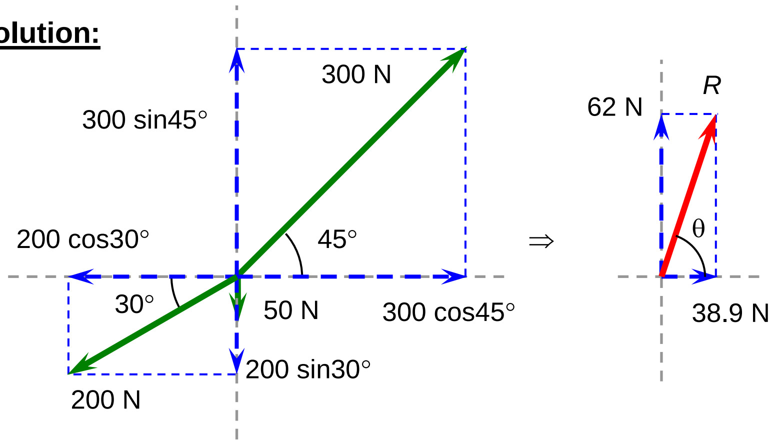
Example 12



Three forces, $\vec{F}_1 = 300$ N, $\vec{F}_2 = 200$ N and $\vec{F}_3 = 50$ N act at a point as shown on the right.

(a) Resolve the three forces into their *x*- and *y*-components,
(b) Calculate the resultant force acting at the point.

Solution:



(a) Resolving $\vec{F}_1 = 300$ N into its components,

$F_{1x} = 300\cos 45°$ $\quad\quad$ $F_{1y} = 300\sin 45°$

$F_{1x} = 212$ N $\quad\quad$ $F_{1y} = 212$ N

Therefore, the *x*- and *y*-components of $\vec{F}_1$ are both 212 N.

Resolving $\vec{F}_2 = 200$ N into its components,

$F_{2x} = 200\cos 30°$ $\quad\quad$ $F_{2y} = 200\sin 30°$

$F_{2x} = 173$ N $\quad\quad$ $F_{2y} = 100$ N

Therefore, the *x*- and *y*-components of $\vec{F}_2$ are 173 N and 100 N respectively.

For $\vec{F}_3 = 50$ N, it is already in the vertical direction.

As such, it will not have any horizontal component. $F_{3y} = F_3 = 50$ N

Therefore, there is no horizontal component for $\vec{F}_3$ and its vertical component is 50 N.

(b) Resultant horizontal component $= F_{Rx} = F_{1x} - F_{2x}$

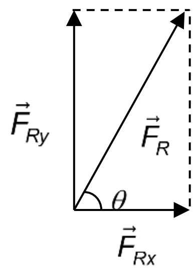
$= 300\cos 45° - 200\cos 30°$

$= 38.9$ N (in the positive *x*-direction)

Resultant vertical component $= F_{Ry} = F_{1y} - F_{2y} - F_3$

$= 300\sin 45° - 200\sin 30° - 50$

$= 62$ N (in the positive *y*-direction)



Magnitude of resultant force $F_R{}^2 = F_{Rx}{}^2 + F_{Ry}{}^2$

$$F_R = \sqrt{(38.9)^2 + (62)^2}$$
$$= 73 \text{ N}$$

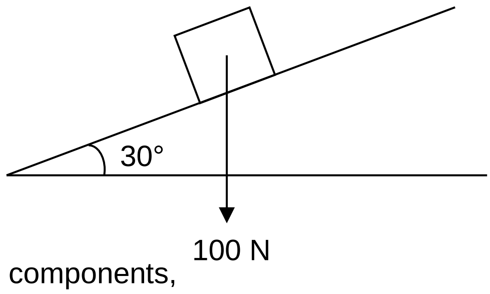
Direction, $\theta = \tan^{-1}\left(\frac{62}{38.9}\right)$

$= 57.9° = 58°$

Therefore, the resultant force acting at the point is 73 N at an angle of 58° above the horizontal, as shown in the diagram.

Example 13

A body of weight 100 N rests on a plane which is inclined at 30° to the horizontal. Calculate the components of the weight parallel to and perpendicular to the plane.



Solution:

Resolving the weight into its components,

$W_{//} = 100\sin 30°$

$W_{//} = 50 \text{ N}$

$W_{\perp} = 100\cos 30°$

$W_{\perp} \approx 87 \text{ N}$

Therefore, the component of the weight parallel to the slope is 50 N, while the component of the weight perpendicular to the slope is 87 N.

Definitions of SI Base Units — Appendix 1

The **metre** is $(299\ 792\ 458)^{-1}$ of the distance light travels in one second.

The **kilogram** is equal to the mass of the International Prototype kilogram (a platinum-iridium cylinder) kept in Sevres, Paris.

The **second** is defined in terms of 9 192 631 770 periods of a particular wavelength of light emitted by a caesium atom.

The **ampere** is the steady current flowing in two straight, infinitely long and parallel conductors of circular cross-section, placed one metre apart in a vacuum, which will produce a force of 2×10^{-7} N acting perpendicularly on a metre length of conductor.

One **kelvin** is exactly $(273.16)^{-1}$ of the temperature interval between absolute zero and the triple point of water.

One **mole** is the amount of any substance containing the same number of elementary units (atoms or molecules) as there are atoms found in 0.012 kg of carbon-12.

The **candela** is the luminous intensity, in the perpendicular direction, of a surface of 1/600 000 square metre of a blackbody at the temperature per square metre.

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4 Douglas C. Giancoli, *Physics - Principles with Applications (6th edition)*, Prentice Hall