1 **(a)** Express $\dfrac{33-8x}{x^2+2x-15}+2$ as a single algebraic fraction. Hence, without using a calculator, solve exactly the inequality $\dfrac{33-8x}{x^2+2x-15}>-2$. [4]

(b) Using your answer to part **(a)**, find the set of values of x for which $\dfrac{33-8\mathrm{e}^{2x}}{\mathrm{e}^{4x}+2\mathrm{e}^{2x}-15}>-2$. [2]

2 The sum of the first n terms of a sequence, u_r, is given by $\displaystyle\sum_{r=1}^{n}u_r=1-\frac{n}{(n+1)!}$.

(a) Find u_n in terms of n, for $n\geq 2$, expressing your answer as a single algebraic fraction. [2]

(b) Show that $\displaystyle\sum_{r=5}^{n}u_r<\frac{1}{30}$ for all $n\geq 5$. [2]

(c) Explain why $\displaystyle\sum_{r=1}^{\infty}u_r$ is a convergent series. [1]

3 The functions f and g are defined by

$$\mathrm{f}:x\mapsto\frac{ax-6}{x-3}\quad\text{for } x\in\mathbb{R},\ x\neq 3,$$

$$\mathrm{g}:x\mapsto \mathrm{e}^{-x}\quad\text{for } x\in\mathbb{R},\ x\geq\ln 3.$$

The function f is such that $\mathrm{f}(x)=\mathrm{f}^{-1}(x)$ for all x in the domain of f.

(a) Find the value of a. [3]

(b) State the exact value of $\mathrm{f}^{6}(\pi)$. [1]

(c) Find the exact range of fg. [3]

4 **(a)** Given that **a**, **b** and **c** are non-zero vectors such that $(\mathbf{a}+\mathbf{b})\times(\mathbf{a}+\mathbf{c})=\mathbf{b}\times\mathbf{c}$, and $\mathbf{b}\neq\mathbf{c}$, find the relationship between **a**, **b** and **c**. [4]

(b) It is given instead that **a**, **b** and **c** satisfy the equation $\mathbf{a}+\mathbf{b}+\mathbf{c}=\mathbf{0}$ with $|\mathbf{a}|=2$, $|\mathbf{b}|=3$ and $|\mathbf{c}|=4$. Find the value of

$$\mathbf{a}\cdot\mathbf{b}\ +\ \mathbf{b}\cdot\mathbf{c}\ +\ \mathbf{c}\cdot\mathbf{a}.$$ [3]

5 It is given that $\mathrm{f}(n)=\dfrac{n}{5^{n-1}}$ where n is a positive integer.

(a) By considering $\mathrm{f}(r)-\mathrm{f}(r+1)$, find an expression for $\displaystyle\sum_{r=2}^{n}\frac{4r-1}{5^r}$. [3]

(b) Hence find an expression for $\displaystyle\sum_{r=1}^{n}\frac{4r+6}{5^{r+1}}$. [3]

6 **(a)** Find the exact value of $\displaystyle\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{2\sin^{-1} x}{\sqrt{1-x^2}}\,dx$. [3]

(b) Find the exact value of $\displaystyle\int_0^{\frac{\pi}{3}} \left|\cos 2x\right| dx$. [3]

(c) Find $\displaystyle\int \frac{1}{-x^2+2kx+3k^2}\,dx$, where k is a positive constant. [4]

7 **(a)** The curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{ax^2+bx+c}{x+d},$$

and a, b, c and d are constants, and $a \neq 0$.

Given that C has asymptote $y = x+1$, find the value of a and show that $b = d+1$. [2]

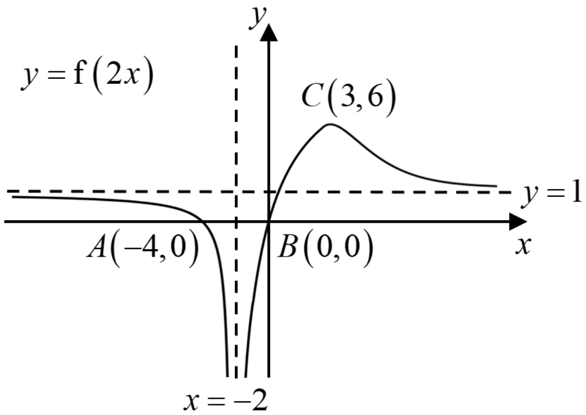
If f is an increasing function for all $x \in \mathbb{R}$, $x > -d$, show that $c < d$. [3]

(b) It is further given that $c = 1$ and $d = 2$.

(i) Sketch C. [3]

(ii) By sketching a suitable graph in the same diagram in part **(b)(i)**, find the number of real roots to the equation $\left(\dfrac{x^2+3x+1}{x+2}\right)^2 + 4x^2 = 16$. [2]

8 **(a)** The diagram shows the graph with equation $y = \mathrm{f}(2x)$. The graph passes through the points $A(-4,0)$, $B(0,0)$ and $C(3,6)$, and has asymptotes $x = -2$ and $y = 1$.



On separate clearly labelled diagrams, deduce the graphs of

(i) $y = \mathrm{f}(2x-2)$, [2]

(ii) $y = \mathrm{f}(x)$. [2]

(b) The curve C_1 undergoes the transformations in the order given below:

1. A translation of 2 units in the negative x direction.
2. A stretch parallel to the x axis, factor 2, y axis invariant.
3. A translation of 1 unit in the positive y direction.

The resulting curve C_2 has equation

$$y=\frac{x^2+9x+22}{x+4}\ ,\ \ x\in\mathbb{R},\ \ x\neq -4.$$

Find, in the simplest form, the equation for C_1. [4]

9 Find the area of the region bounded by the graphs of $y=2x^2+3$ and $y=-2x+7$. [3]

State the area of the region bounded by the 2 graphs if both graphs are translated 7 units in the negative y-direction. [1]

The region R is bounded by $y=2x^2+3$, $y=-2x+7$, the x-axis and the y-axis. Find the exact volume of the solid generated when R is rotated 2π about the y-axis. [5]

10 Given that $z=2-\mathrm{i}$ is a root of the equation $4z^4-12z^3+17z^2+pz+q=0$, where p and q are real, find p and q. [4]

Using the values of p and q found, find the other roots of the equation $4z^4-12z^3+17z^2+pz+q=0$ in exact form. [4]

11 Dendrologists are specialised scientists who study trees and woody plants. Their work is diverse and can encompass various activities related to the identification, classification, biology, and ecology of trees. A group of dendrologists are studying the growth of 2 species of trees, codenamed Tree *Vee* and Tree *Jay*.

In the 1$^{\text{st}}$ year, the height of Tree *Vee* and Tree *Jay* are both H cm.

In the 2$^{\text{nd}}$ year, Tree *Vee*'s height increases by s cm and subsequently, the increase in height every year is 10% less than the previous year's increase. Show that the height of Tree *Vee* in the 4$^{\text{th}}$ year is given by $(H+2.71s)$ cm. [1]

Show that the height of Tree *Vee* in the n^{th} year is given by $\left[H+10s\left(1-0.9^{n-1}\right)\right]$ cm. [3]

Hence, write down in terms of H and s, the theoretical maximum height (in cm) of Tree *Vee*. [1]

In the 2$^{\text{nd}}$ year, Tree *Jay*'s height increases by t cm and subsequently, the increase in height every year is 0.5 cm less than the previous year's increase. Show that the height of Tree *Jay* in the 10$^{\text{th}}$ year is given by $(H-18+9t)$ cm. [2]

It is now given that $t=20$.

After the 10$^{\text{th}}$ year, Tree *Jay*'s height increases at a constant rate of 7 cm per year. Express Tree Jay's height (in cm) in the n^{th} year (where $n\geq 11$) in terms of H and n. [2]

It is further given that $s=30$, and the 1$^{\text{st}}$ year is the year 2024. Find the years in which the heights of Tree *Vee* and Tree *Jay* are within 7 cm of each other, **after 2034**. [3]

12 Game developers closely monitor the number of people playing their game. Understanding player numbers and behaviour can not only help in optimising in-game purchases, advertisement placements, and other revenue-generating aspects, it can also help the company manage server loads and ensure the game runs smoothly without performance issues.

Two game developers are interested in the number of players playing the mobile game “Mobile Saga”. They attempt to model the number of players x, in **hundred thousands**, at time t months after the launch of the game using a differential equation. On the day of the launch, there were 55 000 players.

(a) One game developer suggests that x and t are related by the differential equation $\dfrac{\mathrm{d}x}{\mathrm{d}t}=\dfrac{3}{5}x-kt^2$, where k is a positive constant.

(i) By substituting $x=u\mathrm{e}^{\frac{3}{5}t}$, show that the differential equation can be written as $\dfrac{\mathrm{d}u}{\mathrm{d}t}=-kt^2\,\mathrm{e}^{-\frac{3}{5}t}$. [2]

(ii) Hence show that $x=\dfrac{5k}{3}t^2+\dfrac{50k}{9}t+\dfrac{250k}{27}+\left(\dfrac{11}{20}-\dfrac{250k}{27}\right)\mathrm{e}^{\frac{3}{5}t}$. [4]

(iii) Company A intends to place an advertisement in the game only if there are more than 76 000 players playing the game. Given that $k=\dfrac{1}{10}$, find the length of time for which Company A will place an advertisement in “Mobile Saga”, giving your answer correct to the nearest month. [2]

(b) The other game developer suggests that x and t are related by the differential equation $\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2}=-\dfrac{10}{(1+t)^3}$. Given further that there were 180 000 players playing “Mobile Saga” after 1 month, find x in terms of t. [4]