1 Introduction

In physics, 'work' has a meaning different from normal everyday usage. In physics, work is a method of energy transfer. Doing positive work on a system means transferring energy into the system while doing negative work means transferring energy out of the system.

Energy comes in many different forms - kinetic, potential, thermal, electromagnetic radiation - even mass is a form of energy as put forth by Einstein's famous $E = mc^2$ equation. Some of these energies are always in a state of flux or flow. Thus a system's total energy can change because of work done or the energy flows. That naturally leads to the concept of power which is the rate of flow or transfer or transformation of energy.

2 Work

Definition of work

Consider a box of mass m on a horizontal *frictionless* surface. Initially, the box was stationary. A constant force F applied on the box caused it to accelerate and acquire a final velocity v and KE = ½ mv^2.

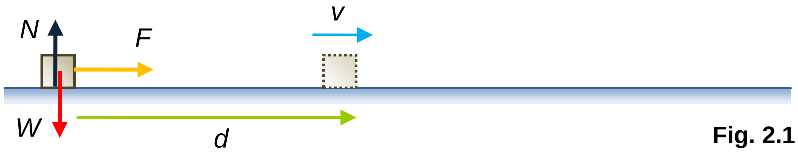


Fig. 2.1

Of the three forces acting on the box, the normal force N and the weight W do not contribute to the acceleration and gain in KE. Let's work out the connection between F and the gain in KE:

- F is a net force $\therefore F = ma$
- F constant $\therefore$ a also constant. Straight line motion with constant a implies $v^2 = u^2 + 2as$
- $v^2 = 2as$ since $u = 0$
- $v^2 = 2(F/m)d$
- $\therefore \frac{1}{2} mv^2 = Fd$

The product Fd is the *work done* by F.

Formally, work done WD is defined as follows

> Work done by a force is defined as the product of the force and the displacement *in the direction* of the force.

Work done(scalar) by a force on a system is defined as the product of the force and the displacement *in the direction* of the force.

Basic cases of work

Case 1 - $\vec{F}$ & $\vec{d}$ in same direction, $WD = Fd$

This case has been considered above. F & d are magnitudes.

Case 2 - $\vec{F}$ & $\vec{d}$ in opposite direction, $WD = -Fd$

In Fig. 2.2, $\vec{F}$ is to the left on a box with initial velocity $\vec{v}$ to the right, causing deceleration.

The box loses KE and the work done by $\vec{F}$ is negative.

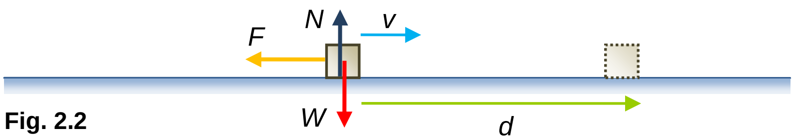


Fig. 2.2

Basic cases:

1. $\vec{F}$ & $\vec{d}$ in same direction, $WD = Fd$
2. $\vec{F}$ & $\vec{d}$ in opposite direction, $WD = -Fd$
3. $\vec{F}$ & $\vec{d}$ are perpendicular, $WD = 0$

Case 3 - $\boxed{\vec{F}\ \&\ \vec{d}\ \text{are perpendicular, } WD = 0}$

Referring to Fig. 2.2, if $\vec{F}$ = N vertically upwards or $\vec{F}$ = W vertically downwards, it does not contribute to the change in KE of the box and WD = 0.

General cases of work

In general, $\vec{F}$ can be at any angle to $\vec{d}$. However, the force can always be resolved into a component parallel to $\vec{d}$ and another component perpendicular to $\vec{d}$ as shown in Fig. 2.3 and 2.4. The component perpendicular to $\vec{d}$ does no work (see basic case 3). Work is only done by the component parallel to $\vec{d}$.

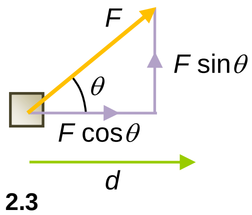


Fig. 2.3

$\boxed{\text{Work} = (F\cos\theta)d}$

See basic case 1

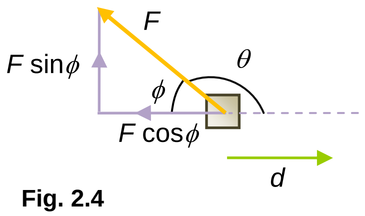


Fig. 2.4

$\boxed{\text{Work} = -(F\cos\phi)d}$

Negative because horizontal force component is opposite to $\vec{d}$ (see basic case 2).

In Fig.2.4, since -(F cosϕ)d = Fd cosθ, the general formula for work done:

$$\boxed{WD = \vec{F}\cdot\vec{d} = Fd\cos\theta}$$

General formula for work = Fd cosθ where θ is the angle between $\vec{F}$ and $\vec{d}$.

where θ is the angle between $\vec{F}$ and $\vec{d}$. Fd cosθ can be thought of as the product of the force component magnitude (F cosθ) and the displacement magnitude d. It can also be thought of as the product of displacement component magnitude (d cosθ) and the force magnitude F.

It is important to note that when the vector arrows are joined such that they both point away or towards the joining point as shown in Fig. 2.5, the angle θ is the *angle between* them.

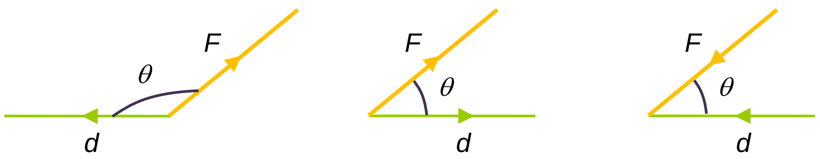


Fig. 2.5

In Fig. 2.6, when one vector points towards but the other points away from the joining point, the angle α is *not* the *angle between* the vectors. In this case, the *angle between* the vectors is given by 180° - α.

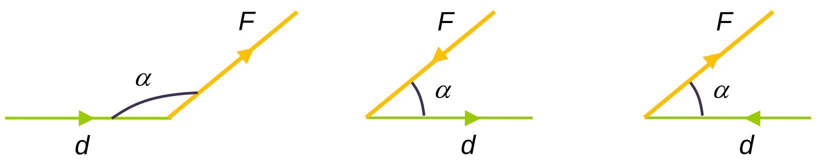


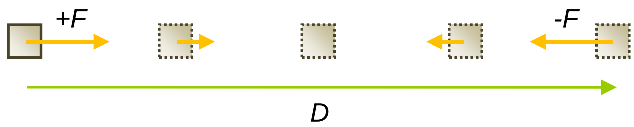
Fig. 2.6

Work Done by Non-constant Force

The formula $WD = Fd \cos\theta$ is only applicable for constant magnitude F.

Consider an object that is initially at rest being acted upon by a single force only. The force is initially $+F$ but its magnitude decreases to zero before increasing again in the opposite direction. The force is then removed when it reaches $-F$.

WD = Fd cos*θ* is only applicable for *constant* magnitude *F*.



Taking to the right as positive.

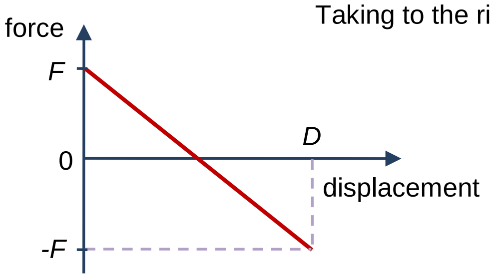


Fig. 2.7

In this case, the work done by the non-constant force is given by the area under the force-displacement graph. For the first half of the journey, the work done is positive while negative for the second half. Furthermore, the total work done is zero.

Work done for a non-constant force can be found from area under *F*-*d* graph.

+ and - Work Done

Positive and negative work done by a force on a system corresponds to the transfer of energy *to* the system and *out* of the system respectively.

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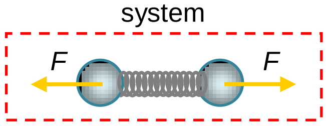
In Fig. 2.7, the object accelerates to the right until $D/2$. Thereafter it decelerates to a stop. In other words, the kinetic energy of the object increases until $D/2$, then decreases to zero at the end. This can be accounted for by looking at the work done on the object. For the first half of the journey, positive work done causes the kinetic energy to increase while in the second half an equal amount of negative work done causes the kinetic energy to drop to zero.

In general, work done tends to cause the total energy of a system to change. Since the total energy may consist of various forms of energy such as kinetic energy, potential energy, internal energy(thermal) etc, one or more forms of energy possessed by the system may change as a result of work done.

In understanding the relationship between work done and energy, it is critical to have a clear idea of the system in consideration. Consider two spheres connected by a light compressed spring and isolated from its surrounding. When released:

system



The net force on system = 0.
No work done on the system, so total energy of the system is constant.
Internally, there is conversion of elastic PE to KE of the spheres.

Conclusion about relationship between work and energy depends on the chosen system.

system - sphere only

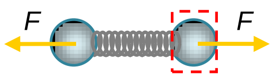


Fig. 2.8

The net force on system = *F*.
F does work on the system, so KE of system increases. Sphere by itself does not have elastic PE.

For a rigid system, total work done on it = change in its KE

Work Done On/By Gas

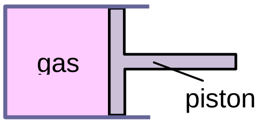


Fig. 2.9

In Fig. 2.9, if the gas expands, the force exerted by the gas on the piston and external atmosphere is in the direction of the displacement. Thus positive work is done on the piston and atmosphere or energy is transferred from the gas to them.

If on the other hand, the atmosphere outside pushes the piston inwards, the force by the gas does negative work on the external environment which means that energy is removed from the environment. However, the environment does positive work on the gas and we can say that there is energy transfer from the environment to the gas.

Gas expands: We say 'work is done by the gas on its surrounding' or 'negative work is done by the surrounding on gas'.

Gas compressed: We say 'work is done on the gas by surrounding' or 'negative work is done by gas on surrounding'.

When the expansion or compression happens while the pressure P *of the gas is constant*, then the absolute value of the work done on or by the gas system is $|P\Delta V|$, where ΔV is the change in volume of the system. If the pressure is not constant but varies with volume, then $WD = \int PdV$ or the area under P-V graph.

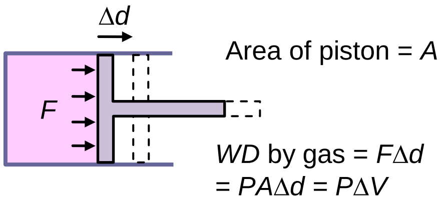


Fig. 2.10

Absolute value of work done on or by a gas = $|P\Delta V|$ if P is *constant*.

3 Energy

Forms of Energy

Energy exists in many forms - kinetic, potential, heat, radiant, electrical and nuclear binding energy. Radiant energy refers to the energy of electromagnetic waves or of the photons. Electrical energy carried by electric current is the energy made available from conversion of electric potential energy. Nuclear binding energy is the energy needed to separate the protons and neutrons inside a nucleus and it corresponds to the energy given out when the protons and neutrons come together to form the nucleus.

The various forms of energy can be confusing sometimes. *Mechanical energy* refers to the sum of KE and PE of an object as a single body while *internal energy* refers to the sum of KE of the randomly moving molecules and the PE between the molecules in the object. Sound energy can be conceived as KE and PE of the air molecules due to the sound waves. Elastic potential energy can be thought of as fundamentally electric potential energy between molecules.

Mechanical energy of an object = KE + PE

Potential Energy

Potential energy can be sub-divided into different types - e.g. gravitational, electric and elastic. In general, potential energy (PE) can be stored in a system with

(a) at least two bodies which

(b) exert forces - on each other - that depend only on their positions.

For a system of two masses connected by a spring, greater separation corresponds to greater elastic PE. Similarly, a pair of masses or charges will change their gravitational and electric PE respectively when the separation is changed. It does not matter what is the actual path or what previous values of PE a system had, the amount of PE depends only on the current separation.

Potential energy (PE) can be stored in a system with at least 2 bodies which exert forces on each other in a way that depend only on their positions.

The *change* in PE (for the mentioned types) is always equal to the work done by an external agent in changing the separation without at the same time changing other forms of energy that the system possesses. If positive work is done by the external agent, then PE of the system will be increased while negative work leads to a decrease.

$\Delta PE = WD$ by external force when no change to other forms of system's energies.

Principle of Conservation of Energy (PCOE)

The principle states that energy can neither be created nor destroyed; it can only be transferred from one system to another or converted from one form to another.

As such, the total energy of the whole Universe is constant. However for a small part of the Universe such as a rock, its total energy can increase or decrease due to the surrounding giving it energy or taking away energy from it. For instance, on a hot day, heat energy flows into the rock, thus causing its internal thermal energy to increase. As another example, consider a toy car with a wound up spring. When released on a horizontal surface, the car converts its stored elastic PE into KE in such a way that its total energy is constant. In this case, there is no transfer of energy between the car and its surrounding but there is energy transformation within it.

In the toy car example, its total energy consists of KE, elastic PE, gravitational PE, thermal energy and some other forms of energy. In fact, it is very difficult to keep its total energy constant because there are many processes that can cause energy to flow in or out of the car system. For instance, there will be friction in the moving mechanism inside that will produce heat, some of which will dissipate to the surrounding. Consequently, instead of all elastic PE becoming KE, some will be lost to the surrounding and so the total energy will decrease.

The car as a system is a so called non-isolated system. *An isolated system is one where it is not able to exchange energy, work or matter with its surrounding*. Notice that work is a form of energy transfer and matter is equivalent to energy as put forth by Einstein's $E = mc^2$ equation. On a practical level, we can only isolate a system to a greater or lesser extent but never achieving full isolation of a system.

An alternative statement of the principle of conservation of energy is thus:

The principle states that the total energy of an *isolated* system must be conserved.

Here the word conserve means 'remain constant in time'. In contrast, 'conserving energy' when used by environmentalists refers to efficient and responsible use of Earth's limited energy sources such as fossil fuel, coal and gas.

PCOE states that energy can neither be created nor destroyed; it can only be transferred from one system to another or converted from one form to another.

An isolated system is one where it is not able to exchange energy, work or matter with its surrounding.

Alternative PCOE states that the total energy of an *isolated* system must be conserved.

4 Power

Power is the rate of work done or rate of energy transfer/transformation.

$P = \dfrac{dW}{dt}$ or $\dfrac{dE}{dt}$ where W and E are work and energy respectively

P above is called *instantaneous* power because it can vary from moment to moment. Contrast that with the *average* power for a certain time interval $t_2 - t_1$ as shown in Fig. 4.1.

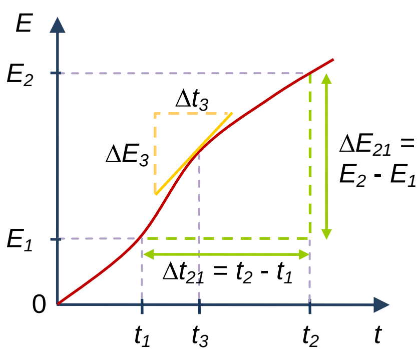


Fig. 4.1

Instantaneous power $P = \dfrac{dE}{dt}$ at t_3 is the gradient of the tangent which $= \dfrac{\Delta E_3}{\Delta t_3}$

Average power between t_1 & t_2 is $= \dfrac{\Delta E_{21}}{\Delta t_{21}}$

Instantaneous power is $P = \dfrac{dW}{dt}$ or $\dfrac{dE}{dt}$

Average power is $P_{ave} = \dfrac{\Delta W}{\Delta t}$ or $\dfrac{\Delta E}{\Delta t}$

$P = Fv$ formula

Earlier, work done $W = Fx\cos\theta$ (F, θ constant and x is displacement)

$$P_{ave} = \frac{\Delta W}{\Delta t} = \frac{Fx\cos\theta}{\Delta t} = F\cos\theta\frac{x}{\Delta t} = (F\cos\theta)v_{ave}$$

If Δt tends to zero, P, v_{ave} and F all become instantaneous values.

Hence $P = Fv\cos\theta$

Or $P = Fv$ in frequent cases where $\vec{F}$ and $\vec{v}$ are in the same direction.

Average power $P_{ave} = Fv_{ave}$ and instantaneous power $P = Fv$ for $\vec{F}$ and $\vec{v}$ in the same direction.

Example

A barge is being pulled along a canal by two horses on the banks. At a particular moment when the barge is moving at a velocity of 0.5 m s^{-1}, the horses are doing work on the barge at a rate of 800 W.

(i) Find the tension *T*.

(ii) If the resistive force *R* is 800 N at this moment, find the rate of increase of kinetic energy.

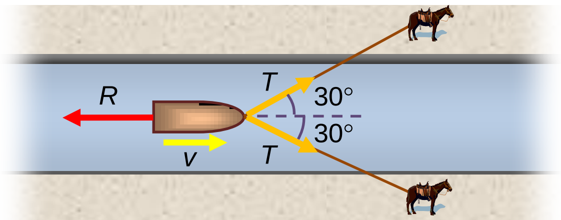


Fig. 4.2

(i) Rate of work done by the horses $= Fv\cos\theta$

$$= 2(T\cos30°)v$$

$$800 = 2(T\cos30°)0.5$$

$$\underline{T = 923 \text{ N}}$$

(ii) For the barge, rate of increase of KE

= rate of energy input from horses - rate of energy loss due to resistive force

$= 800 - Rv$

$= 800 - (800)0.5$

$\underline{= 400 \text{ J s}^{-1}}$

R is doing negative work since its direction is opposite to the direction of displacement. Hence R is associated with a rate of energy loss of the barge.

5 Efficiency

In physics, efficiency = $\dfrac{\text{useful energy or power output}}{\text{energy or power input}} \times 100\%$

$$\boxed{\eta = \left(\frac{E_{useful}}{E_{in}} \text{ or } \frac{P_{useful}}{P_{in}}\right) \times 100\%}$$

Efficiency,

$$\eta = \left(\frac{E_{useful}}{E_{in}} \text{ or } \frac{P_{useful}}{P_{in}}\right) \times 100\%$$

A scenario involving efficiency is best visualised as follows:

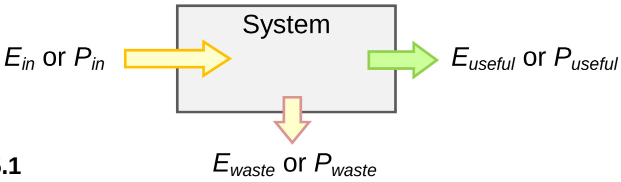


Fig. 5.1

Efficiency is easy to calculate, but before any calculation is made, make sure one is clear what is the system and which energy or power is the *useful* part. Just like in many other situations, a fuzzy idea of what system is being considered will lead to fuzzy identification of system properties such as forces, mass, energies etc.

Important things to watch out for calculating efficiency:

1. what is the system?
2. which output is useful?

Consider a hypothetical device which is powered by running water entering and leaving horizontally. Every second, 4 kg of water flows into it at a speed of 3.0 m s^{-1} and out of it at 1.0 m s^{-1}. When running, this device generates 4 W of heat and 12 W of light.

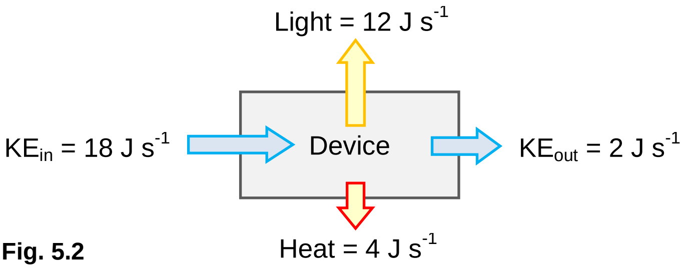


Fig. 5.2

What is the efficiency of the device?

The answer depends on what energy output is considered useful. If the device is meant to provide illumination only, then η = (12/18)×100% = 66.7%.

However, if the device is using the output heat for heating up the interior of a home in winter, then η = (16/18)×100% = 88.9%.