JC1 H2 Mathematics (9758)
Term 4 Revision Topical Quick Check
Chapter 7 Differentiation
Chapter 8 Applications of Differentiation

1 SAJC Promo 9758/2022/Q3b / JPJC Promo 9758/2022/Q2aiii

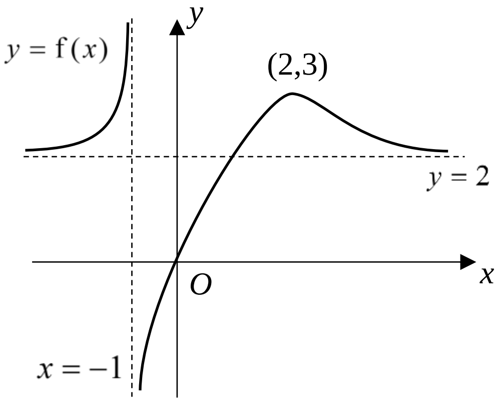
Differentiate the following with respect to *x*.

(i) $\ln\left(\dfrac{2x}{\sqrt{x^2+1}}\right)$ [2]

(ii) $\sec 3x \sin^{-1} 2x$ [3]

2 DHS Promo 9758/2022/Q1b

The graph of $y=\mathrm{f}(x)$ has a maximum turning point at (2,3) and passes through the origin. The lines $x=-1$ and $y=2$ are asymptotes to the graph, as shown in the diagram below.



Sketch the graph of $y=\mathrm{f}'(x)$, showing clearly the axial intercepts and the asymptotes. [3]

3 MI PU2 P1 Promo 9758/2022/Q7

A curve *C* has parametric equations

$$x=\cos 2\theta, \qquad y=\sin\theta, \qquad \text{for } 0\le\theta\le\frac{1}{2}\pi .$$

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{1}{4\sin\theta}$. [3]

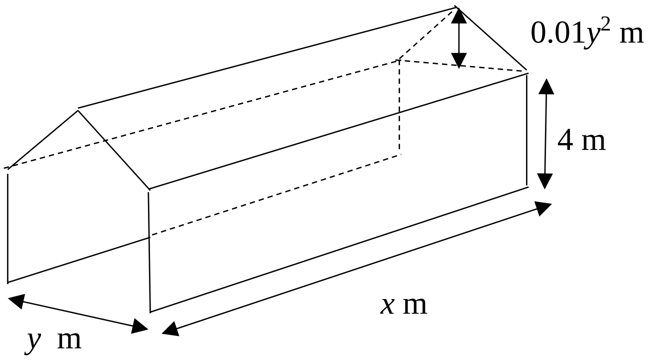
(ii) Sketch *C*, showing clearly the features of the curve at the points where $\theta=0$ and $\theta=\frac{1}{2}\pi$. [2]

(iii) The tangent to the curve *C* at the point where $\theta=p$ is parallel to the line $2y+x=0$. Find the equation of this tangent. [4]

(iv) The tangent from part **(iii)** meets the *x*-axis at *P* and the *y*-axis at *Q*. Find the area of the triangle *OPQ*. [3]

4 RVHS Promo 9758/2022/Q11(i)-(iii)

To ease food security concerns, the government is building tents at the community garden plots to allow urban farming to take place more efficiently among residents. The organizing committee is building a tent consisting of two rectangular pieces on the roof and two rectangular vertical sides as shown in the diagram below. The base of the tent has sides x m by y m, and covers a total floor area of 40 m^2. The vertical sides of the tent are 4 m tall, and the roof adds another $0.01y^2$ m to the overall height of the tent.



(Diagram not drawn to scale)

(i) The total external surface area of the tent is denoted by A m^2. Show that A is given by

$$A = 8x + 40\sqrt{\frac{16}{25x^2} + 1}\ .$$ [3]

(ii) Suppose that A has a stationary value at some x, show that x satisfies the equation $25x^6 + 16x^4 - 256 = 0$. [3]

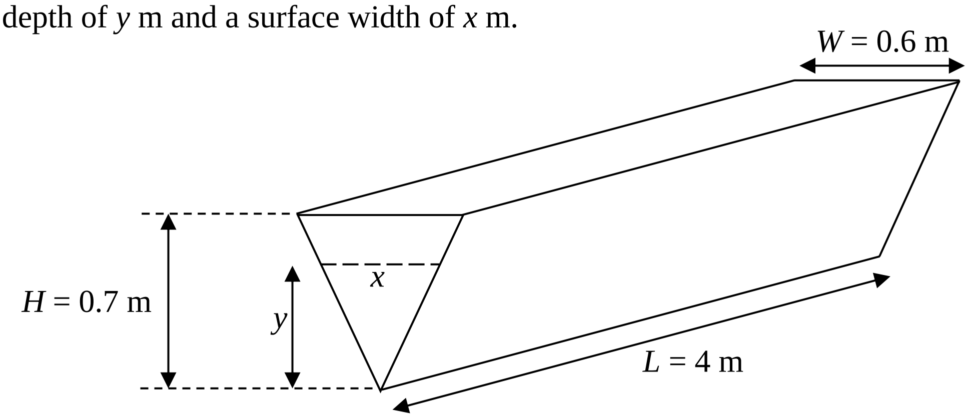
(iii) The design team decides that the material for the tent costs \$3.10 per m^2, estimate the minimum total cost of the material for the whole tent. [3]

5 MJC Prelim 9740/2008/P1/Q13

A piece of wire of length d units is cut into two pieces. One piece is bent to form a circle of radius r units, and the other piece is bent to form a regular hexagon. Prove that, as r varies, the sum of the areas enclosed by the two shapes is a minimum when the radius of the circle is approximately $0.076d$ units. [7]

6 JPJC Promo 9758/2022/Q5

The diagram shows a V-shaped tank with dimensions $L = 4$ m, $W = 0.6$ m and $H = 0.7$ m. The tank is initially empty. Water is pumped into the tank at a rate of 0.0025 m^3/s. At any instant from the start of water flowing into the tank, the water in the tank has a depth of y m and a surface width of x m.



(i) Find the rate of change of the water depth when $y = 0.4$ m, leaving your answer to 4 decimal places. [4]

(ii) Find, to the nearest second, the time taken to completely fill up the tank from the instant when $y = 0.4$ m. [2]

Answer Key

No.	**Year**	**JC**	**Answers**
1	2022	SAJC	**(i)** $\frac{1}{x} - \frac{x}{x^2+1}$ **(ii)** $\sec 3x \left(\frac{2}{\sqrt{1-4x^2}} + 3\tan 3x \sin^{-1} 2x \right)$
2	2022	DHS	
3	2022	MI	**(iii)** $y = -\frac{1}{2}x + \frac{3}{4}$ **(iv)** $\frac{9}{16}$
4	2022	RVHS	**(iii)** \$177.53
6	2022	JPJC	**(i)** $\frac{\mathrm{d}y}{\mathrm{d}t} = 0.0018$ m/s (4 d.p) **(ii)** 226 seconds