Name:______________________________ Class:__________

JURONG PIONEER JUNIOR COLLEGE

JC2 Preliminary Examination 2024

MATHEMATICS
Higher 2

Paper 1

9758/01
9 September 2024
3 hours

Candidates answer on the Question Paper.

Additional materials: List of Formulae (MF 26)

READ THESE INSTRUCTIONS FIRST

Write your name and civics class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given by [ ] at the end of each question or part question.

For Candidate's Use	For Examiner's Use
Question Number	Marks Obtained
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
Total Marks	/ 100

This document consists of **6** printed pages.

[Turn over

1 The first four terms of a sequence are given by $v_1 = -9$, $v_2 = 7$, $v_3 = 47$ and $v_4 = 141$. It is given that v_n is a cubic polynomial in n. Find v_n in terms of n. [3]

2 Without using a calculator, solve the inequality $\dfrac{11x-19}{x^2+2x-8} \le 2$. [4]

Hence solve $\dfrac{11\mathrm{e}^{-x}-19}{\mathrm{e}^{-2x}+2\mathrm{e}^{-x}-8} \le 2$, leaving your answer in the exact form. [2]

3 The region R is bounded by the lines $y=0$, $y=\dfrac{5}{2}\sqrt{3}$ and the curve $x^2=\sqrt{25-y^2}$. R is rotated about the y-axis through π radians. Using the substitution $y=5\sin\theta$, find the exact volume generated. [6]

4 **(i)** A curve C has equation $y=\dfrac{x-3}{(x-2)(x+5)}$. Sketch C, giving the equations of the asymptotes, the coordinates of the stationary point(s) and the point(s) where C crosses either axis. [4]

(ii) Describe a sequence of transformations that maps C onto the graph of $y=\dfrac{2x-3}{(x-1)(2x+5)}$. [3]

5 The function f is defined by $\mathrm{f}: x \mapsto \mathrm{e}^{(x+c)^2}$, $x\in\mathbb{R}$, $x\le k$, $c>0$.

(i) Find the largest value of k in terms of c for which the function f^{-1} exists. [1]

For the rest of the question, use the value of k found in part **(i)**.

(ii) Find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [4]

(iii) Sketch, on the same diagram, the graphs of f and f^{-1} and state the relationship between the graphs. [3]

(iv) The function g is defined by $\mathrm{g}: x\mapsto \ln x$ for $x\in\mathbb{R}$, $x>0$. Give a reason why the composite function gf exists. Find $\mathrm{gf}(x)$. [2]

6 Referred to the origin O, points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively such that angle $AOB = 30°$ and $|\mathbf{a}| = 4|\mathbf{b}|$.

(i) Point C has position vector $m\mathbf{a} + n\mathbf{b}$, where m and n are positive integers. Find the area of triangle ABC in terms of m, n and $|\mathbf{b}|$. [4]

Point D is the mid-point of OA and point E lies on OB such that $OE : EB = 3 : 2$.

(ii) Find the position vectors $\overrightarrow{OD}$ and $\overrightarrow{OE}$, giving your answers in terms of $\mathbf{a}$ and $\mathbf{b}$. [2]

(iii) Show that the vector equation of the line BD can be written as $\mathbf{r} = \lambda\mathbf{a} + (1 - 2\lambda)\mathbf{b}$, where λ is a parameter. By finding the vector equation of the line AE in a similar form in terms of a parameter μ, find, in terms of $\mathbf{a}$ and $\mathbf{b}$, the position vector of the point F where the lines BD and AE meet. [4]

7 The plane p has equation $\mathbf{r} = \begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -5 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ a \\ 1 \end{pmatrix}$, and the line l has equation $\dfrac{x-4}{3} = \dfrac{y-1}{5}, z = 0$, where a is a constant and λ and μ are parameters.

(a) Find the value of a such that l and p do not meet in a unique point. [4]

(b) In the case when $a = 7$,

(i) find the coordinates of the point at which l and p intersect. [3]

(ii) find the Cartesian equation of line l', the reflection of l in p. [5]

8 Selena decides to borrow \$200 000 from the bank to fund her home purchase on 1 January 2025. On the first day of each month, 0.5% interest is added to the amount owed with the first interest amount added on 1 January 2025. On the last day of each month, she makes a repayment of \$*x* to the bank, starting from 31 January 2025.

(i) Show that the amount of money Selena owes the bank at the end of n months is $200\,000(1.005)^n - 200x\left(1.005^n - 1\right)$. [3]

(ii) If she repays the bank \$1500 a month on the last day of each month, on which date will she fully repay the loan? What is the amount of the last repayment? [4]

(iii) If she wants to fully repay the loan in 10 years, how much will she need to repay the bank every month? [2]

9 **(a)** One of the roots of the equation $z^4 - z^3 - 9z^2 + sz + t = 0$, where s and t are real, is $1+2\mathrm{i}$. Find the other roots of the equation and the values of s and t. [5]

(b) The complex number w is such that $w = a + \mathrm{i}b$, where a and b are positive real numbers. The complex conjugate of w is denoted by w^*. Given that $\dfrac{w^3}{\mathrm{i}w^*}$ is purely imaginary, find w in terms of a. [5]

10 A Red Lion skydiver jumps off from a helicopter. He leaves the helicopter with zero speed, and his speed v ms^{-1} satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 2\mathrm{e}^{-0.1t}.$$

(i) Find v in terms of t. Hence find the exact time the skydiver takes to reach a speed of 10 ms^{-1}. [5]

(ii) According to this model, explain what happens to the speed of the skydiver eventually. [1]

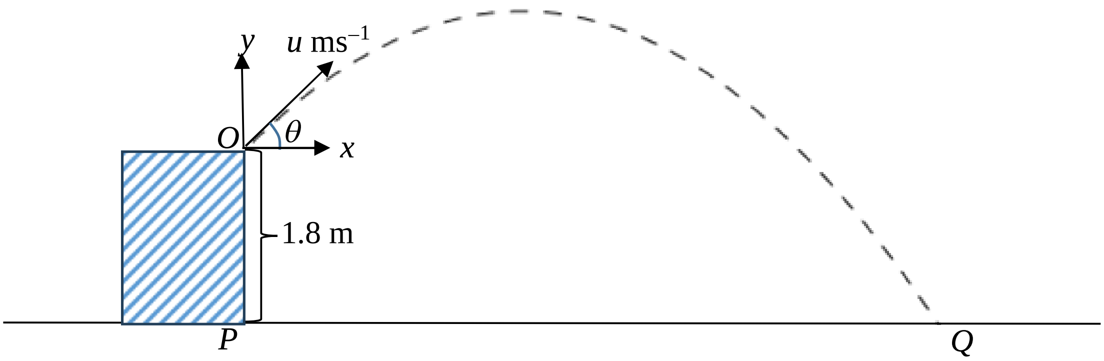
Another Red Lion skydiver jumps off the helicopter and opens his parachute. His speed is 18 ms^{-1} immediately after opening the parachute. His speed t seconds after he opens his parachute is w ms^{-1} and satisfies the differential equation

$$-2\frac{\mathrm{d}w}{\mathrm{d}t} = (w-3)(w+2).$$

(iii) Find w in terms of t. [6]

(iv) Sketch the graph of w against t. State the speed that the skydiver will not fall below. [2]

11



The diagram shows an object being projected from the top of a platform at O at an angle of projection θ made with the horizontal, where $0<\theta<\frac{\pi}{2}$, with an initial speed of u ms^{-1}. The x-y plane contains the trajectory of the object, with the equation of the trajectory, referred to the horizontal and vertical axes through O, given by

$$y = x\tan\theta - \frac{10x^2}{2u^2\cos^2\theta},$$

where x m and y m are the distances travelled to the right and upwards from O respectively as shown in the diagram above.

The point P is 1.8 m vertically below O on the horizontal ground and Q is the point where the object lands on the ground.

(i) If the angle of projection of the object is $\frac{\pi}{4}$ and it lands at a distance of 15 m on the ground from P, find its initial speed of projection. [3]

It is given that $u=10$ for the rest of the question.

(ii) Show that at Q, x satisfies the equation

$$x^2 - 10x\sin 2\theta - 18\cos 2\theta - 18 = 0.$$ [3]

(iii) By using differentiation, express x in terms of $\tan 2\theta$ at the stationary value of x. [3]

(iv) Hence calculate θ to 2 decimal places and find the stationary value of x. [4]