Xinmin Secondary School
Math Department
Additional Mathematics: Calculus Practice

Name: ______________________________ (   )   Date: ___________

Routine Differentiation; Equation of Tangents and Normal

1 **XMS 2020 AM P2**

A curve has equation $y = 4\left(\sqrt{2x-3}\right)^3$, for $x > \frac{3}{2}$. It meets the line $x = 2$ at the point A.

(i) Find $\frac{dy}{dx}$. [1]

(ii) Find the equation of the normal to the curve at point A. [3]

(iii) Explain why $y = 4\left(\sqrt{2x-3}\right)^3$ is an increasing function. [1]

2 **MFSS 2022 AM P1**

A curve has the equation $y = \frac{2-x}{3x-4}$, $x \neq \frac{4}{3}$.

(i) Find an expression for $\frac{dy}{dx}$. [2]

(ii) Find the coordinates of the points on the curve where the normal is parallel to the line $2y - 16x = 3$. [5]

3 **PHS 2022 AM P1**

A curve has the equation $y = (x-3)\sqrt{2x+3}$, where $x > -\frac{3}{2}$.

(a) Show that $\frac{dy}{dx}$ can be expressed in the form $\frac{kx}{\sqrt{2x+3}}$ and state the value of k. [4]

(b) Find the equation of the tangent when $x = 11$. [3]

(c) Find the rate of change of x at the instant when $x = 11$, given that y is increasing at a rate of 5 units per second at this instant. [2]

4 **JSS 2022 AM P1**

It is given that $f(x) = Ax(e^{kx})$, where A and k are constants.

Find the exact values of A and k such that $f'(x) + 2ke^{kx} + 6f(x) = 0$. [6]

5 **PHS 2023 AM P1**

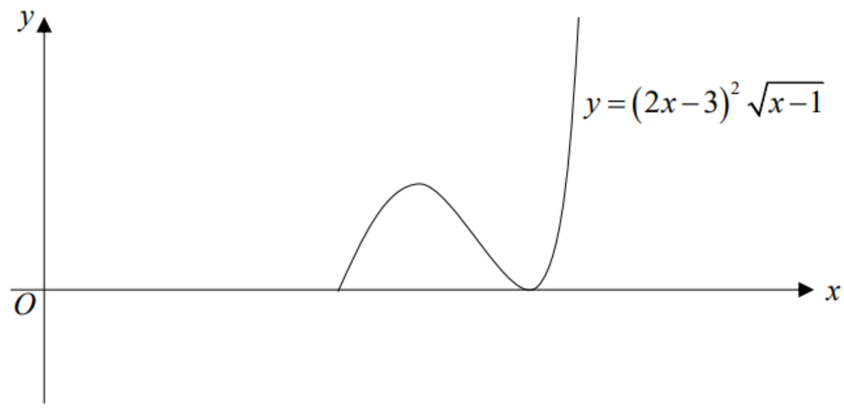
(a) The equation of a curve is $y = \ln\left(xe^{-3x}\right)$.

The normal to the curve at the point P has a gradient of $\frac{1}{2}$. Find the coordinates of P. [4]

(b) The normal to the curve at P meets the x-axis at Q.
Find the area of triangle OQP, where O is the origin. [3]

Differentiation: Increasing and Decreasing Function

1 **XMS 2022 AM P2**



The diagram shows part of the graph $y = (2x-3)^2\sqrt{x-1}$.

Find the range of values of x for which y is increasing. [5]

2 **PHS 2023 AM P1**

A curve has the equation $y = 3 + \left(\frac{x}{2} - 1\right)^4$. The point (p, q) is the stationary point on the curve.

(a) Determine the coordinates of the stationary point (p, q). [4]

(b) **(i)** Justify whether y is increasing or decreasing for values of x less than p. [2]

(ii) Hence infer whether y is increasing or decreasing for values of x greater than p. [1]

(c) What do the results of **part (b)** imply about the stationary point? [1]

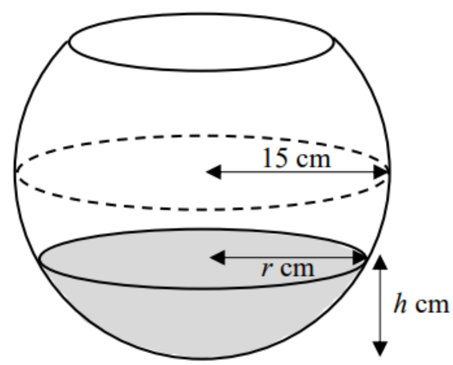
Differentiation: Connected Rate of Change

1 **XMS 2020 AM P1**

A round-bottomed container was initially filled with liquid that forms the shape of a hemisphere with radius 15 cm. However, there is a small crack at the bottom of the container and the liquid starts to drip out at a constant rate of 2π cm^3/s.

The volume, V cm^3, of the liquid in the container is given by $V = \pi h^2\left(15 - \frac{h}{3}\right)$, where h cm is the distance from the surface of the liquid to the bottom of the container.

At any instant, the surface of the liquid is in the shape of a circle with radius r cm.



(i) At $h = 12$ cm, find

(a) the value of $\frac{\mathrm{d}V}{\mathrm{d}h}$ in terms of π, [2]

(b) the rate at which the distance from the surface of the liquid to the bottom of the container is decreasing. [2]

(ii) Show that $r = \sqrt{30h - h^2}$. [2]

(iii) Hence, find the rate of change of r when $h = 12$. [3]

(iv) Using your answer in part (iii), find the rate of change in the area of the surface of the liquid when $h = 12$. [2]

2 **XMS 2022 AM P2**

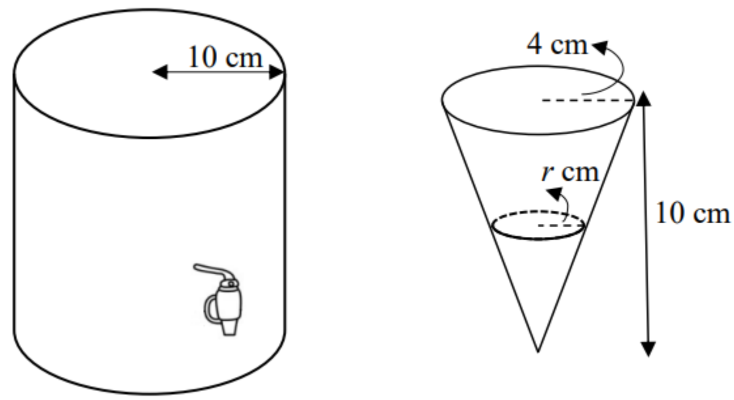


Diagram I **Diagram II**

Diagram I shows a water dispenser, in the shape of a cylinder of radius 10 cm, which dispenses water at a constant rate into an empty conical cup, as shown in diagram II, of radius 4 cm and height 10 cm. The depth of the water in the dispenser decreases at a rate of 0.0015 cm/s.
After t seconds, radius of the horizontal surface of the water in the conical cup is r cm.

(a) Show that the volume of water in the conical cup increases at a rate of $\frac{3\pi}{20}$ cm^3/s. [2]

(b) Express the volume of water in the conical cup in terms of r. [2]

(c) At the instant where the volume of the water in the conical cup is $\frac{20\pi}{3}$ cm^3, find the rate of change in the radius of the horizontal surface of the water in the conical cup. [4]

3 **MFSS 2022 AM P1**

An ice cube retains its shape during melting. When its length is x mm, the surface area, A, is decreasing at a rate of 10 mm^2/s. The volume, V, of the ice cube changes at the rate of $-45\,\text{mm}^3$/s,

(i) find the value of x. [5]

(ii) find the rate of change of x for the **value found in (i)**. [1]

4 FMSS 2022 AM P1

The total surface area of a spherical ice-ball is decreasing at a rate of $2\,\text{cm}^2/\text{s}$. Find the rate of change of the volume when its radius is 0.5 cm. [5]

[Volume of sphere = $\frac{4}{3}\pi r^3$; total surface area of sphere = $4\pi r^2$]

5 JSS 2022 AM P1

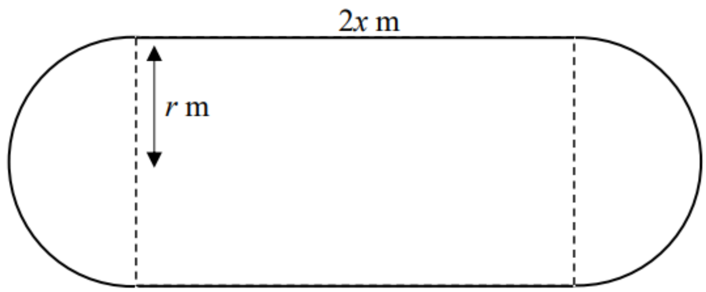
A curve has equation $y = \dfrac{4}{\sqrt{x+3}}$. A point (x, y) is moving along the curve.

Find the coordinates of the point at the instant where the y-coordinate is decreasing at a rate twice of the rate of increase of the x-coordinate. [5]

Differentiation: Maxima and Minima Problems

1 XMS 2022 AM P1

A gardener wants to use 40 m of fence to form a flower bed, which is made up of a rectangle of length $2x$ m and two identical semicircles of radius r m.



(a) Express r in terms of x. [2]

(b) Show that the total area, A m^2, of the flowerbed is given by [2]

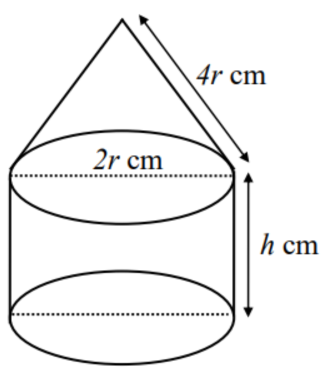
$$A = \frac{400 - 4x^2}{\pi}.$$

(c) Given that x can vary, find the value of x which gives a stationary value of A. [2]

(d) The gardener's wife claimed that he can optimise the length of the fencing to obtain a maximum area by forming only a circular flowerbed of radius $\frac{20}{\pi}$ m. Do you agree with her? Explain your answer with relevant workings. [2]

2 MFSS 2022 AM P2

The diagram shows a **solid** made up by a right circular cone and a cylinder of diameter $2r$ cm. The slant height of the cone is $4r$ cm and height of the cylinder is h cm.



(i) Given that the total surface area of the solid is 300 cm^2, express h in terms of r. [2]

(ii) Show that the volume, V cm^3, of the solid is given by $V = 150r + \left(\frac{\sqrt{15}}{3} - \frac{5}{2}\right)\pi r^3$. [3]

(iii) Given that r can vary, find the stationary value of V and determine whether this value of V is maximum or minimum. [5]

3 **FMSS 2022 AM P1**

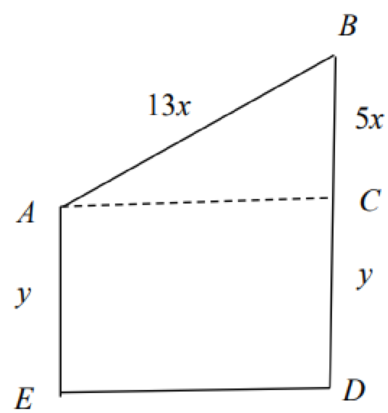
The curve $y = ax + \dfrac{b}{2x-1}$ has a stationary point at $P(2,7)$.

(a) Find the value of a and b. [4]

(b) Find the other stationary point of the curve. [3]

(c) Determine the nature of each stationary point. [2]

4 **PHS 2022 AM P1**



A piece of wire, l cm long, is bent to form the shape $ABCDE$ as shown in the diagram. $ACDE$ is a rectangle with $AE = y$ cm and ΔABC is a right-angled triangle with $AB = 13x$ cm and $BC = 5x$ cm.

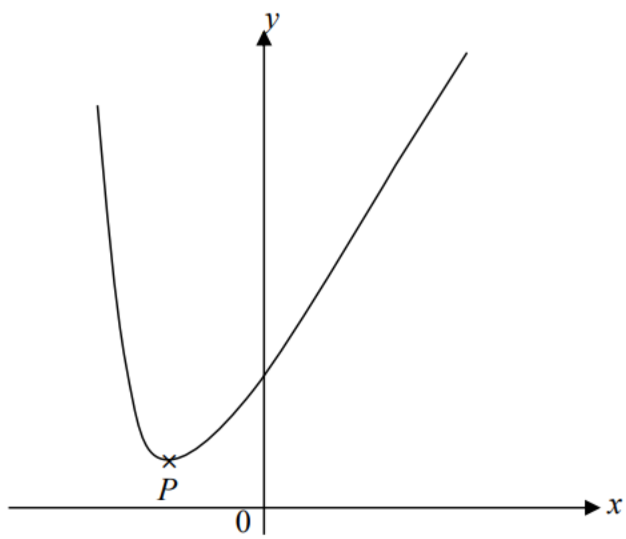
(a) Express l in terms of x and y. [1]

(b) Given that the area enclosed is 96 cm^2, show that $l = 25x + \dfrac{16}{x}$. [3]

(c) Find the value of x for which l has a stationary value and determine the nature of this stationary value. [3]

5 JSS 2022 AM P1

The diagram below shows part of the graph of $y = \dfrac{x^2 + 2x + 5}{x + 3}$.

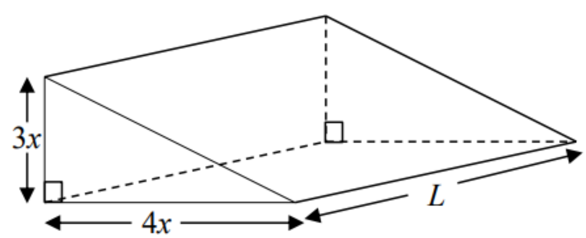


Find the coordinates of the minimum point P. [7]

6 JSS 2022 AM P2

The figure below shows a right-angled triangular prism.

The height and base of the triangular faces of the prism are $3x$ meters and $4x$ meters respectively. The length of the prism is L meters.



(a) Given that the volume of the prism is 240 m^3, show that the surface area of the prism, A m^2, is given by

$$A = 12x^2 + \frac{480}{x}.$$ [4]

(b) Given that x and L can vary, find the value of x for which A has a stationary value and determine whether this value of A is maximum or minimum. [5]