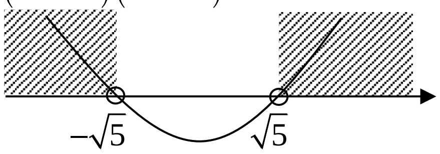
PU2 MATHEMATICS

Paper 8865/01

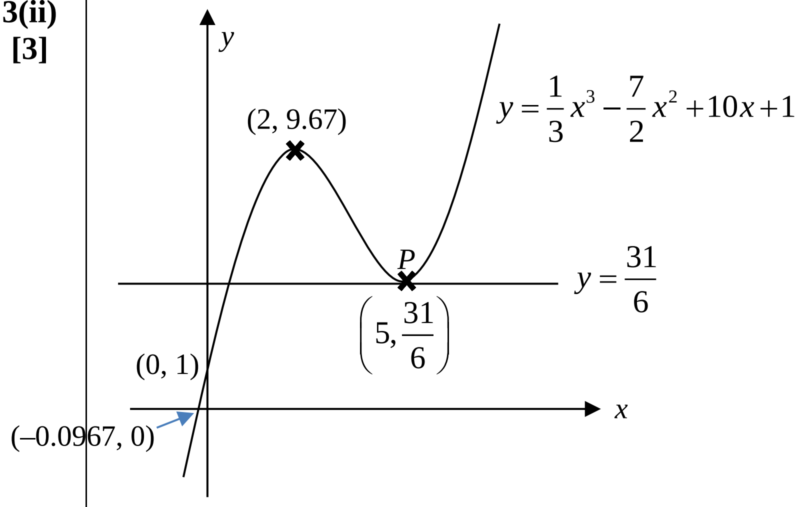
Section A: Pure Mathematics

Qn	**Solution**
1(i) **[3]**	Let x, y and z be the cost of one serving of beans, vegetables and nuts respectively. $\text{Amy's}\ : 2x+3y+z=9.1-----(1)$ $\text{Betty's}\ : 3x+4y+2z=14-----(2)$ $\text{Cathy's}\ : 3x+3y=8.1-----(3)$ Using GC, $x=1.2$, $y=1.5$, $z=2.2$ The cost of one serving of beans, vegetables and nuts is \$1.20, \$1.50 and \$2.20 respectively.
1(ii) **[1]**	<u>Method 1</u> Original Price of the salad = $\dfrac{6.32}{0.8}=7.9$ 1.2 + 1.5a + 2.2 = 7.9 a = 3 <u>Method 2</u> 0.8(1.2) + 0.8(1.5a) + 0.8(2.2) = 6.32 a = 3

Qn	**Solution**
2(a)(i) **[2]**	<u>Method 1</u> Substitute x = 3 and y = p into $p^2x-5y=8$ $3p^2-5p=8$ $3p^2-5p-8=0$ $p=-1$ or $p=\dfrac{8}{3}$ (rejected) <u>Method 2</u> Substitute x = 3 and y = p into $x^2+9y^2+3x=30+xy$ $9+9p^2+9=30+3p$ $9p^2-3p-12=0$ $3p^2-p-4=0$ $p=-1$ or $p=\dfrac{4}{3}$ (rejected)

Qn	Solution
2(ii) **[3]**	Substitute $p = -1$ into $p^2x - 5y = 8$ $x - 5y = 8$ $x = 8 + 5y - - - - - (1)$ Subst (1) into $x^2 + 9y^2 + 3x = 30 + xy$ $(8+5y)^2 + 9y^2 + 3(8+5y) = 30 + (8+5y)y$ $64 + 80y + 25y^2 + 9y^2 + 24 + 15y = 30 + 8y + 5y^2$ $29y^2 + 87y + 58 = 0$ $y = -1$(1st set of solution) or $y = -2$ $x = 8 + 5(-2) = -2$ The other set of solution is $x = -2$ and $y = -2$.
2(iii) **[2]**	$x^2 > 5$ $x^2 - 5 > 0$ $(x+\sqrt{5})(x-\sqrt{5}) > 0$  $x < -\sqrt{5}$ or $x > \sqrt{5}$

Qn	Solution
3(i) **[3]**	$\frac{dy}{dx} = x^2 - 7x + 10$ $\left.\frac{dy}{dx}\right\|_{x=5} = 5^2 - 7(5) + 10 = 0$ When $x = 5, y = y = \frac{1}{3}(5)^3 - \frac{7}{2}(5)^2 + 10(5) + 1 = \frac{31}{6}$ Equation of tangent at $P: y - y_1 = m(x - x_1)$ $y - \frac{31}{6} = 0(x-5)$ $y = \frac{31}{6}$

Qn	Solution
3(ii) **[3]**	
3(iii) **[2]**	Method 1 (Using GC) Required area $=\int_{\frac{1}{2}}^{5}\left(\left(\frac{1}{3}x^3-\frac{7}{2}x^2+10x+1\right)-\frac{31}{6}\right)\mathrm{d}x$ $=11.4$ (3 s.f.) Method 2 (Algebraic) Required area $=\int_{\frac{1}{2}}^{5}\left(\left(\frac{1}{3}x^3-\frac{7}{2}x^2+10x+1\right)-\frac{31}{6}\right)\mathrm{d}x$ $=\int_{\frac{1}{2}}^{5}\left(\frac{1}{3}x^3-\frac{7}{2}x^2+10x-\frac{25}{6}\right)\mathrm{d}x$ $=\left[\frac{1}{3}\left(\frac{x^4}{4}\right)-\frac{7}{2}\left(\frac{x^3}{3}\right)+10\left(\frac{x^2}{2}\right)-\frac{25}{6}x\right]_{\frac{1}{2}}^{5}$ $=\left(\frac{5^4}{12}-\frac{7\left(5^3\right)}{6}+5\left(5^2\right)-\frac{25}{6}(5)\right)-$ $\left(\frac{\left(\frac{1}{2}\right)^4}{12}-\frac{7\left(\frac{1}{2}\right)^3}{6}+5\left(\frac{1}{2}\right)^2-\frac{25}{6}\left(\frac{1}{2}\right)\right)$ $=10.416-(-0.973)$ $=11.4$ (3 s.f.)

Qn	Solution
4(a)(i) **[2]**	$\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{1}{3(2x-1)^2}\right]=\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{1}{3}\times\frac{1}{(2x-1)^2}\right]$ $=\frac{1}{3}\times\frac{\mathrm{d}}{\mathrm{d}x}\left[(2x-1)^{-2}\right]$ $=\frac{1}{3}(-2)(2x-1)^{-3}(2)$ $=-\frac{4}{3}(2x-1)^{-3}=-\frac{4}{3(2x-1)^3}$
4(a)(ii) **[2]**	$\frac{\mathrm{d}}{\mathrm{d}x}\left(4\mathrm{e}^{x^3}\right)=4\times\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{x^3}\right)$ $=4\left(3x^2\mathrm{e}^{x^3}\right)$ $=12x^2\mathrm{e}^{x^3}$
4(b)(i) **[3]**	$\int\left(x-\frac{2}{x}\right)^2\mathrm{d}x=\int x^2-2(x)\left(\frac{2}{x}\right)+\left(\frac{2}{x}\right)^2\mathrm{d}x$ $=\int x^2-4+4x^{-2}\,\mathrm{d}x$ $=\frac{x^3}{3}-4x+4\left(\frac{x^{-1}}{-1}\right)+c$ $=\frac{x^3}{3}-4x-\frac{4}{x}+c$
4(b)(ii) **[2]**	$\int\mathrm{e}^{\frac{x+1}{4}}\mathrm{d}x=\int\mathrm{e}^{\frac{1}{4}x+\frac{1}{4}}\mathrm{d}x=\frac{\mathrm{e}^{\frac{1}{4}x+\frac{1}{4}}}{\frac{1}{4}}+c=4\mathrm{e}^{\frac{x+1}{4}}+c$

Qn	Solution
5(i) **[2]**	Using GC, $\frac{\mathrm{d}}{\mathrm{d}t}\left[15\ln(t+1)-5t+10\right]\Big\|_{t=5}=-2.5$ The population of the moth is <u>decreasing at a rate of 2.5 thousand moths per year at the 5th year</u> after the natural bird predator is introduced.
5(ii) **[5]**	$P=15\ln(t+1)-5t+10$ $\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{15}{t+1}-5$ To find stationary point,

<table>
<tr><th>Qn</th><th>Solution</th></tr>
<tr><td></td><td>

$\frac{dP}{dt}=0$
$\Rightarrow \frac{15}{t+1}-5=0$
$\Rightarrow \frac{15}{t+1}=5$
$\Rightarrow 15=5(t+1)$
$\Rightarrow 5t=10$
$\Rightarrow t=2$
Performing first derivative test:

<table>
<tr><td>t</td><td>2^-
(1.99)</td><td>2</td><td>2^+
(2.01)</td></tr>
<tr><td>$\frac{dP}{dt}$</td><td>>0
(0.0167)</td><td>0</td><td><0
(-0.0166)</td></tr>
<tr><td>Slope</td><td>/</td><td>—</td><td>\</td></tr>
</table>

Hence P is a <u>maximum</u> at $t=2$
The maximum value of $P=15\ln(2+1)-5(2)+10=16.5$ The maximum population is 16 500 (3 s.f.).
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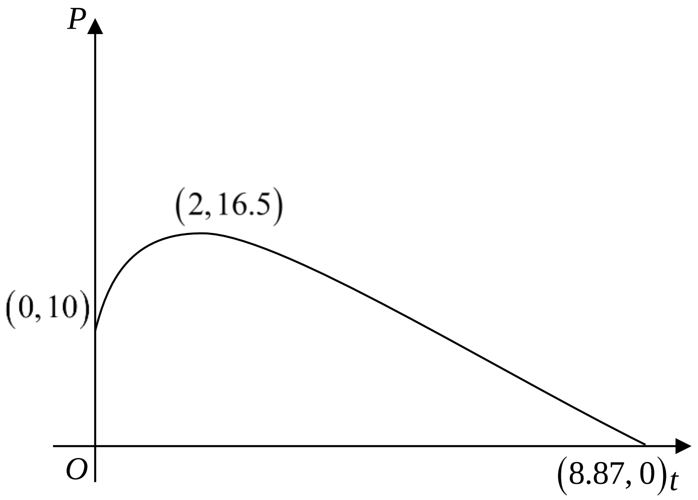
<tr><td>5(iii)
[2]</td><td>



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<tr><td>5(iv)
[2]</td><td>

The rate of change of the population of the natural bird predators is 0 initially, i.e. $Q=0$ when $t=0$.

$0=b(3(0)+1)-a\times 2^0$
$\Rightarrow 0=b-a$
$\Rightarrow a=b$
</td></tr>
<tr><td>5(v)
[1]</td><td>

Using GC,
$\int_0^4 5(3t+1)-5\times 2^t \, dt$=31.798=31.8 (3s.f.)
Population is 31.8 hundred (or 3180)
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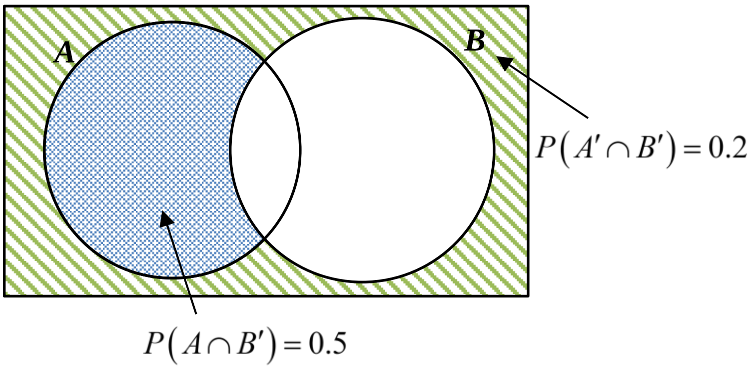
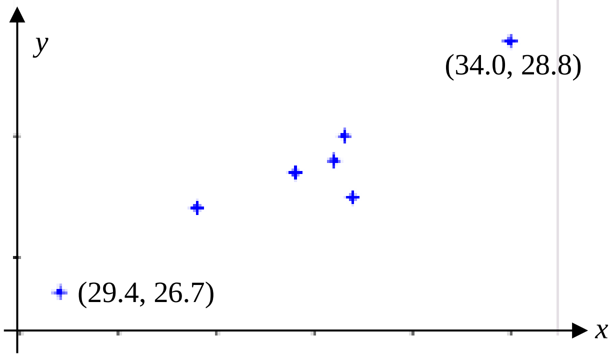
Section B: Probability and Statistics

Qn	Solution
6(i) **[2]**	Vowels: A, O, U, I, E Consonants: F, V, R, T There can only be one arrangement for vowels (V) and consonants(C) to alternate: V, C, V, C, V, C, V, C, V, No. of ways $=5!\times 4!=2880$
6(ii) **[3]**	Complement: all the 4 consonants are together V V V V V [C C C C] No. of ways for all 4 consonants together $=6!\times 4!$ No. of ways $=9!-6!\times 4!$ $=345600$

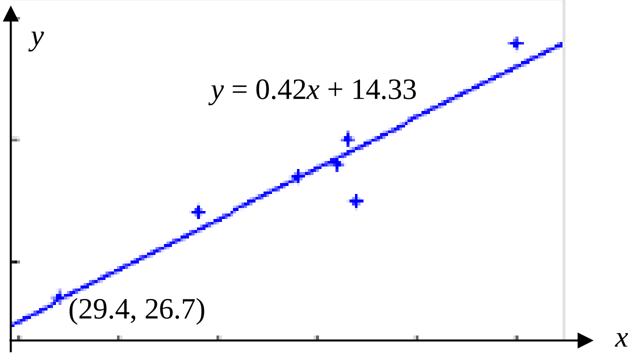
Qn	Solution
7(i) **[2]**	Let X be the number of faulty mechanical pencils, out of 15. Then $X\sim \mathrm{B}(15, 0.07)$ $\mathrm{P}\left(X<\frac{15}{4}\right)=\mathrm{P}(X<3.75)$ $=\mathrm{P}(X\le 3)$ $=0.98247=0.982$ (3s.f.)
7(ii) **[3]**	$\mathrm{P}\left(X\ge 1 \mid X<\frac{15}{4}\right)=\mathrm{P}(X\ge 1 \mid X\le 3)$ $=\frac{\mathrm{P}(X\ge 1\cap X\le 3)}{\mathrm{P}(X\le 3)}$ $=\frac{\mathrm{P}(1\le X\le 3)}{\mathrm{P}(X\le 3)}$ $=\frac{\mathrm{P}(X\le 3)-\mathrm{P}(X=0)}{\mathrm{P}(X\le 3)}$ $=0.65729=0.657$ (3s.f.)

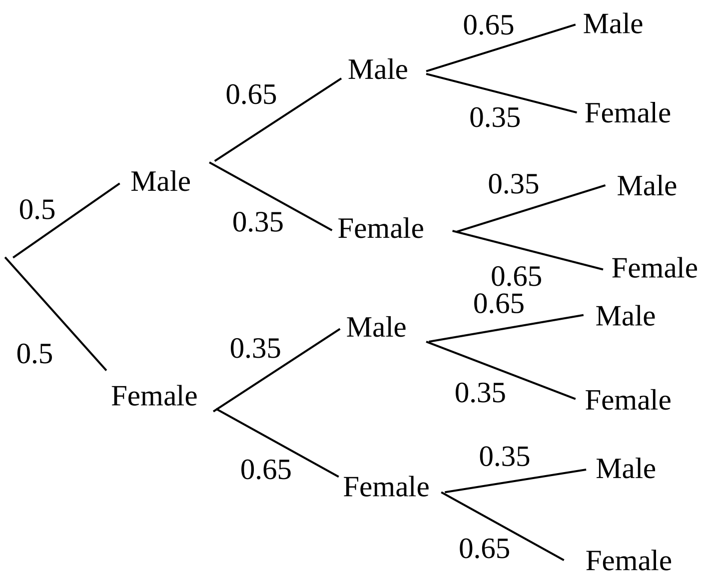
Qn	Solution
8(i) **[2]**	 $\mathrm{P}(B) = 1 - \mathrm{P}(A' \cap B') - \mathrm{P}(A \cap B')$ $= 1 - 0.2 - 0.5$ $= 0.3$
8(ii) **[3]**	**<u>Method 1:</u>** Given A and B are independent, A and B' are independent. $\mathrm{P}(A \cap B') = \mathrm{P}(A) \times \mathrm{P}(B') = 0.5$ $\mathrm{P}(A) = \frac{0.5}{\mathrm{P}(B')}$ $= \frac{0.5}{1-0.3}$ $= \frac{5}{7}$ -- **<u>Method 2:</u>** Given A and B are independent, A' and B' are independent. $\mathrm{P}(A' \cap B') = \mathrm{P}(A') \times \mathrm{P}(B') = 0.2$ $\mathrm{P}(A') = \frac{0.2}{\mathrm{P}(B')}$ $= \frac{0.2}{1-0.3}$ $= \frac{2}{7}$ $\mathrm{P}(A) = 1 - \mathrm{P}(A')$ $= 1 - \frac{2}{7}$ $= \frac{5}{7}$ --

Qn	Solution
	<u>Method 3:</u> Given A and B are independent, $\mathrm{P}(A\cap B)=\mathrm{P}(A)\times\mathrm{P}(B)$ $\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$ $=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A)\times\mathrm{P}(B)$ $0.8=\mathrm{P}(A)+0.3-\mathrm{P}(A)\times 0.3$ $0.5=0.7\mathrm{P}(A)$ $\mathrm{P}(A)=\frac{5}{7}=0.714$ (3s.f.)
8(iii) **[1]**	$\mathrm{P}(A'\mid B)=\mathrm{P}(A')$ (since A and B and independent) $=1-\frac{5}{7}$ $=\frac{2}{7}=0.286$ (3s.f.) **Alternatively,** $\mathrm{P}(A'\mid B)=\frac{\mathrm{P}(A'\cap B)}{\mathrm{P}(B)}$ $=\frac{\mathrm{P}(A')\times\mathrm{P}(B)}{\mathrm{P}(B)}$ (since A and B and independent) $=\mathrm{P}(A')$ $=\frac{2}{7}=0.286$ (3s.f.)

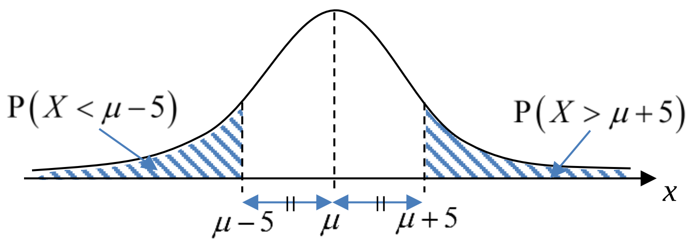
Qn	Solution
9(i) **[2]**	
9(ii) **[2]**	$r=0.94517=0.945$ (3 s.f.) <u>Since r = 0.945 is close to 1</u>, there is a <u>strong positive linear correlation</u> between the <u>maximum ambient temperature</u> and the <u>mean ambient temperature</u>.
9(iii) **[2]**	Using GC, $y=0.42x+14.326$ $y=0.42x+14.33$ (2 dp)

(34.0, 28.8)

Qn	Solution
	 **Note:** It is recommended to sketch the regression line at the same time the scatter diagram is sketched in part **(i)**.
9(iv) **[2]**	When $x = 31$, $y = 0.42\left(31\right) + 14.326$ $y = 27.346 = 27.3$ (3 s.f.) Since $r = 0.945$ is close to 1 and $x = 31$ is within the data range, the estimate is reliable.

Qn	Solution
10(i) **[3]**	
10(ii) **[2]**	P(all three offspring are of same gender) = P(MMM) + P(FFF) $= 0.5 \times 0.65 \times 0.65 + 0.5 \times 0.65 \times 0.65$ $= 0.4225$

Qn	Solution
10(iii) **[3]**	P(first offspring is a male| at least two female offspring) $= \dfrac{\text{P(first offspring is a male} \cap \text{at least two female offspring)}}{\text{P(at least two female offspring)}}$ $= \dfrac{\text{P(MFF)}}{\text{P(MFF)+P(FMF)+P(FFM)+P(FFF)}}$ $= \dfrac{0.5\times0.35\times0.65}{0.5\times0.35\times0.65+0.5\times0.35\times0.35+0.5\times0.65\times0.35+0.5\times0.65\times0.65}$ $=0.2275$
10(iv) **[2]**	<u>Method 1</u> Required probability $=0.4225^2\times(1-0.4225)\times3$ $=0.309$ (3 s.f.) <u>Method 2</u> Let Y be the number of families with all three offspring of the same gender out of 3 families. Y~B(3, 0.4225) P(Y = 2) = 0.309 (3 s.f.)

Qn	Solution
11(i) **(a)** **[1]**	 Let X be the mass of one packet of Espresso Roast coffee beans. Then $X\sim \mathrm{N}(\mu,\sigma^2)$. $\mathrm{P}(X>\mu+5)=\mathrm{P}(X<\mu-5)=0.15$ $\mathrm{P}(X<\mu+5)=1-\mathrm{P}(X>\mu+5)$ $=1-0.15$ $=0.85$
11(i) **(b)** **[3]**	$\mathrm{P}(X<\mu-5)=0.15$ $\mathrm{P}\left(\dfrac{X-\mu}{\sigma}<\dfrac{\mu-5-\mu}{\sigma}\right)=0.15$ $\mathrm{P}\left(Z<\dfrac{-5}{\sigma}\right)=0.15$ $\quad$ $Z\sim \mathrm{N}(0,1)$ Using GC, $-\dfrac{5}{\sigma}=-1.0364$ $\sigma=4.8242=4.8$ (1d.p.)

Qn	Solution
11(ii) **[3]**	$X \sim \mathrm{N}\left(200, 5^2\right)$ Given $\mathrm{P}\left(X_1 + X_2 + X_3 < k\right) = 0.3$. $\mathrm{E}\left(X_1 + X_2 + X_3\right) = 3 \times 200 = 600$ $\mathrm{Var}\left(X_1 + X_2 + X_3\right) = 3 \times 5^2 = 75$ $X_1 + X_2 + X_3 \sim \mathrm{N}\left(600, 75\right)$ Using GC, $k = 595.46 = 595$ (3s.f.)
11(iii) **[3]**	Let Y be the mass of one packet of Filter Roast coffee beans. Then $Y \sim \mathrm{N}\left(205, 8^2\right)$. We want to find $\mathrm{P}\left(X < Y\right) = \mathrm{P}\left(X - Y < 0\right)$ $\mathrm{E}\left(X - Y\right) = 200 - 205 = -5$ $\mathrm{Var}\left(X - Y\right) = 5^2 + 8^2 = 89$ $X - Y \sim \mathrm{N}\left(-5, 89\right)$ $\mathrm{P}\left(X - Y < 0\right) = 0.70194 = 0.702$ (3s.f.)
11(iv) **[3]**	**Method 1** We want to find $\mathrm{P}\left(0.95\left(X_1 + X_2 + Y\right) > 580\right)$ i.e. $\mathrm{P}\left(X_1 + X_2 + Y > 610.53\right)$ $\mathrm{E}\left(X_1 + X_2 + Y\right) = 2 \times 200 + 205 = 605$ $\mathrm{Var}\left(X_1 + X_2 + Y\right) = 2 \times 5^2 + 8^2 = 114$ $X_1 + X_2 + Y \sim \mathrm{N}\left(605, 114\right)$ $\mathrm{P}\left(0.95\left(X_1 + X_2 + Y\right) > 580\right) = 0.30225 = 0.302$ (3s.f.) **Method 2** Mass of one packet of ground Espresso Roast coffee beans: $0.95X$ Mass of one packet of ground Filter Roast coffee beans: $0.95Y$ Total mass of 2 packets of ground Espresso Roast coffee beans and 1 packet of ground Filter Roast coffee beans: $0.95\left(X_1 + X_2 + Y\right)$ We want to find $\mathrm{P}\left(0.95\left(X_1 + X_2 + Y\right) > 580\right)$ $\mathrm{E}\left(0.95\left(X_1 + X_2 + Y\right)\right) = 0.95\left(2 \times 200 + 205\right) = 574.75$

Qn	Solution
	$\text{Var}\left(0.95\left(X_1+X_2+Y\right)\right)=0.95^2\left(2\times5^2+8^2\right)=102.885$ $0.95\left(X_1+X_2+Y\right)\sim \text{N}\left(574.75,102.885\right)$ $\text{P}\left(0.95\left(X_1+X_2+Y\right)>580\right)=0.30237=0.302$ (3s.f.)

Qn	Solution
12(i) **[2]**	A random sample means that every day between May and July for the past three years has **equal probability of being selected** to form the sample. The <u>selection of one day</u> is <u>independent</u> of the selection of another day.
12(ii) **[2]**	Unbiased estimate of population mean, $\bar{x}$ $=\dfrac{2104.2}{60}=35.07$ Unbiased estimate of population variance, s^2 $=\dfrac{1}{60-1}\left[74698.8-\dfrac{\left(2104.2\right)^2}{60}\right]=15.331\approx15.3$
12(iii) **[1]**	Let X be the mid-day temperature of a randomly chosen day between May and July for the past three years. Let μ be the population mean mid-day temperature between May and July for the past three years. H_o: $\mu=33.9$ H_1: $\mu>33.9$
12(iv) **[3]**	Under H_o, since n = 60 is large, by Central Limit Theorem, $\bar{X}\sim \text{N}\left(33.9,\dfrac{15.331}{60}\right)$ approximately. Use z-test at 5% level of significance, Using GC, the test statistics $\bar{x}=35.07$ gives $z_{calc}=2.3146$ and p-value = = 0.010317 ≈0.0103 Using GC, since p-value = 0.010317 $\approx0.0103\leq$ 0.05 Since p-value = 0.0103 $\leq$ 0.05, we reject H_0 and conclude that there is sufficient evidence at 5% level of significance to conclude that the climate is getting hotter.
12(v) **[2]**	Does not indicate that the mean mid-day temperature at this place is getting hotter => Do not reject Ho. p-value > $\dfrac{\alpha}{100}$ 0.010317 > $\dfrac{\alpha}{100}$

Qn	Solution
	$0 < \alpha < 1.03$ (3 s.f.)
12(vi) **[3]**	Let Y be the mid-day temperature of a randomly chosen day between May and July. $Y \sim \mathrm{N}\left(33.9,\ 2.3^2\right)$ $\mathrm{H_o}$: $\mu = 33.9$ $\mathrm{H_1}$: $\mu > 33.9$ Under $\mathrm{H_o}$, $\bar{Y} \sim \mathrm{N}\left(33.9, \frac{2.3^2}{60}\right)$. Test Statistic, $Z = \frac{\bar{Y} - 33.9}{\sqrt{\frac{2.3^2}{60}}} \sim \mathrm{N}(0,1)$ Use z-test at 5% level of significance. Since the test indicated that the climate at this place is hotter ⇨ Reject $\mathrm{H_o}$ <u>Method 1 (Using $\bar{Y}$ distribution):</u> Critical value: 34.388 Rejection region: $\bar{y} \geq 34.388$ Since we reject $\mathrm{H_o}$, test statistic value is within the rejection region. $\therefore k \geq 34.4$ (3 s.f.) <u>Method 2 (Using standardized test statistic):</u> Corresponding test statistic value: $z = \frac{k - 33.9}{\sqrt{\frac{2.3^2}{60}}}$ Critical value: 1.6449 Rejection region: $z \geq 1.6449$ Since we reject $\mathrm{H_o}$, test statistic value is within the rejection region. $z = \frac{k - 33.9}{\sqrt{\frac{2.3^2}{60}}} \geq 1.6449$ $k \geq 1.6449 \times \sqrt{\frac{2.3^2}{60}} + 33.9$ $k \geq 34.388$ $k \geq 34.4$ (3 s.f.)