BEDOK VIEW SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2021

CANDIDATE NAME			
REGISTER NUMBER		CLASS	

ADDITIONAL MATHEMATICS **4049/02**
Secondary 4 Express
Paper 2

31 August 2021
2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

Setter: Ms S. Ang **Parent's / Guardian's Signature:** ……………………..…

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta ABC = \frac{1}{2}ab \sin C$$

1 Prove that $2x^2 - 5x + 24 > 0$ for all real values of *x*. Hence, or otherwise, find the range of values of *x* for which $\dfrac{3x^2 - 16x + 5}{2x^2 - 5x + 24} < 0$. [7]

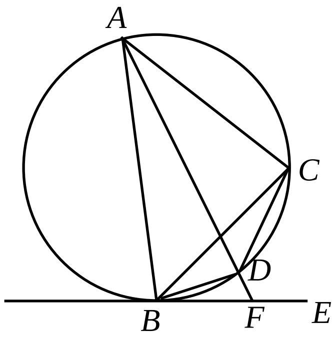
2 **(a)**

By considering the general term in the binomial expansion of $\left(x^3 - \frac{1}{2x^2}\right)^{10}$,

explain why the power is a multiple of 5. [3]

2 **(b)** Find the coefficient of x^5 in the expansion of $\frac{2}{x^5}\left(x^3 - \frac{1}{2x^2}\right)^{10}$. [3]

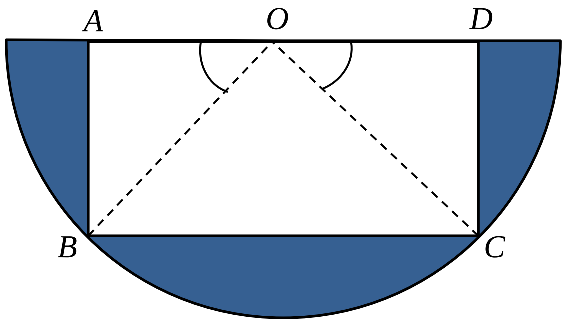
3 In the figure below, line *BFE* is a tangent to the circle at *B*. *FA* is a angle bisector of angle *BAC* and cuts the circle at *D*.



(a) Show that angle *DBC* = angle *DBF*. [3]

(b) Explain whether $\frac{BD}{AB} = \frac{DF}{BF}$. [2]

4 The diagram below shows a rectangle *ABCD* inside a semicircle with centre *O* and radius 6 cm. The angles *AOB* and *COD* are each equal to θ, where θ is measured in degrees.



(a) Show that the perimeter, *P* cm, of the rectangle is given by

$P = 12\sin\theta + 24\cos\theta$. [2]

(b) Express *P* in the form $R\sin(\theta + \alpha)$. [2]

4 **(c)** State the maximum value of P and find the corresponding value of θ. [2]

(d) Show that the area of the shaded region is $18(\pi - 2\sin 2\theta)$ cm^2. [3]

5 The table below shows the values of y corresponding to the values of x.

x	1	2	3	4	5
y	1	1.6	2	2.28	2.5

It is known that x and y are related by an equation of the form $\frac{a}{y} = \frac{b}{x} + 1$, where a and b are constants.

(a) Draw a straight line graph of $\frac{1}{y}$ against $\frac{1}{x}$ on the graph paper in the next page. [2]

(b) Use the graph to estimate the value of a and of b. [3]

(c) It is found that the value of $\frac{1}{x}$ is rather small. Another straight line graph is proposed by plotting the values of x in the horizontal axis.

(i) State the variable to be plotted in the vertical axis. [2]

(ii) Explain what does b represent in this new straight line graph. [1]

[Turn Over

6 A curve is such that $y = \dfrac{1+\sin x}{\cos x}$.

(a) Show that $\dfrac{dy}{dx} = \sec^2 x + \dfrac{\sin x}{\cos^2 x}$. [2]

(b) Hence find $\displaystyle\int \frac{2\sin x}{\cos^2 x}\,dx$. [3]

[Turn Over

6 **(c)** There is another curve such that $z = ky$. The gradient of the normal to this curve at $x = 0$ is $-\frac{1}{2}$. Find the value of k. [3]

(d) Explain whether there are other points on the curve which also have a gradient of the normal of $-\frac{1}{2}$. [2]

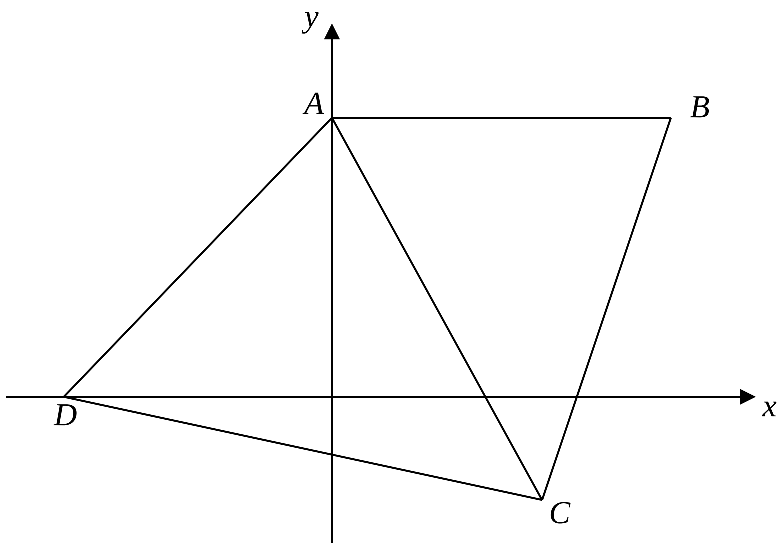
7 **(a)** Variables *x* and *y* are related by the form $(ey)^n = e^{x-a}$. It is known that when $x = 3$, $y = 1$ and $x = 7$, $y = e^2$. Find the value of *a* and of *n*. [4]

(b) Find the equation of the straight line which must be drawn on $y = \ln(x+1)$ to obtain a solution to the equation $e^5 (x+1)^2 = e^{2x}$. [3]

[Turn Over

7 **(c)** Given $\log_k 16 = m$, show that $\log_4 k = \dfrac{2}{m}$. Hence solve the equation $\log_k 16 - 1 = \log_4 k$. [5]

8 The diagram below shows a quadrilateral *ABCD* in which *A* is (0, 6) and *AB* is parallel to the *x*-axis. *D* is a point on the *x*-axis such that the equation of *DC* is $x+5y=-6$. *AC* is perpendicular to the line $2y=x+7$.



(a) Find the coordinates of point *C*. [4]

8 **(b)** Given that the area of triangle *ACD* is 1.5 times that of triangle *ABC*, find

(i) the coordinates of point *B*, [3]

(ii) the perpendicular distance from *D* to *AC*, leaving your answer in the form $a\sqrt{b}$. [3]

9 **(a)** A curve is such that $\frac{d^2y}{dx^2} = 9e^{-3x} - 2x$ and passes through the point $\left(2,\ e^{-6}\right)$.

(i) Given that $\frac{dy}{dx} = 2$ when $x = 0$, find the equation of curve. [5]

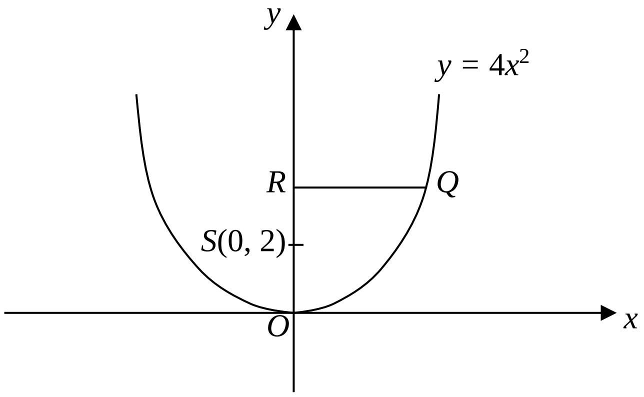
(ii) Find the equation of the tangent at the point $\left(2,\ e^{-6}\right)$, leaving your answer in the exact form. [3]

9 **(b)** Given that $\int_0^3 \mathrm{f}(x)\ \mathrm{d}x = \int_3^4 \mathrm{f}(x)\ \mathrm{d}x = 5$, find the value of *k* for which

$$\int_0^4 \frac{2}{5}\mathrm{f}(x)\ \mathrm{d}x + \int_4^3 \left[\mathrm{f}(x) - \frac{k(2x-5)}{(4x-10)^2} \right] \mathrm{d}x = -1 + \frac{3}{8}\ln 3$$

. [5]

10 The diagram below shows part of the curve $y = 4x^2$. The point R lies on the y-axis and the point Q is on the curve such that RQ is parallel to the x-axis and has a length of p units.



(a) Given that the point S is (0, 2), show that the area, A, of triangle RQS is $2p^3 - p$. [3]

10 **(b)** The point R moves along the y-axis and the point Q moves along the curve such that RQ remains parallel to the x-axis and p increases at a rate of 0.03 units per second. Find, when $p = 2$,

(i) the rate of change of A, [4]

(ii) the rate of change of the distance of point R from the x-axis as it travels along the y-axis. [3]

END OF PAPER