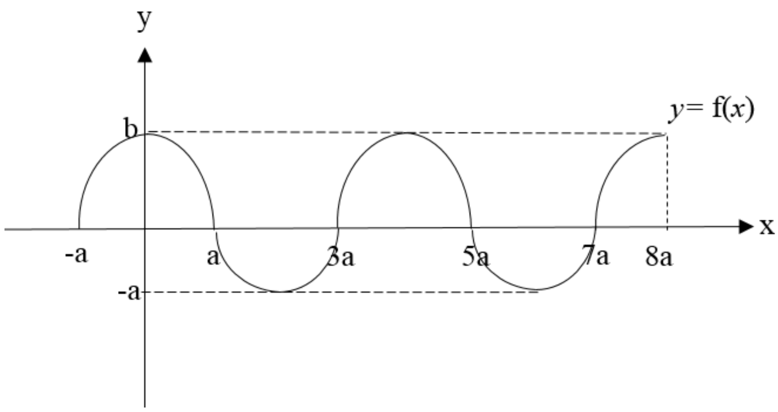
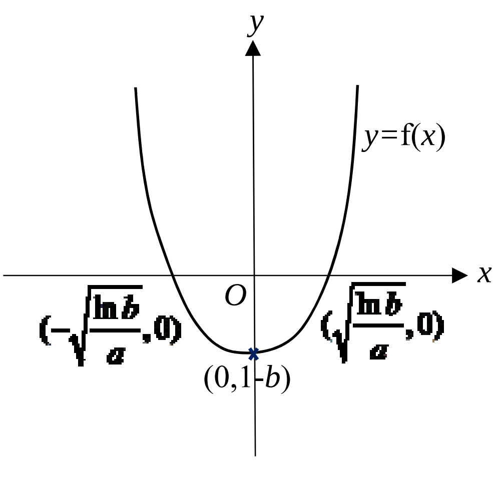
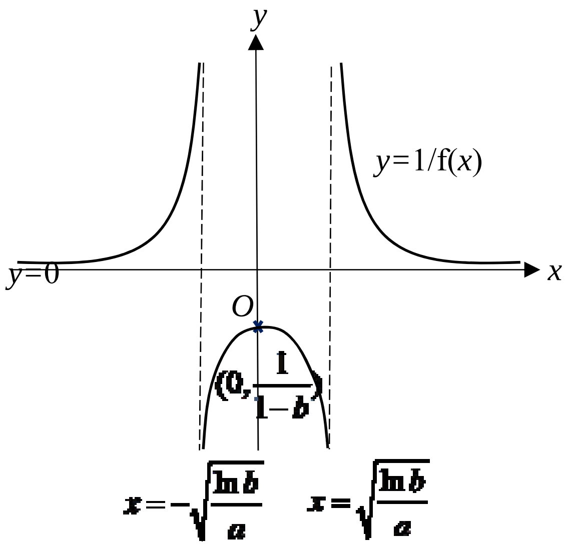
2017 Prelim Paper 1 Solutions

1	$V=\frac{4}{3}\pi r^3$ $\frac{\mathrm{d}V}{\mathrm{d}t}=4\pi r^2\frac{\mathrm{d}r}{\mathrm{d}t}$ When V=20, $20=\frac{4}{3}\pi r^3$ $r=\left(\frac{15}{\pi}\right)^{\frac{1}{3}}$ When $r=\left(\frac{15}{\pi}\right)^{\frac{1}{3}}$, $\frac{\mathrm{d}V}{\mathrm{d}t}=\lambda$. $\lambda=4\pi\left(\frac{15}{\pi}\right)^{\frac{2}{3}}\frac{\mathrm{d}r}{\mathrm{d}t}$ $\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{\lambda}{4\pi}\left(\frac{\pi}{15}\right)^{\frac{2}{3}}$ Surface Area, $A=4\pi r^2$ $\frac{\mathrm{d}A}{\mathrm{d}t}=8\pi r\frac{\mathrm{d}r}{\mathrm{d}t}$ When $r=\left(\frac{15}{\pi}\right)^{\frac{1}{3}}, \frac{\mathrm{d}r}{\mathrm{d}t}=\frac{\lambda}{4\pi}\left(\frac{\pi}{15}\right)^{\frac{2}{3}}$. $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}$ $=8\pi\left(\frac{15}{\pi}\right)^{\frac{1}{3}}\frac{\lambda}{4\pi}\left(\frac{\pi}{15}\right)^{\frac{2}{3}}$ $=2\lambda\left(\frac{\pi}{15}\right)^{\frac{1}{3}}\ \mathrm{cm}^2/\mathrm{s}$
2	$BC = BD + DC$ $=\frac{h}{\tan\frac{\pi}{3}}+\frac{h}{\tan\left(\frac{\pi}{4}+x\right)}$
	BC $=\frac{h}{\sqrt{3}}+\frac{h}{\frac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}}$

	$= \frac{h\sqrt{3}}{3} + \frac{h(1-\tan x)}{1+\tan x}$ $\approx \frac{h\sqrt{3}}{3} + \frac{h(1-x)}{1+x}$ $= \frac{h\sqrt{3}}{3} + h(1-x)(1+x)^{-1}$ $= \frac{h\sqrt{3}}{3} + h(1-x)[1+(-1)x+\frac{(-1)(-2)}{2!})x^2+...]$ $= \frac{h\sqrt{3}}{3} + h(1-x)[1-x+x^2+...]$ $= \frac{h\sqrt{3}}{3} + h(1-2x+2x^2+...)$ $\approx h\left(1+\frac{\sqrt{3}}{3} - 2x + 2x^2\right)$
3 (i)	

(ii)	$\int_{3a}^{4a} \mathrm{f}(x)\ \mathrm{d}x$ $=\int_{-a}^{0} b\sqrt{1-\frac{x^2}{a^2}}\ \mathrm{d}x$ $=b\int_{\pi}^{\frac{\pi}{2}} \sqrt{1-\frac{a^2\cos^2\theta}{a^2}}\ (-a\sin\theta)\ \mathrm{d}\theta$ $=ab\int_{\frac{\pi}{2}}^{\pi} \sin^2\theta\ \mathrm{d}\theta$ $=ab\int_{\frac{\pi}{2}}^{\pi} \frac{1-\cos 2\theta}{2}\ \mathrm{d}\theta$ $=\frac{ab}{2}\left[\theta-\frac{\sin 2\theta}{2}\right]_{\frac{\pi}{2}}^{\pi}$ $=\frac{ab}{2}\left[\pi-\frac{\pi}{2}\right]$ $=\frac{\pi}{4}ab$
4 (i)	$y=\mathrm{e}^{ax^2}-b=\mathrm{e}^{\left(\sqrt{a}x\right)^2}-b$ If $\mathrm{f}(x)=\mathrm{e}^{x^2}$, then $\mathrm{f}\left(\sqrt{a}x\right)=\mathrm{e}^{\left(\sqrt{a}x\right)^2}$ and so $y=\mathrm{f}(x)\to y=\mathrm{f}\left(\sqrt{a}x\right)\to y=\mathrm{f}\left(\sqrt{a}x\right)+b$ Hence the sequence of transformations are: 1. Scale by a factor of $\frac{1}{\sqrt{a}}$ parallel to the *x*-axis, 2. Translate the resulting curve by *b* units in the negative *y*-direction.
(ii)	



5 (i)

Since $\mathbf{u}+\mathbf{v}-\mathbf{w}$ is perpendicular to $\mathbf{u}-\mathbf{v}+\mathbf{w}$,

$$(\mathbf{u}+\mathbf{v}-\mathbf{w})\cdot(\mathbf{u}-\mathbf{v}+\mathbf{w})=0$$

$$\mathbf{u}\cdot\mathbf{u}-\mathbf{u}\cdot\mathbf{v}+\mathbf{u}\cdot\mathbf{w}$$
$$+\mathbf{v}\cdot\mathbf{u}-\mathbf{v}\cdot\mathbf{v}+\mathbf{v}\cdot\mathbf{w}$$
$$-\mathbf{w}\cdot\mathbf{u}+\mathbf{w}\cdot\mathbf{v}-\mathbf{w}\cdot\mathbf{w}=0$$

Since $\mathbf{u}\cdot\mathbf{u}=|\mathbf{u}|^2, \mathbf{v}\cdot\mathbf{v}=|\mathbf{v}|^2, \mathbf{w}\cdot\mathbf{w}=|\mathbf{w}|^2$, and $\mathbf{u}\cdot\mathbf{v}=\mathbf{v}\cdot\mathbf{u}, \mathbf{u}\cdot\mathbf{w}=\mathbf{w}\cdot\mathbf{u}, \mathbf{v}\cdot\mathbf{w}=\mathbf{w}\cdot\mathbf{v}$,

$$|\mathbf{u}|^2-|\mathbf{v}|^2-|\mathbf{w}|^2+2\mathbf{v}\cdot\mathbf{w}=0$$

Since $\mathbf{u},\mathbf{v},\mathbf{w}$ are unit vectors, $|\mathbf{u}|=1, |\mathbf{v}|=1, |\mathbf{w}|=1$,

$$1-1-1+2\mathbf{v}\cdot\mathbf{w}=0$$

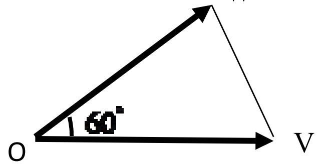
$$\mathbf{v}\cdot\mathbf{w}=\frac{1}{2}$$

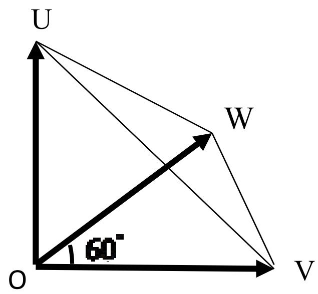
$$|\mathbf{v}||\mathbf{w}|\cos\theta=\frac{1}{2}$$

$$\cos\theta=\frac{1}{2}$$

Hence, $\theta=60^\circ$

5 (ii)



	Area of $\triangle$OVW $=\left(\frac{1}{2}(OV)(OW)\sin 60^\circ\right)$ $=\left(\frac{1}{2}\right)(1)(1)\left(\frac{\sqrt{3}}{2}\right)$ $=\frac{\sqrt{3}}{4}$ units^2
5 (iii)	 Since $\mathbf{u}$ and $\mathbf{v}\times\mathbf{w}$ are parallel, we have $OU \perp OV, OU \perp OW$. Volume of $OUVW$ $=\frac{1}{3}(\text{Area of }\triangle\text{OVW})(OU)$ $=\frac{1}{3}\left(\frac{\sqrt{3}}{4}\right)(1)$ $=\frac{\sqrt{3}}{12}$ units^3
6 (a)	**(i)** Using integration by parts, $\int e^x \cos nx \, dx$ $u = e^x$ $\quad$ $\frac{dv}{dx} = \cos nx$ $\frac{du}{dx} = e^x$ $\quad$ $v = \frac{\sin nx}{n}$ $= e^x\left(\frac{\sin nx}{n}\right) - \int \frac{e^x}{n}\sin nx \, dx$ $u = e^x$ $\quad$ $\frac{dv}{dx} = \sin nx$ $\frac{du}{dx} = e^x$ $\quad$ $v = -\frac{\cos nx}{n}$

$$= e^x\left(\frac{\sin nx}{n}\right) - \frac{1}{n}\left[-\frac{e^x \cos nx}{n} + \int \frac{e^x \cos nx}{n}\,dx\right]$$

$$= e^x\left(\frac{\sin nx}{n}\right) + \frac{1}{n}\left(\frac{e^x \cos nx}{n}\right) - \frac{1}{n^2}\int e^x \cos nx\,dx$$

Rearranging,

$$\left(1+\frac{1}{n^2}\right)\int e^x \cos nx\,dx = e^x\left(\frac{\sin nx}{n}\right) + \frac{1}{n}\left(\frac{e^x \cos nx}{n}\right)$$

$$\int e^x \cos nx\,dx = \left(\frac{n^2}{1+n^2}\right)\left[e^x\left(\frac{\sin nx}{n}\right) + \frac{1}{n}\left(\frac{e^x \cos nx}{n}\right)\right] + c$$

where c is a constant

(ii) $\int_{\pi}^{2\pi} e^x \cos nx\,dx = \left(\frac{n^2}{1+n^2}\right)\left[e^x\left(\frac{\sin nx}{n}\right) + \frac{1}{n}\left(\frac{e^x \cos nx}{n}\right)\right]_{\pi}^{2\pi}$

$$= \left(\frac{n^2}{1+n^2}\right)\left\{e^{2\pi}\left[\left(\frac{\sin 2n\pi}{n}\right) + \frac{1}{n}\left(\frac{\cos 2n\pi}{n}\right)\right] - e^{\pi}\left[\left(\frac{\sin n\pi}{n}\right) + \frac{1}{n}\left(\frac{\cos n\pi}{n}\right)\right]\right\}$$

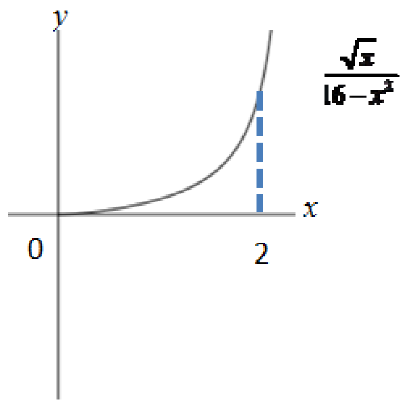
For any positive integer n, $\sin 2n\pi = 0$ and $\cos 2n\pi = 1$

If n is odd, $\sin n\pi = 0$ and $\cos n\pi = -1$

$$\int_{\pi}^{2\pi} e^x \cos nx\,dx = \left(\frac{n^2}{1+n^2}\right)\left[e^{2\pi}\left(0+\frac{1}{n^2}\right) - e^{\pi}\left(0-\frac{1}{n^2}\right)\right]$$

$$= \left(\frac{1}{1+n^2}\right)\left(e^{2\pi} + e^{\pi}\right) \text{ (Ans)}$$

6 (b)



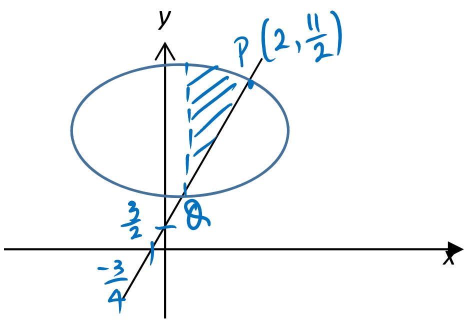
$$y = \frac{\sqrt{x}}{16-x^2} \Rightarrow y^2 = \frac{x}{\left(16-x^2\right)^2}$$

Hence volume required

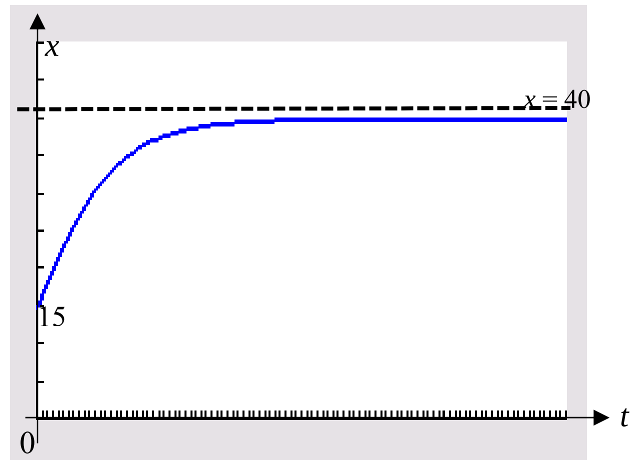
	$= \pi r^2 h - \pi \int_0^2 y^2 \mathrm{d}x$ $= \pi \left(\frac{\sqrt{2}}{12}\right)^2 (2) - \pi \int_0^2 \frac{x}{\left(16-x^2\right)^2} \mathrm{d}x$ $= \pi \left(\frac{\sqrt{2}}{12}\right)^2 (2) - \frac{\pi}{(-2)} \int_0^2 \frac{-2x}{\left(16-x^2\right)^2} \mathrm{d}x$ $= \pi \left(\frac{\sqrt{2}}{12}\right)^2 (2) + \frac{\pi}{2} \left[\frac{\left(16-x^2\right)^{-1}}{-1}\right]_0^2$ $= \frac{4}{144}\pi + \frac{\pi}{2}\left[-\frac{1}{12} + \frac{1}{16}\right]$ $= \frac{5\pi}{288} \text{ units}^3$
7 (i)	$\frac{r\mathrm{e}^{\mathrm{i}\theta}}{r\mathrm{e}^{\mathrm{i}\theta} - r}$ $= \frac{\mathrm{e}^{\mathrm{i}\theta}}{\mathrm{e}^{\mathrm{i}\left(\frac{\theta}{2}\right)}\left(\mathrm{e}^{\mathrm{i}\left(\frac{\theta}{2}\right)} - \mathrm{e}^{-\mathrm{i}\left(\frac{\theta}{2}\right)}\right)}$ $= \frac{\mathrm{e}^{\mathrm{i}\left(\frac{\theta}{2}\right)}}{2\mathrm{i}\sin\left(\frac{\theta}{2}\right)}$ $= \frac{\cos\left(\frac{\theta}{2}\right) + \mathrm{i}\sin\left(\frac{\theta}{2}\right)}{2\mathrm{i}\sin\left(\frac{\theta}{2}\right)}$ $= \frac{1}{2} + \frac{1}{2\mathrm{i}}\cot\left(\frac{\theta}{2}\right)$ $= \frac{1}{2} - \frac{1}{2}\left(\cot\left(\frac{\theta}{2}\right)\right)\mathrm{i}$
(ii)	$z = 2\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{3}\right)}$

(iii)	$\dfrac{4w^2}{(w-1)^2}-\dfrac{4w}{w-1}+4=0$ $\left(\dfrac{2w}{w-1}\right)^2-2\left(\dfrac{2w}{w-1}\right)+4=0$ Let $z=\dfrac{2w}{w-1}$, then $z^2-2z+4=0$ From (ii) the solutions are $z=2e^{i\left(\frac{\pi}{3}\right)}$ or $z=2e^{-i\left(\frac{\pi}{3}\right)}$ Since $z=\dfrac{2w}{w-1}$ $zw-z=2w$ $w(z-2)=z$ $w=\dfrac{z}{z-2}$ Part (i) result can be used as $z=2e^{i\left(\frac{\pi}{3}\right)}$, where $r=2$ with $\theta=\dfrac{\pi}{3},\ \theta=-\dfrac{\pi}{3}$. $w=\dfrac{1}{2}-\dfrac{1}{2}\left(\cot\dfrac{\pi}{6}\right)i$ or $w=\dfrac{1}{2}-\dfrac{1}{2}i\cot\left(-\dfrac{\pi}{6}\right)$ $w=\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i$ or $w=\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$
8 (i)	Distance travelled per lap is in AP: $a = 2(30) = 60$, $d = 2 \times 3 = 6$. Given total distance travelled > 3000 $\dfrac{n}{2}[\,2(60) + (n-1)6] > 3000$ $3n^2 + 57n - 3000 > 0$ $(n + 42.52)(n - 23.52) > 0$ $n < -42.52$ or $n > 23.52$ Since $n \in \mathbf{Z}^+$, least $n = 24$

8 (ii)	Distance of the coach from S just before the runner completes the rth lap $=30+2(3^0)+2(3^1)+2(3^2)+...+2(3^{r-2})$ $=30+2\left(1+3+3^2+....+3^{r-2}\right)$ $=30+2\left(\frac{3^{r-1}-1}{3-1}\right)$ $=30+(3^{r-1}-1)$ $=3^{r-1}+29$
	Distance covered by the athlete after n laps $=\sum_{r=1}^{n}2\left(3^{r-1}+29\right)$ $=2\sum_{r=1}^{n}3^{r-1}+\sum_{r=1}^{n}(58)$ $=2\sum_{r=1}^{n}3^{r-1}+58n$ $=2\left(\frac{3^n-1}{3-1}\right)+58n$ $=\left(3^n-1\right)+58n$
	When D = 8000m $8000=\left(3^n-1\right)+58n$ From GC, $n=8.1254$ Hence the athlete has run 8 complete laps. The athlete has completed 7024 m Hence he still have 8000-7024=976 m On the 9th lap, the coach is $3^{9-1}+29=6590$ m from S. Hence the athlete would be 6590- 976 = 5614 m away from the coach once he finishes 8 km.
9 (i)	$\frac{dx}{d\theta}=2\cos\theta$, $\frac{dy}{d\theta}=-\sqrt{3}\sin\theta$ $\frac{dy}{dx}=\frac{-\sqrt{3}\sin\theta}{2\cos\theta}=-\frac{\sqrt{3}}{2}\tan\theta$ When $\theta=\frac{\pi}{6}$, $x=2$, $y=\frac{11}{2}$, $\frac{dy}{dx}=-\frac{1}{2}$ Equation of normal : $y-\left(\frac{11}{2}\right)=2\left(x-2\right)$ $y=2x+\frac{3}{2}$
(ii)	$x=1+2\sin\theta\cdots\cdots(1)$ $y=4+\sqrt{3}\cos\theta\cdots\cdots(2)$ Substitute equation (1) and (2) into $y=2x+\frac{3}{2}$

	$4+\sqrt{3}\cos\theta = 2(1+2\sin\theta)+\dfrac{3}{2}$ $\dfrac{1}{2}+\sqrt{3}\cos\theta = 4\sin\theta$ $8\sin\theta - 2\sqrt{3}\cos\theta = 1$ At Point Q, $\theta=\alpha$ $8\sin\alpha - 2\sqrt{3}\cos\alpha = 1$ (shown) Using GC: $\alpha=-2.847916$ or $\alpha = 0.52359$ (Reject, same as $\dfrac{\pi}{6}$,point P) Hence, using GC coordinates of Q (0.42105, 2.3421) Q (0.421, 2.34)
(iii)	 when $x=0.42105$ $0.42105=1+2\sin\theta$ $\sin\theta=-0.289475$ $\theta=-0.29368$ or -2.8479 (at point Q) Required Area $=\int_{0.42105}^{2} y_1 \, dx - \int_{0.42105}^{2} y_2 \, dx$ $=\int_{-0.29368}^{\frac{\pi}{6}} \left(4+\sqrt{3}\cos\theta\right) \left(2\cos\theta\right) d\theta - \int_{0.42105}^{2} \left(2x+\dfrac{3}{2}\right) dx$ $=8.9613-6.1911$ $=2.7702 \approx 2.77$ units^2 (3 s.f.)
10 (i)	$\dfrac{dx}{dt} = cx\ln\left(\dfrac{40}{x}\right)$

	$u=\ln\left(\frac{40}{x}\right)$ $=\ln(40)-\ln(x)$ $\frac{\mathrm{d}u}{\mathrm{d}x}=-\frac{1}{x}$ $\frac{\mathrm{d}u}{\mathrm{d}t}=\frac{\mathrm{d}u}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}t}$ $=\left(-\frac{1}{x}\right)cx\ln\left(\frac{40}{x}\right)$ $=-cu$
(ii)	$\frac{\mathrm{d}u}{\mathrm{d}t}=-cu$ $\int\frac{1}{u}\,\mathrm{d}u=-\int c\ \mathrm{d}t$ $\ln\lvert u\rvert=-ct+d$ $\lvert u\rvert=\mathrm{e}^{-ct+d}$ $u=\pm\mathrm{e}^{d}\mathrm{e}^{-ct}$ $=B\mathrm{e}^{-ct},\ B=\pm\mathrm{e}^{d}$ Replace u with $\ln\left(\frac{40}{x}\right)$ $\ln\left(\frac{40}{x}\right)=B\mathrm{e}^{-ct}$ $\frac{40}{x}=\mathrm{e}^{B\mathrm{e}^{-ct}}$ $x=\frac{40}{\mathrm{e}^{B\mathrm{e}^{-ct}}}$ $x=40\mathrm{e}^{-B\mathrm{e}^{-ct}}$
(iii)	When $t=0,\ x=15,$ $15=40e^{-B}$ $\mathrm{e}^{-B}=\frac{3}{8}$ $B=\ln\left(\frac{8}{3}\right)=0.98083=0.981$ When $t=3,\ x=20$

	$20 = 40e^{-Be^{-3c}}$ $e^{-Be^{-3c}} = \frac{1}{2}$ $-Be^{-3c} = \ln\frac{1}{2}$ $\ln\left(\frac{3}{8}\right)\left(e^{-3c}\right) = \ln\left(\frac{1}{2}\right)$ $c = -\frac{1}{3}\ln\left(\frac{\ln\left(\frac{1}{2}\right)}{\ln\left(\frac{3}{8}\right)}\right) = 0.11572 = 0.116$ $x = 40e^{-0.981e^{-0.116t}}$
(iv)	The population of foxes in the long run is 40.
(v)	
11 (i)	$\overrightarrow{OP} = \begin{pmatrix} 0 \\ 5 \\ 30 \end{pmatrix}$ $\overrightarrow{OQ} = \begin{pmatrix} 0 \\ 10 \\ 15 \end{pmatrix}$ $\overrightarrow{OR} = \begin{pmatrix} -10 \\ 10 \\ 30 \end{pmatrix}$ $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \begin{pmatrix} 0 \\ 10 \\ 15 \end{pmatrix} - \begin{pmatrix} 0 \\ 5 \\ 30 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ -15 \end{pmatrix}$ $\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP} = \begin{pmatrix} -10 \\ 10 \\ 30 \end{pmatrix} - \begin{pmatrix} 0 \\ 5 \\ 30 \end{pmatrix} = \begin{pmatrix} -10 \\ 5 \\ 0 \end{pmatrix}$
(ii)	A normal to p

	$\begin{pmatrix}0\\1\\-3\end{pmatrix}\times\begin{pmatrix}-2\\1\\0\end{pmatrix}=\begin{pmatrix}3\\6\\2\end{pmatrix}$ Equation of plane $\mathbf{r}\cdot\begin{pmatrix}3\\6\\2\end{pmatrix}=\begin{pmatrix}0\\10\\15\end{pmatrix}\cdot\begin{pmatrix}3\\6\\2\end{pmatrix}$ $\mathbf{r}\cdot\begin{pmatrix}3\\6\\2\end{pmatrix}=90$ $3x+6y+2z=90$ Or any equivalent equation of plane
(iii)	A normal to the plane $EFGH=\begin{pmatrix}0\\0\\1\end{pmatrix}$ (or any equivalent vector) $\cos\theta=\dfrac{\left\|\begin{pmatrix}0\\0\\1\end{pmatrix}\bullet\begin{pmatrix}3\\6\\2\end{pmatrix}\right\|}{1\times\sqrt{9+36+4}}=\dfrac{\|2\|}{\sqrt{49}}$ $\theta=73.4^0$
(iv)	$\overrightarrow{OS}=\frac{1}{2}\left[\overrightarrow{OQ}+\overrightarrow{OR}\right]=\frac{1}{2}\left[\begin{pmatrix}0\\10\\15\end{pmatrix}+\begin{pmatrix}-10\\10\\30\end{pmatrix}\right]=\begin{pmatrix}-5\\10\\22\frac{1}{2}\end{pmatrix}$

	$\overrightarrow{OT} = \dfrac{4\begin{pmatrix}-5\\10\\22\frac{1}{2}\end{pmatrix}+\begin{pmatrix}0\\5\\30\end{pmatrix}}{4+1}$ $=\frac{1}{5}\begin{pmatrix}-20\\45\\120\end{pmatrix}=\begin{pmatrix}-4\\9\\24\end{pmatrix}$ Hence the coordinates of T are $(-4,\ 9,\ 24)$.
(v)	Equation of the drill line $\mathbf{r}=\begin{pmatrix}-4\\9\\24\end{pmatrix}+\lambda\begin{pmatrix}3\\6\\2\end{pmatrix},\ \lambda\in\mathbb{R}$.
(vi)	Shortlist the possible planes: *ODGC*, *GCBF*, *OABC* Equation of Plane *ODGC* $\mathbf{r}\cdot\begin{pmatrix}0\\1\\0\end{pmatrix}=0$ Equation of Plane *OABC* $\mathbf{r}\cdot\begin{pmatrix}0\\0\\1\end{pmatrix}=0$ Equation of Plane *GCBF* $\mathbf{r}\cdot\begin{pmatrix}1\\0\\0\end{pmatrix}=-20$ If the line of the drill exits from the cuboid, all of the following conditions must be satisfied: $-20\le x\le 0;\ 0\le y\le 10;\ 0\le z\le 30$. The intersection of plane *ODGC* $\begin{pmatrix}-4+3\lambda\\9+6\lambda\\24+2\lambda\end{pmatrix}\cdot\begin{pmatrix}0\\1\\0\end{pmatrix}=0$ $9+6\lambda=0$ $\lambda=-\frac{3}{2}$

Position vector is $\begin{pmatrix} -4+3\left(-\frac{3}{2}\right) \\ 9+6\left(-\frac{3}{2}\right) \\ 24+2\left(-\frac{3}{2}\right) \end{pmatrix} = \begin{pmatrix} -\frac{17}{2} \\ 0 \\ 21 \end{pmatrix}$

Hence the point of intersection has coordinates $\left(-\frac{17}{2},\ 0,\ 21\right)$.

Hence the drill line will exit from the side *ODGC*.

The intersection of plane *OABC*

$$\begin{pmatrix} -4+3\lambda \\ 9+6\lambda \\ 24+2\lambda \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0$$

$$24+2\lambda = 0$$

$$\lambda = -12$$

Position vector is $\begin{pmatrix} -4+3(-12) \\ 9+6(-12) \\ 24+2(-12) \end{pmatrix} = \begin{pmatrix} -40 \\ -63 \\ 0 \end{pmatrix}$

Hence the point of intersection has coordinates $(-40,\ -63,\ 0)$

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Hence the drill line will not exit from the side *OABC*.

The intersection of plane *GCBF*

$$\mathbf{r} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = -20$$

$$\begin{pmatrix} -4+3\lambda \\ 9+6\lambda \\ 24+2\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = -20$$

$$-4+3\lambda = -20$$

$$\lambda = -\frac{16}{3}$$

	Position vector is $\begin{pmatrix} -4+3\left(-\frac{16}{3}\right) \\ 9+6\left(-\frac{16}{3}\right) \\ 24+2\left(-\frac{16}{3}\right) \end{pmatrix} = \begin{pmatrix} -20 \\ -23 \\ \frac{40}{3} \end{pmatrix}$ Hence the point of intersection has coordinates $\left(-20,\ -23,\ \frac{40}{3}\right)$. Hence the drill line will not exit from the side *GCBF*.