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GCE 'O' and 'N' EM Notes

Number and algebra

Positive numbers: greater than 0. eg. 1, 2, 3, 4, 5

Negative numbers: less than 0. eg. -1, -2, -3, -4, -5

Prime numbers: have exactly two factors: 1 and itself. eg. 2, 3, 5, 7, 11

Composite numbers: have more than two factors. Eg. 4, 6, 8, 9, 12

- 1 and 0 are NOT prime numbers.
- 2 is the ONLY even prime number.

Rational numbers: Can be represented in the form of $\frac{P}{Q}$ where $Q \neq 0$.

Irrational numbers: Cannot be represented in the form of $\frac{P}{Q}$.

HCF – Highest Common Factor

- Look at the **lowest** power.

LCM – Lowest Common Multiple

- Look at the **highest** power.

Prime factorisation

- Only can divide by **prime numbers**.

Significant figures (5 rules)

- All non-zero digits are significant. (eg. 211.8 has 4sf)
- All zeros that are found between nonzero digits are significant. (eg. 2**000**7 has 5sf)
- Leading zeros (to the left of the first nonzero digit) are not significant. (eg. **0.00**85 has 2sf)
- Trailing zeros for a whole number that ends with a decimal point are significant. (eg. 32**0** can be 2sf or 3sf)
- Trailing zeros to the right of the decimal place are significant. (eg. 12.**000** has 5sf)

Standard form: expressed as $A \times 10^n$ where A must be $1 \leq A < 10$ and n is an integer.

Common prefixes

Power of 10	English word	SI prefix	Symbol
10^{12}	trillion	tera	T
10^{9}	billion	giga	G
10^{6}	million	mega	M
10^{3}	thousand	kilo	K
10^{-3}	thousandth	milli	m
10^{-6}	millionth	micro	μ
10^{-9}	billionth	nano	n
10^{-12}	trillionth	pico	P

Law of indices

- Zero indices: $a^0 = 1$, $a \neq 0$ and $a < 0$
- Negative indices: $a^{-n} = \frac{1}{a^n}$, $a \neq 0$ and $a < 0$
- Rational indices: $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$, $a \leq 0$
- Law 1: $a^m \times a^n = a^{m+n}$, if $a > 0$
- Law 2: $a^m \div a^n = a^{m-n}$, if $a > 0$
- Law 3: $(a^m)^n = a^{mn}$, if $a > 0$
- Law 4: $a^n \times b^n = (a \times b)^n$, if $a, b > 0$
- Law 5: $a^n \div b^n = \left(\frac{a}{b}\right)^n$, if $a, b > 0$

Inequality

Signs	Definitions	How is it represented on a number line?
$<$	more than	$\blacktriangleleft\!\!\longrightarrow\!\circ$
$>$	less than	$\circ\!\longrightarrow\!\!\blacktriangleright$
$\leq$	more than or equal to	$\blacktriangleleft\!\!\longrightarrow\!\bullet$
$\geq$	less than or equal to	$\bullet\!\longrightarrow\!\!\blacktriangleright$

Basic Four Operations of Algebraic Fractions

- Addition: $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$
- Subtraction: $\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$
- Multiplication: $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$
- Division: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$

  $= \frac{ad}{bc}$

- **Conditions***

1. Addition: If $A < B$, then $A + c < B + c$
2. Subtraction: If $A < B$, then $A - c < B - c$
3. Multiplication: If $A < B$, then $cA < cB$

   If $A < B$, then $-cA > -cB$
4. Division: If $A < B$, then $\frac{A}{c} < \frac{B}{c}$

   If $A < B$, then $\frac{A}{-c} > \frac{B}{-c}$

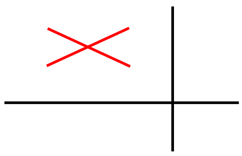
*Note: For multiplication and division of **negative** numbers, the inequality sign must flip (shown in red).

Algebraic expression

- The square of sum: $(a + b)^2 = (a^2 + 2ab + b^2)$
- The square of difference: $(a - b)^2 = (a^2 - 2ab + b^2)$
- The difference of two squares: $a^2 - b^2 = (a + b)(a - b)$

Factorisation methods:

- Divide by HCF
- Algebraic expression
- Grouping
- Completing the square in the form of $y = (x - h)^2 + k$ where **(h, k)** is the turning point.
- Quadratic equation: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Cross product



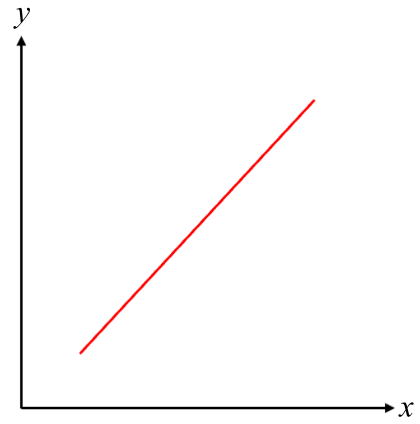
Ratio

- $a : b = \frac{a}{b}$

Proportion

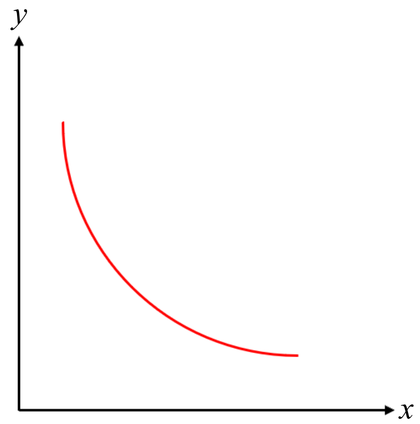
Direct proportion

$y = kx$



Inverse proportion

$y = \frac{k}{x}$



Map scale

- Length → Map : Actual = 1 : n best to change them to cm.
- Area → $1^2 : n^2$

Percentage

- Percentage increase/decrease = $\frac{\text{Increase/Decrease}}{\text{Original Value}} \times 100\%$
- Tax Relief
- Commission
- Profit/Discount
- Income Tax/GST (9%) *

**Note: GST is not always 9%, it changes over the years.*

Number pattern

General term, $T_n = an + b$

where

n is the term number,

a is the common difference between two consecutive terms, and

b is the starting term, which is the value of the sequence when $n = 0$.

For example, the first five terms form a number pattern: 5, 8, 11, 14, 17

Common difference: $8 - 5 = \mathbf{3}$

Now, the equation is $3n + b$.

If $n = 1$, $3(1) + b = 5 \rightarrow b = 5 - 3 = \mathbf{2}$

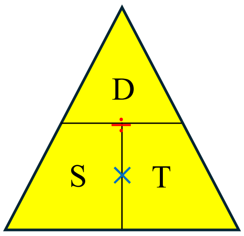
Hence, $\boldsymbol{T_n = 3n + 2}$.

Rate

- Rate is always over time (s).

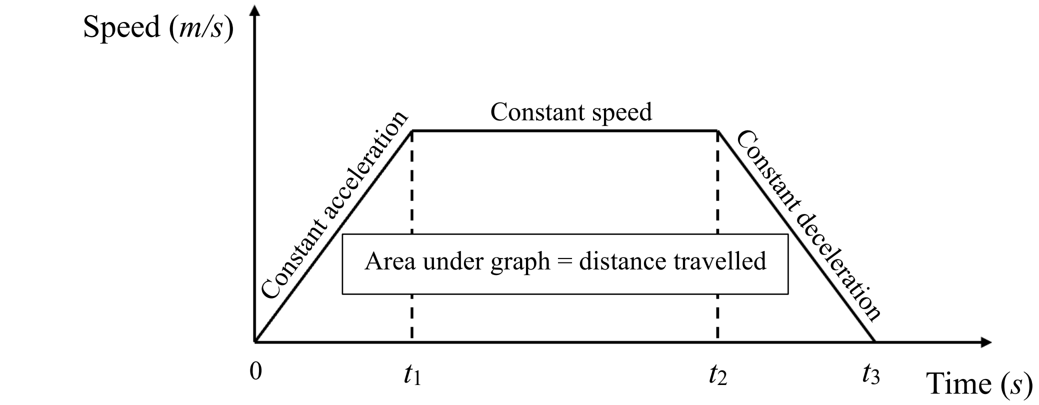
Speed

- Speed (m/s) $= \dfrac{\text{Distance } (m)}{\text{Time } (s)}$
- Average speed $= \dfrac{\text{Total Distance } (m)}{\text{Time } (s)}$



- **Acceleration:** It is an increase in speed over time.
- **Deceleration:** It is a decrease in speed over time.

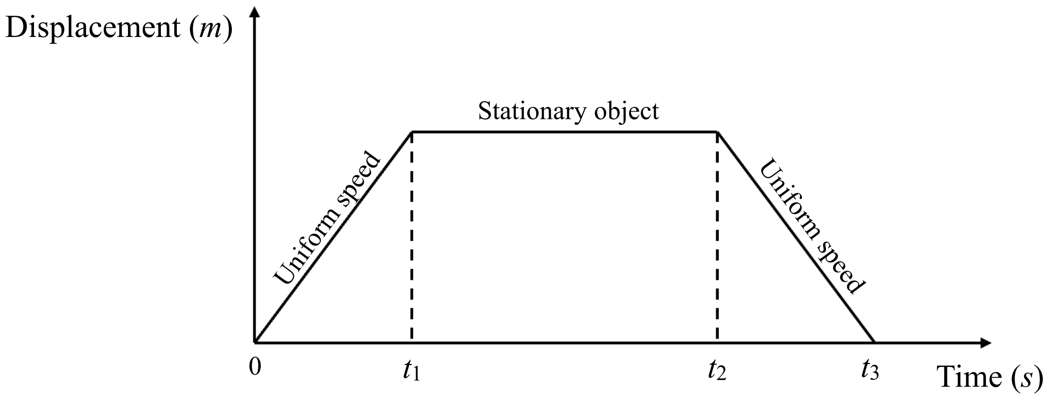
Speed-time graphs



From the speed-time graph above,

- from 0 to t_1, the car is **increasing** speed over time, meaning the car is **moving quickly**
- from t_1 to t_2, the car is at **constant** speed, meaning the speed **does not change**
- from t_2 to t_3, the car is **decreasing** speed over time, meaning the car is **slowing down**
- the **distance** travelled by the car can be determined by finding the **area under the graph**
- to find acceleration and deceleration (negative acceleration) of the car, find the **gradient** of the line.
- gradient for constant speed is **always zero**.

Displacement-time graphs



From the displacement-time graph above,

- from 0 to t_1, the car **does not change** in speed.
- from t_1 to t_2, the car **does not move**.
- from 0 to t_1, the car **does not change** in speed.
- to find the speed of the car, find the **gradient** of the line.
- gradient for stationary object is **always zero**.

Simple interest: It is an interest charge that borrowers pay lenders for a loan.

$I = \frac{PRT}{100}$

where

P is the principal amount

R is the rate of interest

T is the number of years

Compound interest: It is the interest calculated on both the initial principal and all of the previously accumulated interest.

$A = P\left(1 + \frac{r}{100}\right)^n$

where

A is the total amount

P is the principal amount

r is the rate of interest

n is the number of years

Exchange rate: a relative price of one currency expressed in terms of another currency.

For example, the exchange rate between Singapore Dollars (SGD) and US Dollars (USD) is now S$1 = $0.75 USD.

If I want to purchase a bag that costs $300, how much will I need to pay in USD?

S$1 = $0.75 USD

S$300 = $0.75 × $300 = **$225 USD**

Hence, I need to pay **$225 USD** for the same bag in Singapore.

Hire purchase: It is an arrangement made while buying expensive goods.

Hire purchase = Deposit + Monthly payment

When you cannot afford to pay the item in full amount, you pay a deposit to the seller when you first agree to buy the item. **Deposit** is usually a small percentage of the cash price. Then, you pay the remaining amount in small chucks monthly. After you have fully pay including your monthly payment, then you will get the item you want.

Taxation: It is a term for when a taxing authority, usually a government, levies or imposes a financial obligation on its citizens or residents.

Set notation

Set language	**Definition**
$A \cup B$	union of A and B
$A \cap B$	intersection of A and B
$n(A)$	number of elements in set A
$\in$	an element of
$\notin$	not an element of
A'	complement of set A
$\emptyset$	empty set
ξ	universal set
$A \subseteq B$	A is a subset of B
$A \nsubseteq B$	A is a not a subset of B
$A \subset B$	A is a (proper) subset of B
$A \not\subset B$	A is a not a (proper) subset of B

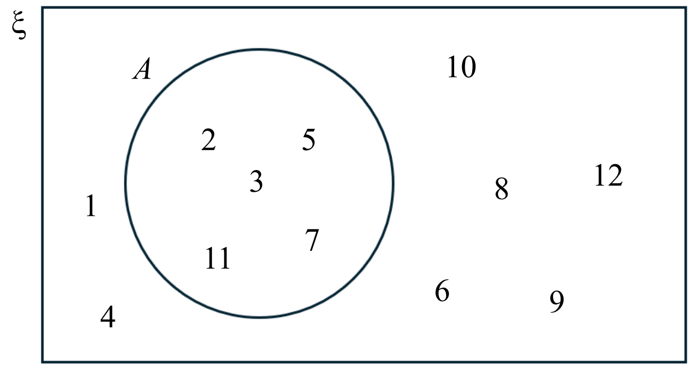
Set: A list of elements. In simple words, sets are **collection of objects** such as pile of books and bunch of keys. The collective nouns "pile" and "bunch" are sets. The words "books" and "keys" are **elements**.

For example, let A be the set of the first five prime numbers.

It will be written like this: $A = \{2, 3, 5, 7, 11\}$. The curly brackets "{…}" are used to show a set.

To find the number of elements in a set, we use this notation: $n(A)$. For set A, the number of elements will be 5.

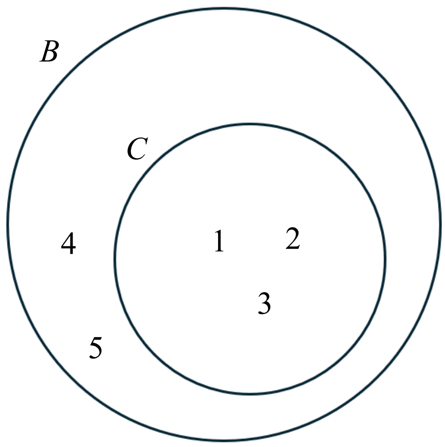
Now let's look at this Venn diagram below.



We can observe from the above Venn diagram that the set of elements belonging to ξ but not to A is called the complementary of the set A, denoted as A'.

Consider the sets $B = \{1, 2, 3, 4, 5\}$ and $C = \{1, 2, 3\}$

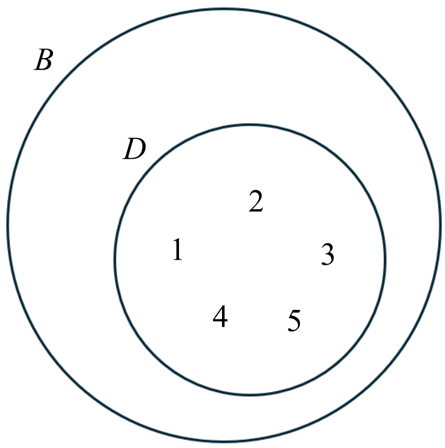
How can we draw a Venn diagram to represent the sets B and C such that we do not repeat the common elements? Since all the elements are distinct, we can draw the Venn diagram as shown below.



We can see that C is completely inside of B, meaning every element of C is an element of B.

Now, consider the sets $B = \{1, 2, 3, 4, 5\}$ and $D = \{1, 2, 3, 4, 5\}$

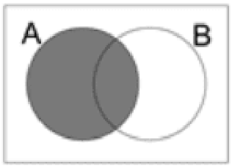
From this information, we can draw the set D completely inside the set B.



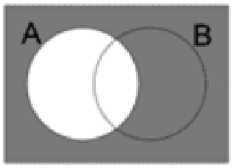
In conclusion, we can say that C and D are subsets of B, and we write $C \subset B$ and $D \subset B$.

In addition, we can also say that C is a proper subset of B, and we write $C \subseteq B$.

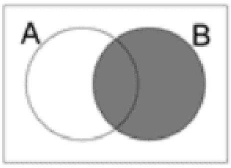
Venn Diagram Set Notation for Single Set:



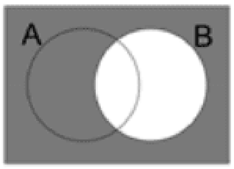
A



A'

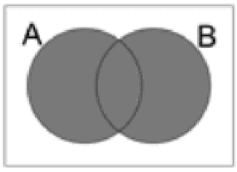


B

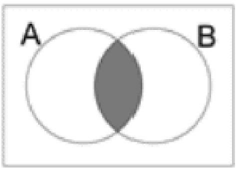


B'

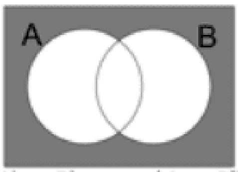
Venn Diagram Set Notation for Double Sets:



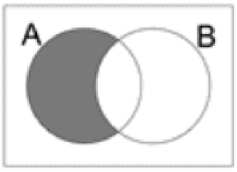
$A \cup B$



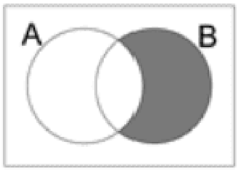
$A \cap B$



$A' \cap B'$ or $(A \cup B)'$

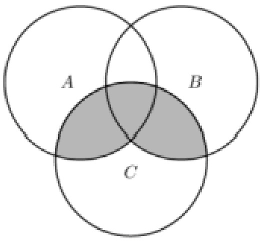


$A \cap B'$

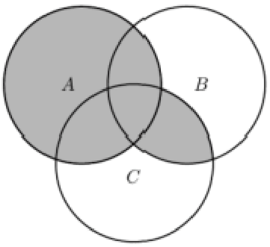


$A' \cap B$

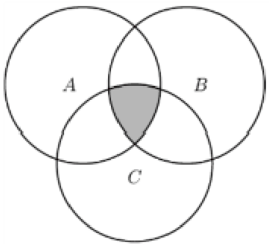
Venn Diagram Set Notation for Triple Sets:



(A U B) ∩ C



A U (B ∩ C)



A ∩ B ∩ C

Images from [*https://jimmymaths.com/venn-diagram-set-notation/*](https://jimmymaths.com/venn-diagram-set-notation/)

Matrices

Let’s look at the problem below.

The table below shows the number of students from two classes, A and B who travel to school by three different transportation: walk, bus and cycle.

	Walk (W)	**Bus (B)**	**Cycle (C)**
Class A	12	15	3
Class B	22	8	10

The table above can be represented by matrices.
We can represent the matrix by a letter such as M. Also, we can label the types of transportation and class names beside the matrix as shown below.

$$\mathbf{M} = \begin{array}{c} \\ \\ \end{array}\begin{array}{ccc} \text{W} & \text{B} & \text{C} \end{array}$$
$$\mathbf{M} = \begin{pmatrix} 12 & 15 & 3 \\ 22 & 8 & 10 \end{pmatrix} \begin{array}{l} \text{Class A} \\ \text{Class B} \end{array}$$

Matrices are read in this order: horizontal by vertical. Just like reading a coordinate: (x, y) where x is the horizontal axis and y is the vertical axis.

3 columns

$$\mathbf{M} = \begin{pmatrix} 12 & 15 & 3 \\ 22 & 8 & 10 \end{pmatrix}$$ 2 rows

Since matrix **M** has 2 rows and 3 columns, we can say that the order of **M** is 2 by 3 or 2×3.

Equal matrices: Must have the same order and their corresponding elements are equal.

Addition and subtraction of matrices: Must have the same order when adding or subtracting.

<u>Addition</u>

$$\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} + \begin{pmatrix} u & v & w \\ x & y & z \end{pmatrix} = \begin{pmatrix} a+u & b+v & c+w \\ d+x & e+y & f+z \end{pmatrix}$$

<u>Subtraction</u>

$$\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} - \begin{pmatrix} u & v & w \\ x & y & z \end{pmatrix} = \begin{pmatrix} a-u & b-v & c-w \\ d-x & e-y & f-z \end{pmatrix}$$

Multiplication of matrices: When a matrix A is multiplied by a scalar k, every element in A is multiplied by k.

$$\text{If } \mathbf{A} = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}, \text{ then } k\mathbf{A} = k\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} = \begin{pmatrix} ka & kb & kc \\ kd & ke & kf \end{pmatrix}$$

Multiplication of two matrices: The number of columns of A must be equal to the number of rows of B.

$$\begin{array}{ccccc} A & \times & B & = & C \\ m \times n & & n \times p & & m \times p \end{array}$$

must be equal

How to multiply two matrices?
Let's multiply **A** = 2 by 3 and **B** = 3 by 2 matrices.
Since the number of columns in matrix **A** and the number of rows in matrix **B** are the same, the multiplication is possible.

Given that $\mathbf{A} = \begin{pmatrix} 7 & 3 & 2 \\ 1 & 4 & 5 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -5 & 3 \\ 9 & -2 \\ 6 & 8 \end{pmatrix}$, find $\mathbf{C} = \mathbf{AB}$.

The order of the product **C** will be 2 by 2.

$$\begin{array}{cccc} & & \text{Column 1} \quad \text{Column 2} & \\ \begin{array}{l} \text{Row 1} \\ \text{Row 2} \end{array} & \begin{pmatrix} 7 & 3 & 2 \\ 1 & 4 & 5 \end{pmatrix} & \begin{pmatrix} -5 & 3 \\ 9 & -2 \\ 6 & 8 \end{pmatrix} & = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \\ & A & B & C \end{array}$$

Step 1: multiply the elements in Row 1 of **A** and in Column 1 of **B**, a_{11}.

$$\begin{array}{ccc} & & \text{Column 1} \quad \text{Column 2} \\ \begin{array}{l} \text{Row 1} \\ \text{Row 2} \end{array} & \begin{pmatrix} 7 & 3 & 2 \\ 1 & 4 & 5 \end{pmatrix} & \begin{pmatrix} -5 & 3 \\ 9 & -2 \\ 6 & 8 \end{pmatrix} \end{array} = \begin{pmatrix} 7 \times (-5) + 3 \times 9 + 2 \times 6 & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$= \begin{pmatrix} 4 & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

Step 2: multiply the elements in Row 1 of **A** and in Column 2 of **B**, a_{12}.

$$\begin{array}{ccc} & & \text{Column 1} \quad \text{Column 2} \\ \begin{array}{l} \text{Row 1} \\ \text{Row 2} \end{array} & \begin{pmatrix} 7 & 3 & 2 \\ 1 & 4 & 5 \end{pmatrix} & \begin{pmatrix} -5 & 3 \\ 9 & -2 \\ 6 & 8 \end{pmatrix} \end{array} = \begin{pmatrix} 4 & 7 \times 3 + 3 \times (-2) + 2 \times 8 \\ a_{21} & a_{22} \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 31 \\ a_{21} & a_{22} \end{pmatrix}$$

Step 3: multiply the elements in Row 2 of **A** and in Column 1 of **B**, a_{21}.

$$\begin{array}{ccc} & & \text{Column 1} \quad \text{Column 2} \\ \begin{array}{l} \text{Row 1} \\ \text{Row 2} \end{array} & \begin{pmatrix} 7 & 3 & 2 \\ 1 & 4 & 5 \end{pmatrix} & \begin{pmatrix} -5 & 3 \\ 9 & -2 \\ 6 & 8 \end{pmatrix} \end{array} = \begin{pmatrix} 4 & 31 \\ 1 \times (-5) + 4 \times 9 + 5 \times 6 & a_{22} \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 31 \\ 61 & a_{22} \end{pmatrix}$$

Step 4: multiply the elements in Row 2 of **A** and in Column 2 of **B**, a_{22}.

$$\begin{array}{cc} & \\ \text{Row 1} & \\ \text{Row 2} & \end{array}\begin{pmatrix} 7 & 3 & 2 \\ \boxed{1 \quad 4 \quad 5} \end{pmatrix}\begin{array}{c}\begin{array}{cc}\text{Column 1} & \text{Column 2}\end{array}\\ \begin{pmatrix} -5 & 3 \\ 9 & -2 \\ 6 & 8 \end{pmatrix}\end{array} = \begin{pmatrix} 4 & 31 \\ 61 & 1 \times 3 + 4 \times (-2) + 5 \times 8 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 31 \\ 61 & 35 \end{pmatrix}$$

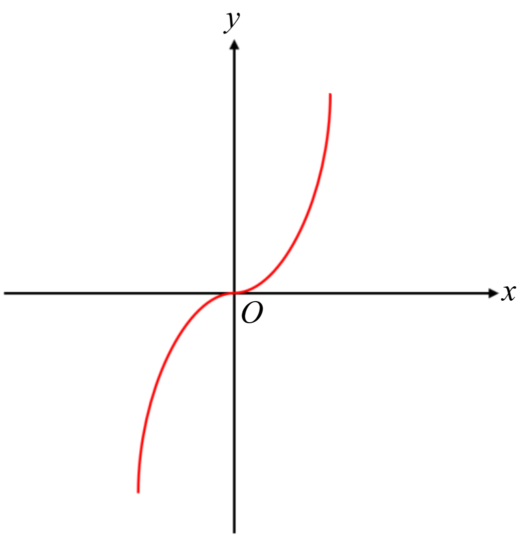
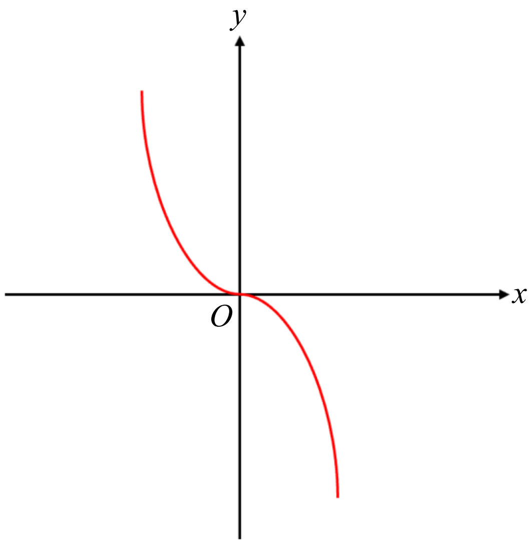
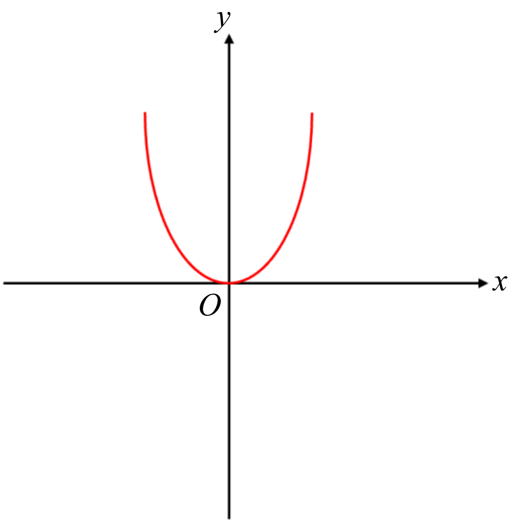
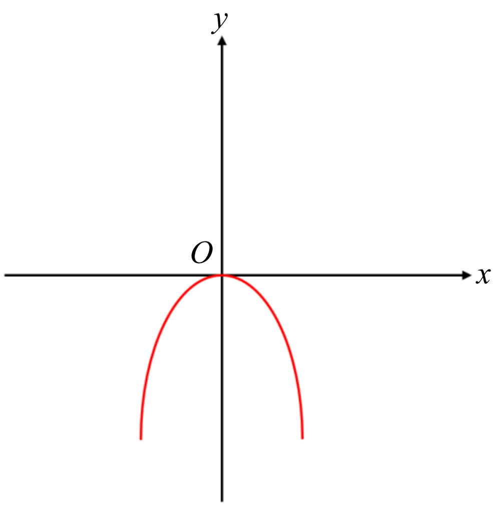
Hence, the complete matrix multiplication is $\mathbf{C} = \begin{pmatrix} 4 & 31 \\ 61 & 35 \end{pmatrix}$.

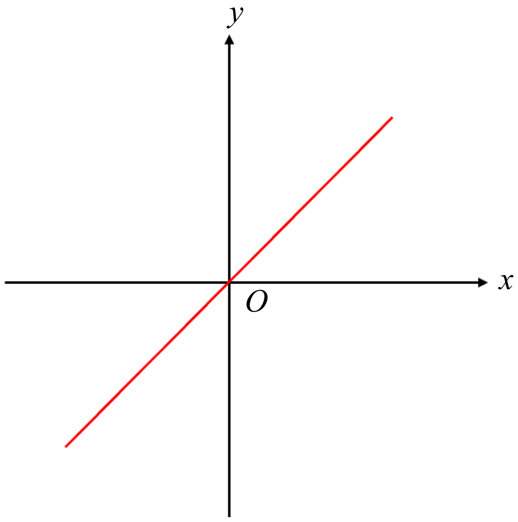
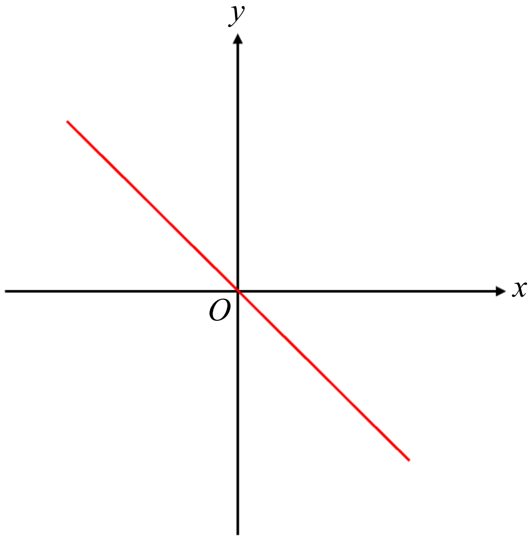
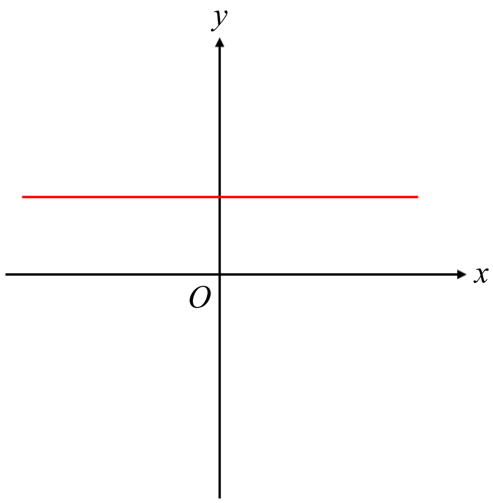
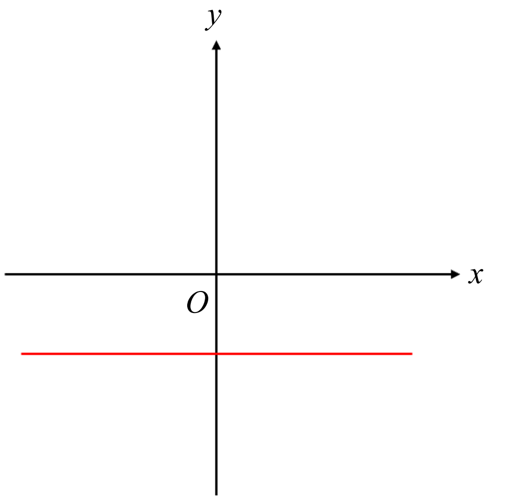
<u>Summary</u>

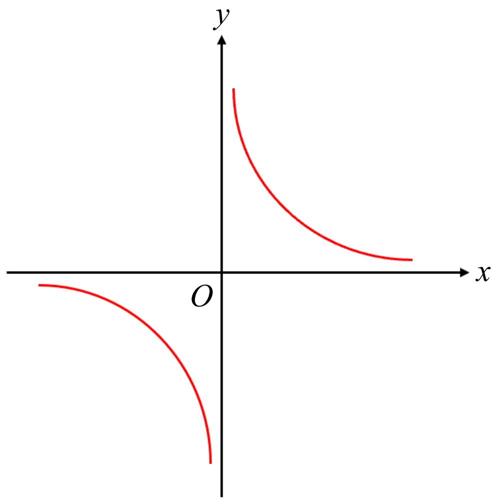
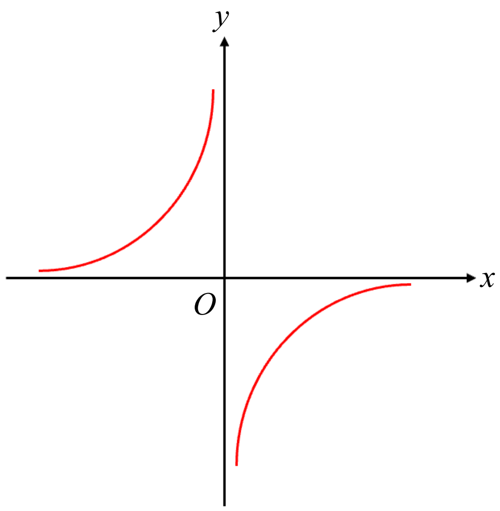
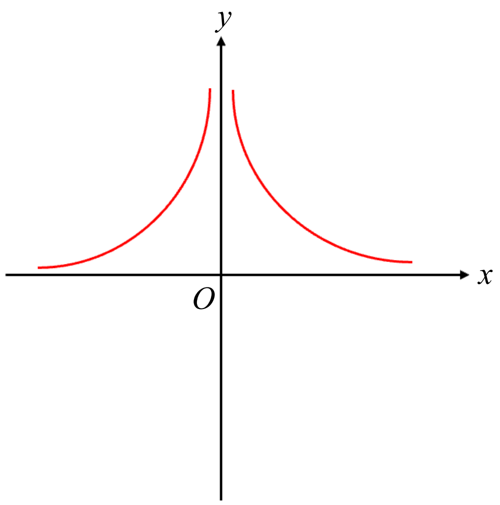
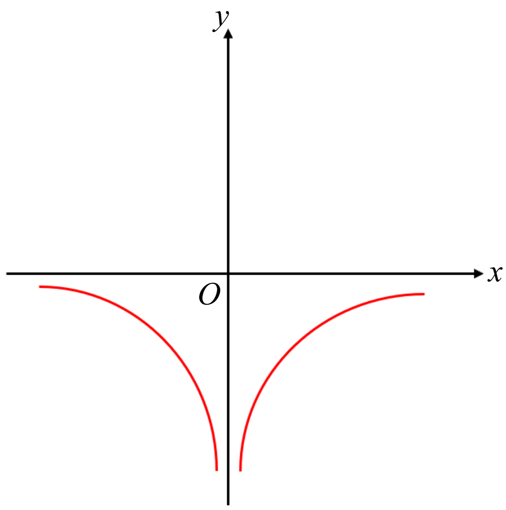
When multiplying two matrices, always check whether both the number of columns in the first matrix and the number of rows in the second matrix are **equal**. Then, if it is possible to multiply, multiply the elements in the rows of the first matrix with the elements in the columns of the second matrix and so on.

Functions and graphs

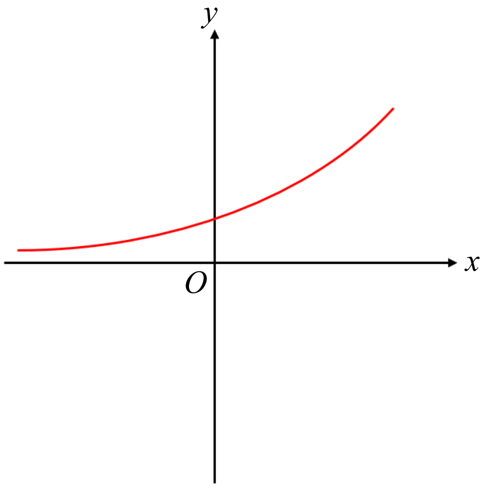
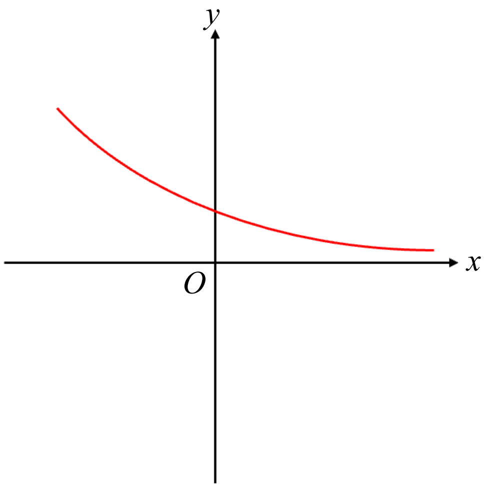
Graphs of power functions $y = ax^n$

$n = 3, a > 0, y = ax^3$	$n = 3, a < 0, y = ax^3$
	
$n = 2, a > 0, y = ax^2$	$n = 2, a < 0, y = ax^2$
	

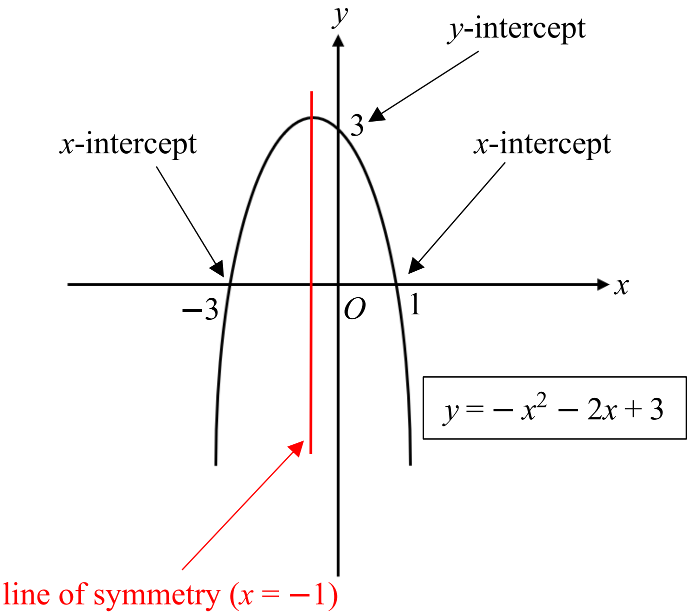
$n = 1, a > 0, y = ax$	$n = 1, a < 0, y = ax$
	
$n = 0, a > 0, y = a$	$n = 0, a < 0, y = a$
	

$n = -1, a > 0, y = \frac{a}{x}$	$n = -1, a < 0, y = \frac{a}{x}$
	
$n = -2, a > 0, y = \frac{a}{x^2}$	$n = -2, a < 0, y = \frac{a}{x^2}$
	

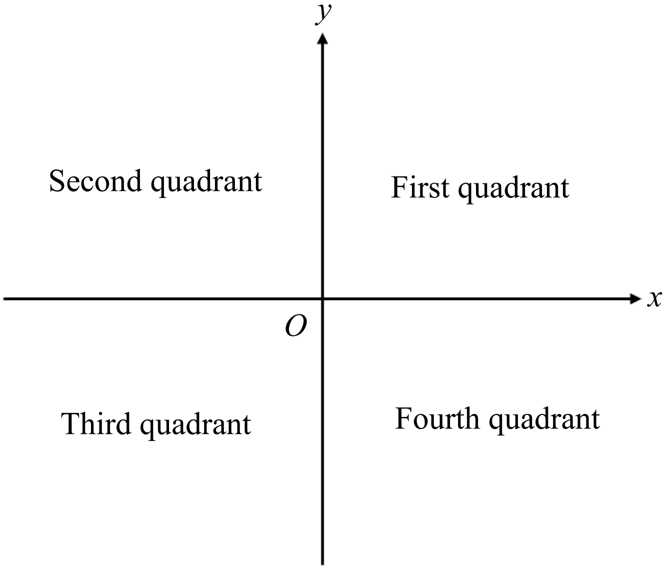
Graphs of power functions $y = ka^x$, where a is a positive integer and $a \neq 1$.

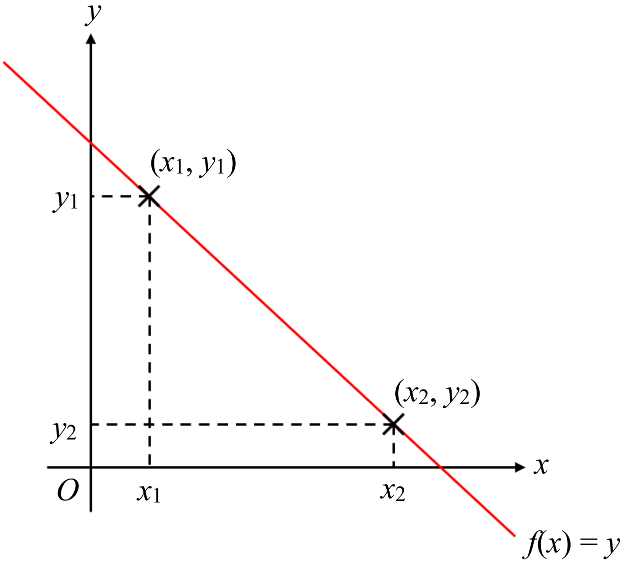
$k > 0, y = ka^x$	$k < 0, y = ka^x$
	

How to read a quadratic graph?



Coordinate Cartesian plane: Consists of horizontal x-axis and vertical y-axis.





General equation of a straight line: $y = mx + c$

where

m is gradient (rise/run),

c is the y-intercept,

y is the function, and

x is the points of the function y.

Length of a line from points (x_2, y_2) to (x_1, y_1): $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Gradient of line from points (x_2, y_2) to (x_1, y_1): $\frac{y_2 - y_1}{x_2 - x_1}$ (rise/run)

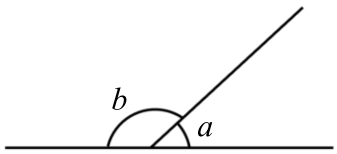
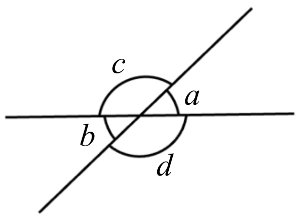
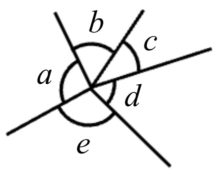
Geometry and measurement

Angles

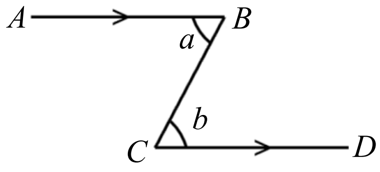
Types of angles

1. Acute angle: $0° < x < 90°$
2. Right angle: $90°$
3. Obtuse angle: $90° < x < 180°$
4. Straight line: $180°$
5. Reflex angle: $180° < x < 360°$

Angle properties

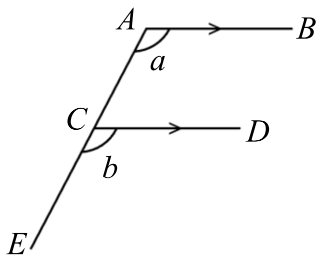
Property 1: Adjacent angles on a straight line are supplementary abbreviation: (adj. ∠s on a str. line)  $\angle a + \angle b = 180°$
Property 2: Vertically opposite angles are equal abbreviation: (vert. opp. ∠s)  $\angle a = \angle b$ $\angle c = \angle d$
Property 3: Angles at a point abbreviation: (∠s at a point)  $\angle a + \angle b + \angle c + \angle d + \angle e = 360°$

Property 4: Alternate angles are equal (Letter 'Z')
abbreviation: (alt. ∠s, *XX*//*XX*)



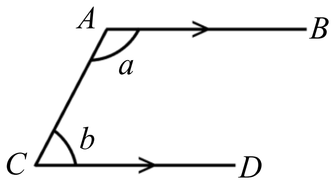
$\angle a = \angle b$ (alt. ∠s, $AB // CD$)

Property 5: Corresponding angles are equal (Letter 'F')
abbreviation: (corr. ∠s, *XX*//*XX*)



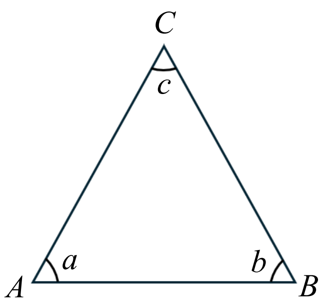
$\angle a = \angle b$ (corr. ∠s, $AB // CD$)

Property 6: Interior angles are supplementary (Letter 'C', 'U' or 'N')
abbreviation: (alt. ∠s, *XX*//*XX*)



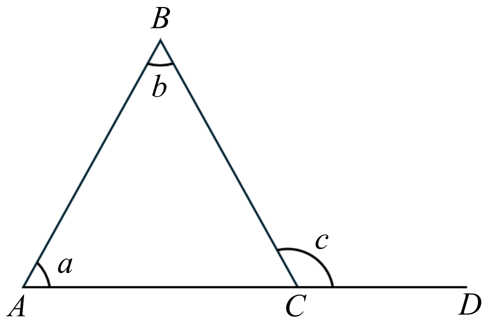
$\angle a + \angle b = 180°$

Property 7: Angles in a triangle are supplementary
abbreviation: (∠s sum of Δ)



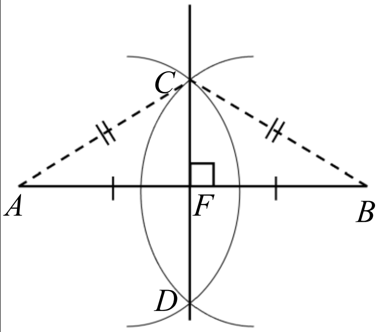
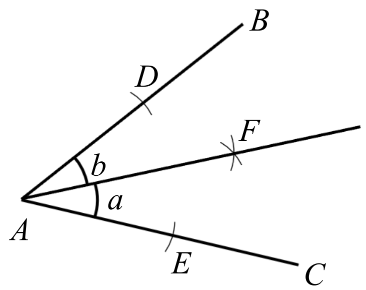
$\angle a + \angle b + \angle c = 180°$

Property 8: Exterior angles
abbreviation: (ext. ∠s)



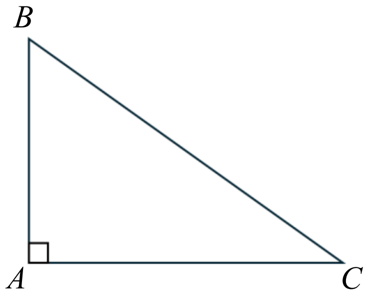
$$\angle a + \angle b = \angle c$$

Perpendicular and Angle bisectors

Perpendicular bisector **Step 1:** Construct an arc with point *A* as the centre and with more than half of *AB* using a compass. **Step 2:** Repeat step 1 with point *B* as the centre. **Step 3:** Label the two intersection points as *C* and *D* where the two arcs meet each other. **Step 4:** Construct a straight line passing through points *C* and *D*. **Step 5:** Label $AF = FB$ and $\angle CFB = 90°$	
Angle bisector **Step 1:** Construct an arc with point *A* on *AB* and with more than half of the same line using a compass. Label this point *D*. **Step 2:** Repeat step 1 with the same point *A* but now construct another arc on *AC*. Label this point *E*. **Step 3:** Construct two arcs with points *D* and *E* as their centres such that these two new arcs intersect at point *E*. **Step 4:** Construct a straight line passing through points *A* and *E*.	 $\angle a = \angle b$

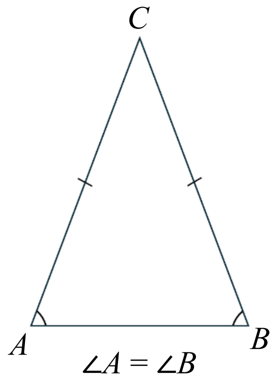
Triangles

Right angle triangle



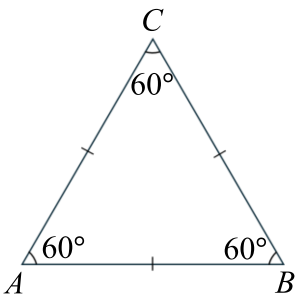
$\angle A = 90°$

Isosceles triangle



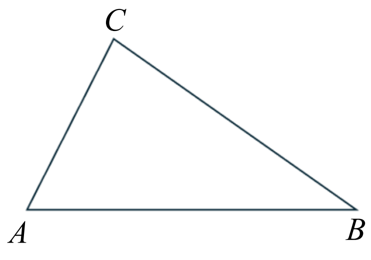
$\angle A = \angle B$
$AC = BC$

Equilateral triangle



$\angle A = \angle B = \angle C = 60°$
$AB = BC = AC$

Scalene triangle



All lengths are different

Polygons

Regular/*n*-sided polygons: Both sides and interior angles (angles inside a shape) are equal.

- n means the number of equal sides of a regular polygon.

$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 7$	$n = 8$	$n = 9$	$n = 10$
Equilateral triangle	Square	Pentagon	Hexagon	Heptagon	Octagon	Nonagon	Decagon

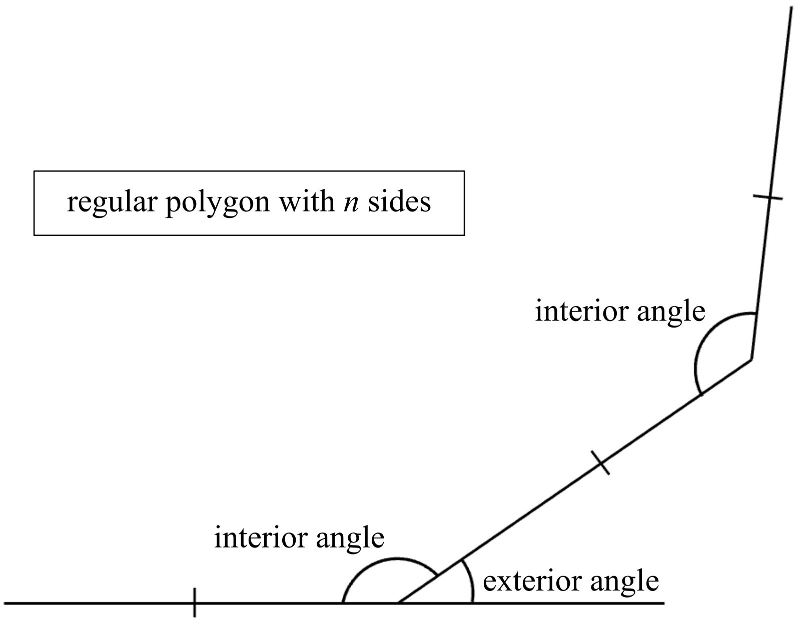
Irregular polygons: All sides and interior angles are not equal.

Sum of all interior angles in a regular polygon = $180° \times (n - 2)$

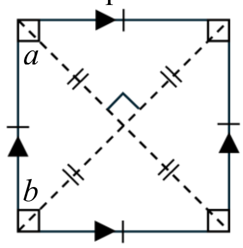
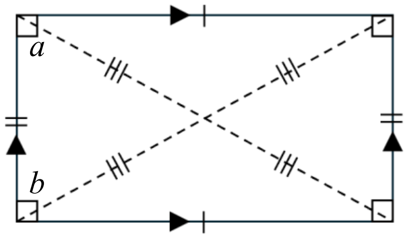
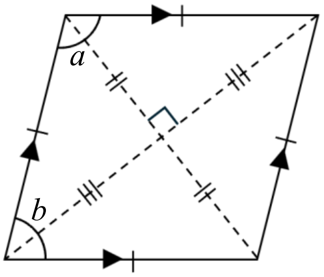
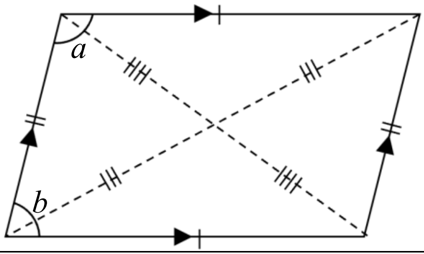
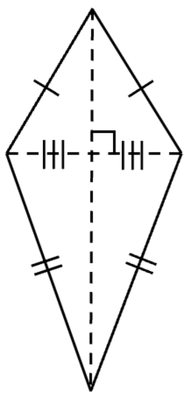
One interior angle of a regular polygon = $\frac{180° \times (n - 2)}{n}$

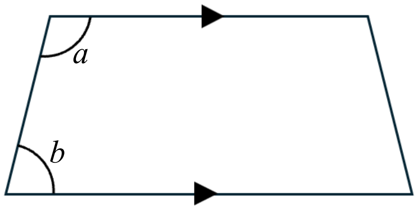
Interior angle + exterior angle = 180° (adj. ∠s on a str. line)

$n \times$ exterior angles = 360°

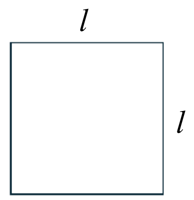
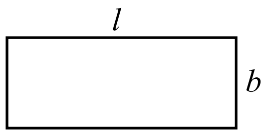
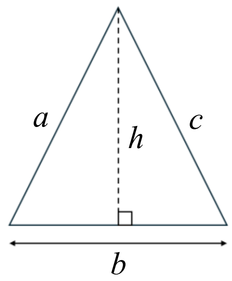
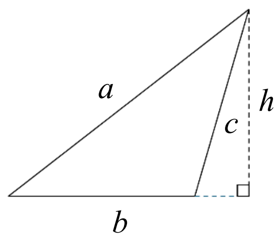
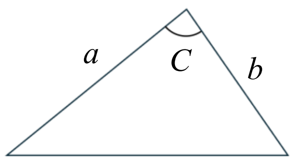
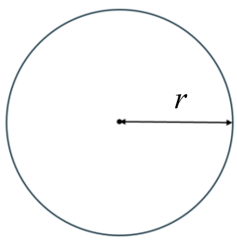


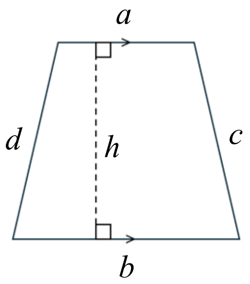
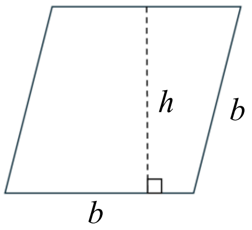
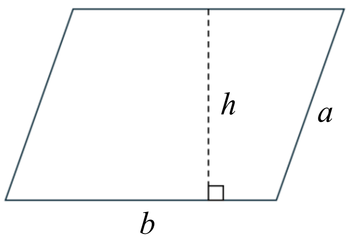
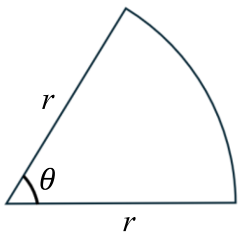
Quadrilateral (four-sided shape) properties proofs

Quadrilateral	Properties
Square 	➢ Opposite sides are parallel ➢ All sides are equal ➢ All angles are right angles ➢ Diagonals are equal ➢ Diagonals bisects each other at right angles ➢ Interior angles are supplementary ($\angle a + \angle b = 180°$)
Rectangle 	➢ Opposite sides are parallel ➢ Opposite sides are equal ➢ All angles are right angles ➢ Diagonals are equal ➢ Diagonals bisect each other ➢ Interior angles are supplementary ($\angle a + \angle b = 180°$)
Rhombus 	➢ Opposite sides are parallel ➢ Opposite angles are equal ➢ All sides are equal ➢ Diagonals bisects each other at right angles ➢ Interior angles are supplementary ($\angle a + \angle b = 180°$)
Parallelogram 	➢ Opposite sides are parallel ➢ Opposite sides are equal ➢ Diagonals bisect each other ➢ Interior angles are supplementary ($\angle a + \angle b = 180°$)
Kite 	➢ Two pairs of adjacent sides are equal ➢ Diagonals are perpendicular ➢ One diagonal (vertical) bisects the other diagonal (horizontal) ➢ One diagonal (vertical) bisects a pair of opposite angles

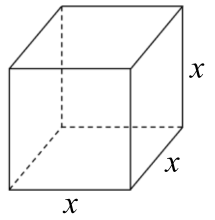
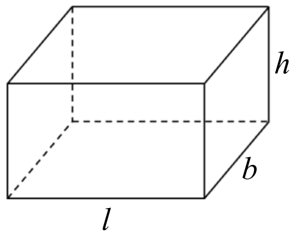
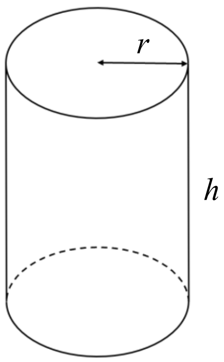
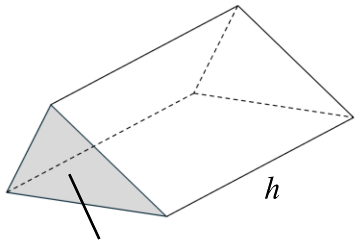
Trapezium  	➢ One pair of parallel lines ➢ Interior angles are supplementary ($\angle a + \angle b = 180°$)

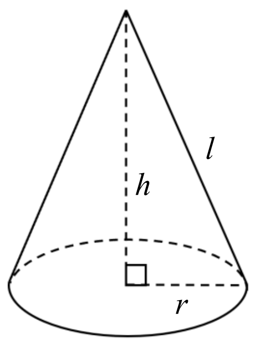
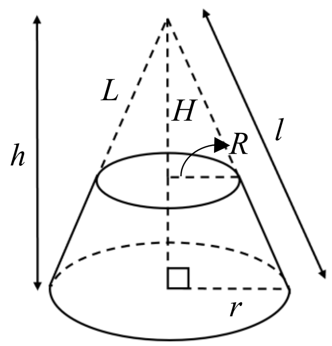
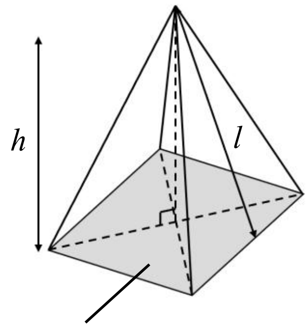
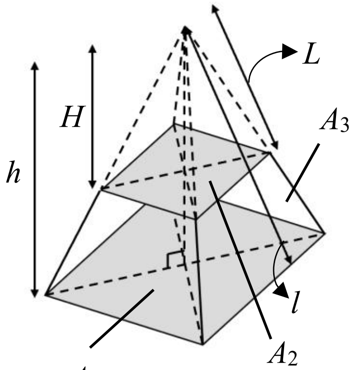
Area and perimeter

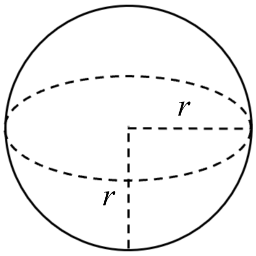
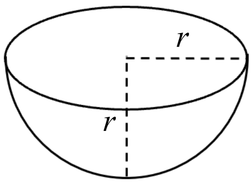
Shape	**Area**	**Perimeter**
Square 	l^2	$4l$
Rectangle 	lb	$2(l + b)$
Triangle   	$\frac{1}{2}bh$ (only for **right angle** triangles) $\frac{1}{2}ab\sin C$ (only for **non-right angle** triangles)	$a + b + c$
Circle 	πr^2	$2\pi r$ or πd

Trapezium 	$\frac{1}{2}(a+b)h$	$a+b+c+d$
Rhombus 	bh	$4b$
Parallelogram 	bh	$2(a+b)$
Sector 	$\frac{\theta}{360°}\times\pi r^2$ where θ is in **degrees** $\frac{1}{2}r^2\theta$ where θ is in **radians**	$\frac{\theta}{360°}\times 2\pi r+2r$ where θ is in **degrees** $r\theta+2r$ where θ is in **radians**

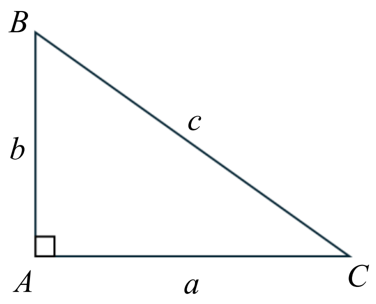
Volume and total surface area

Solid	**Volume**	**Total surface area**
Cube 	x^3	$6x^2$
Cuboid 	lbh	$2(lb) + 2(lh) + 2(bh)$
Cylinder 	$\pi r^2 h$	$2\pi r^2 + 2\pi rh$
Prism 	base area × h	area of all flat surfaces

Cone 	$\frac{1}{3}\pi r^2 h$,	πrl where $l = \sqrt{r^2 + h^2}$
Cone frustum 	$\frac{1}{3}\pi r^2 h - \frac{1}{3}\pi R^2 H$	$\pi rl - \pi RL$ where $L = \sqrt{R^2 + H^2}$
Pyramid 	$\frac{1}{3} \times$ base area $\times h$	base area + all triangular flat faces
Pyramid frustum 	$\frac{1}{3} \times A_1 \times h - \frac{1}{3} \times A_2 \times H$	$A_1 + A_2 +$ all A_3

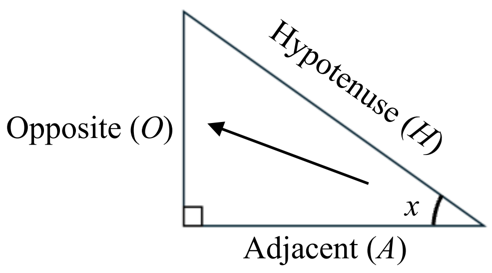
Sphere 	$\frac{4}{3}\pi r^3$	$4\pi r^2$
Hemisphere 	$\frac{2}{3}\pi r^3$	$3\pi r^2$

Pythagoras' Theorem



$a^2 + b^2 = c^2$

Trigonometry ratios



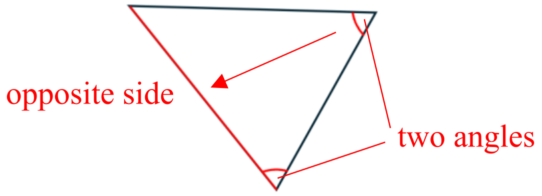
SOH $\rightarrow \sin x = \frac{O}{H}$

CAH $\rightarrow \cos x = \frac{A}{H}$

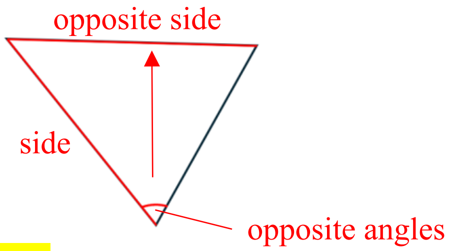
TOA $\rightarrow \tan x = \frac{O}{A}$

Further trigonometry

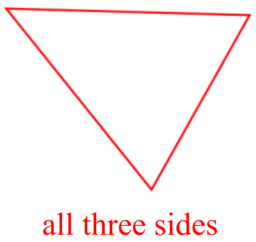
- Sine rule: $\frac{\sin a}{A} = \frac{\sin b}{B} = \frac{\sin c}{C}$ or $\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$
- Use when either two angles and one opposite side or one opposite angle and two sides are given.



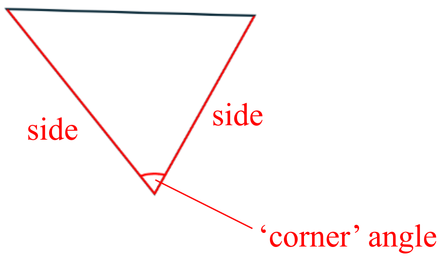
OR



- Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$
- Use when either all three sides or two sides and one ‘corner’ angle are given.



OR



Conversion of radians and degrees

$2\pi \text{ rad} = 360°$

$\pi \text{ rad} = 180°$

$1 \text{ rad} = \frac{180°}{\pi}$

or

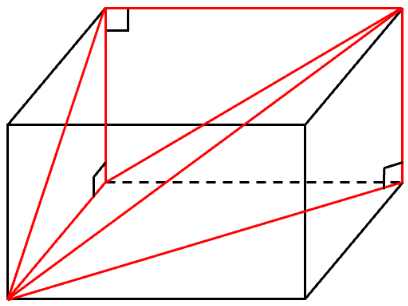
$360° = 2\pi \text{ rad}$

$180° = \pi \text{ rad}$

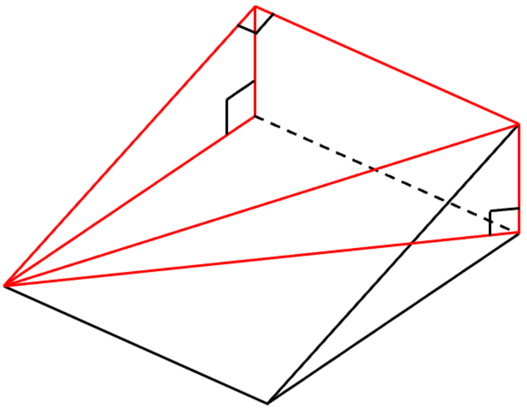
$1° = \frac{\pi}{180°} \text{ rad}$

Right angles in solids

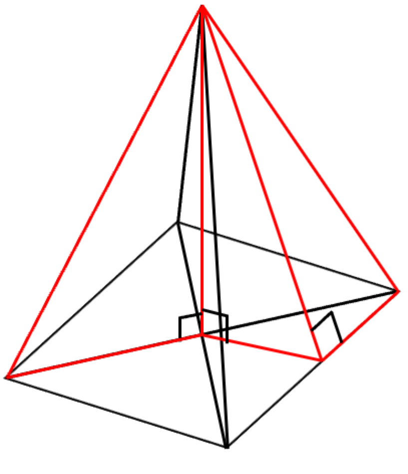
Cuboid



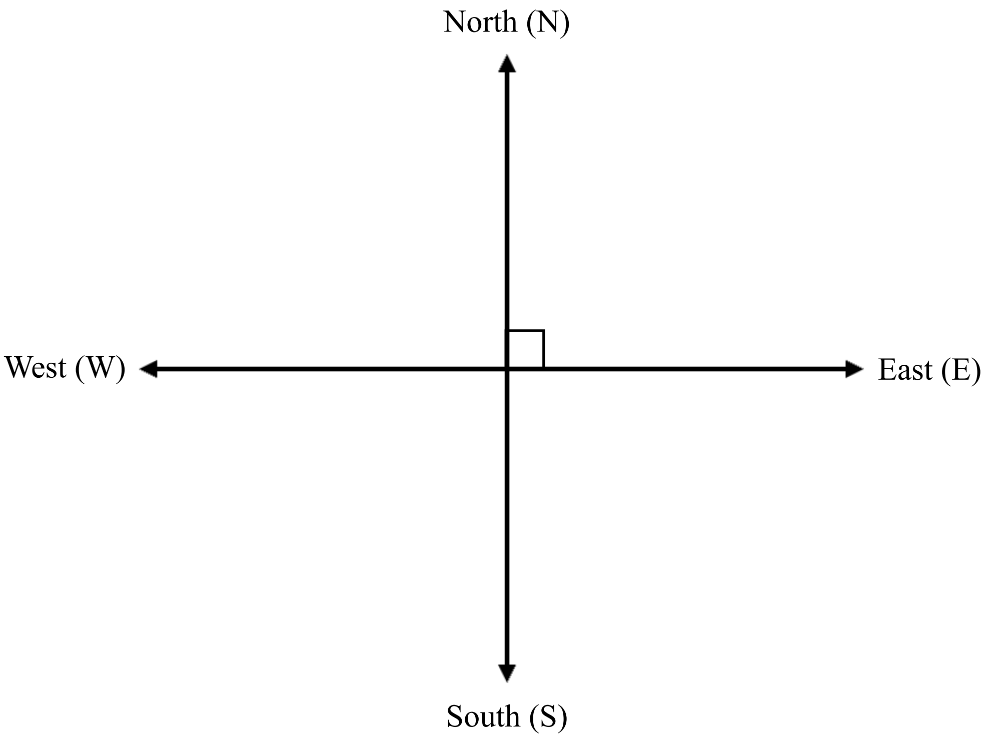
Right angle triangle-based prism



Square-based pyramid

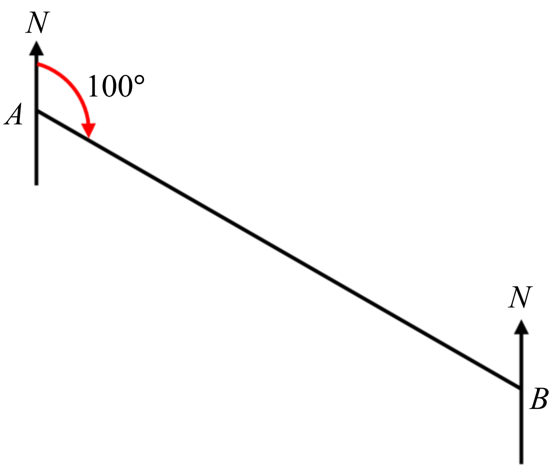


Bearing



Bearing is always in **3-digit number**. eg. 060°, 150°.

The bearing of *B* **from** *A* is 100°.

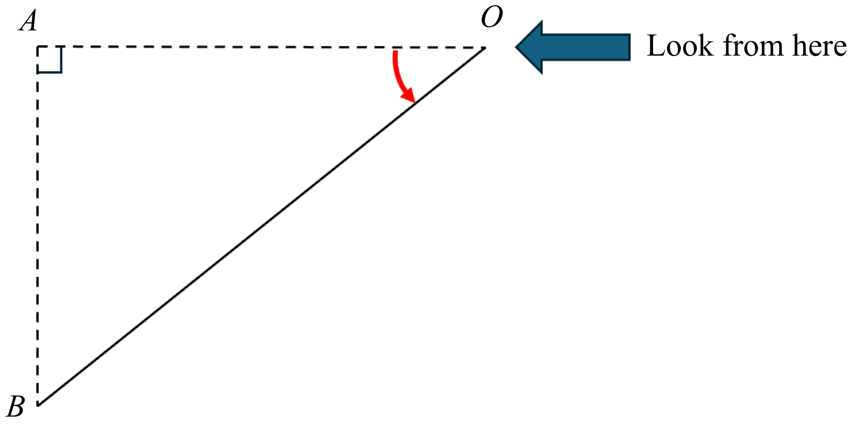


When they say, '**from *A***', start the **clockwise** direction at point ***A***.

Real-life applications of trigonometry ratios

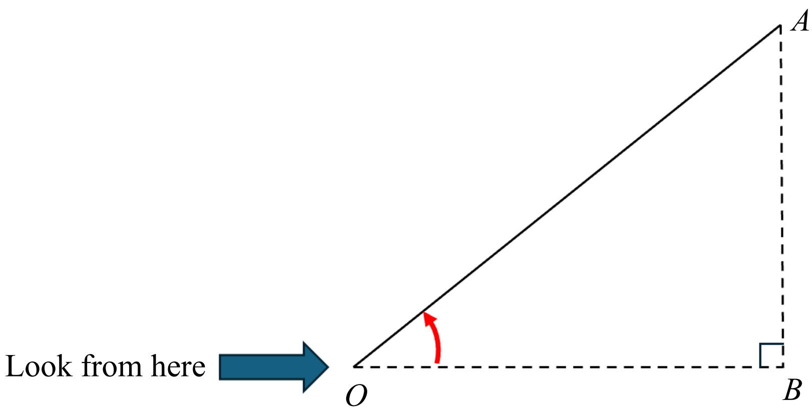
Angle of depression – **look down** from horizontal line

The angle of depression is the angle formed between the horizontal line *OA* and the line from the observer's eye view, *O* to a certain point, *B*, below line *OA*. Angle *OAB* = 90°.

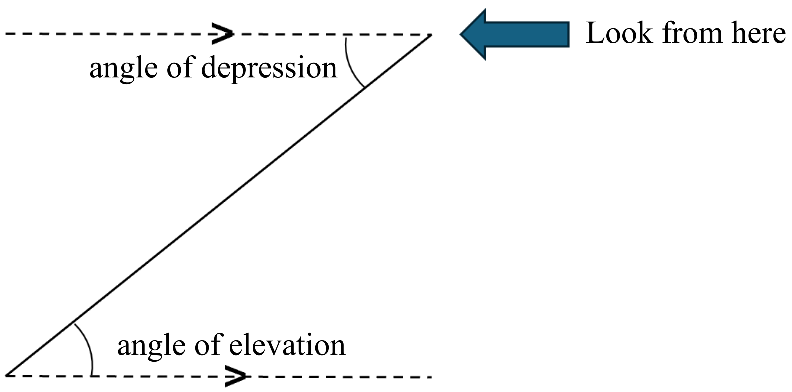


Angle of elevation – **look up** from horizontal line

The angle of elevation is the angle formed between the horizontal line *OB* and the line from the observer's eye view, *O* to a certain point, *A*, above line *OB*. Angle *OBA* = 90°.



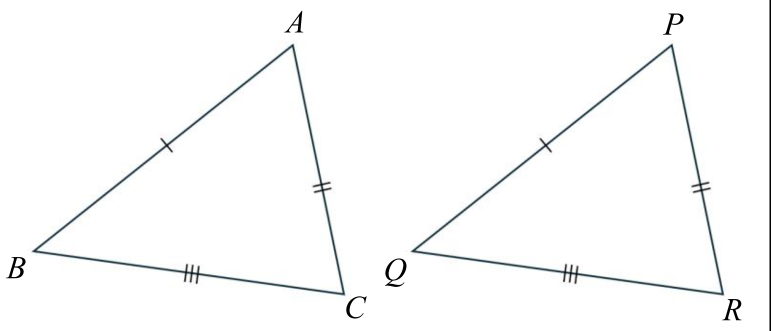
Angle of depression = angle of elevation (alternate angles)

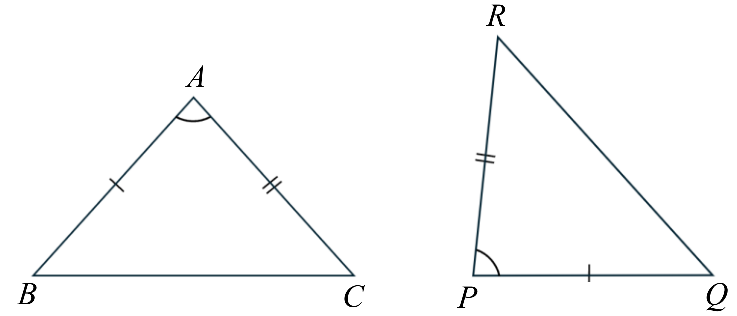


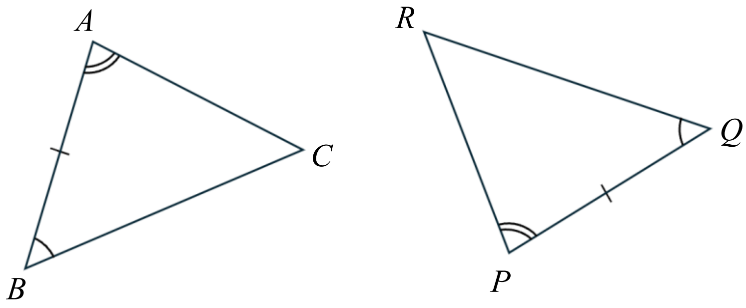
By u/One_Wishbone_4439

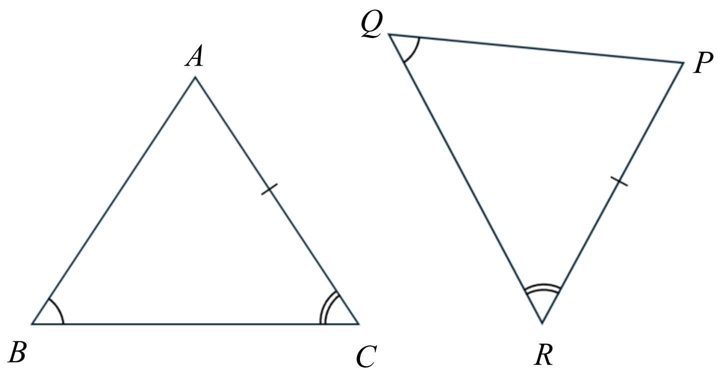
Congruence and similarity

<table>
<tr><th colspan="2">Congruence Tests for triangles</th></tr>
<tr><td>Side – side – side
SSS Congruence Test
➢ $AB = PQ$
➢ $AC = PR$
➢ $BC = QR$

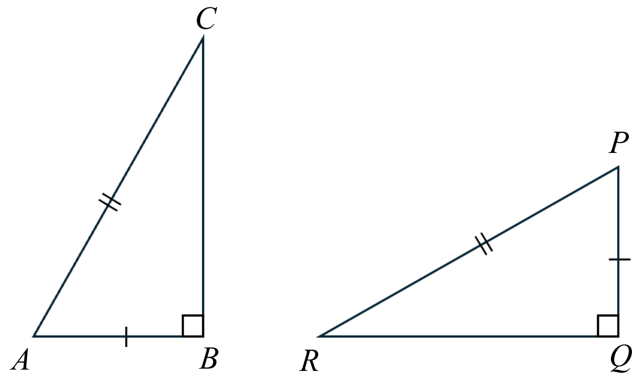
$\therefore \Delta ABC \equiv \Delta PQR$ (SSS Congruence Test)</td><td></td></tr>
<tr><td>Side – angle – side
SAS Congruence Test
➢ $AB = PQ$
➢ $\angle BAC = \angle QPR$
➢ $AC = PR$

$\therefore \Delta ABC \equiv \Delta PQR$ (SAS Congruence Test)</td><td></td></tr>
<tr><td>Angle – side – angle
ASA Congruence Test
➢ $\angle ABC = \angle PQR$
➢ $AB = PQ$
➢ $\angle BAC = \angle QPR$

$\therefore \Delta ABC \equiv \Delta PQR$ (ASA Congruence Test)</td><td></td></tr>
<tr><td>Angle – angle – side
AAS Congruence Test
➢ $\angle ABC = \angle PQR$
➢ $\angle BCA = \angle QRP$
➢ $AC = PR$

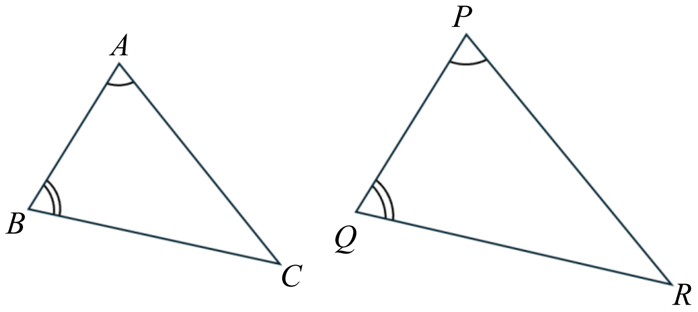
$\therefore \Delta ABC \equiv \Delta PQR$ (AAS Congruence Test)</td><td></td></tr>
</table>

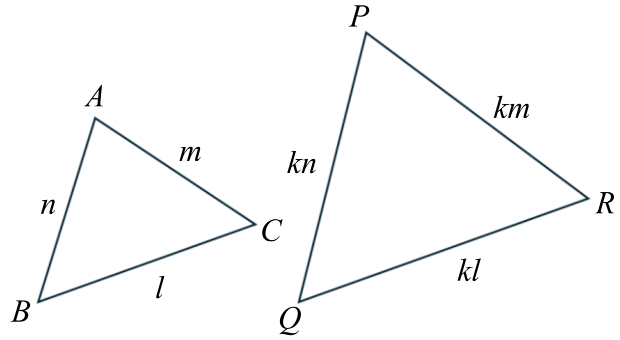
<table>
<tr>
<td>
Right angle – hypotenuse – side
RHS Congruence Test
➢ $\angle ABC = \angle PQR = 90°$
➢ $AC = PR$
➢ $AB = PQ$

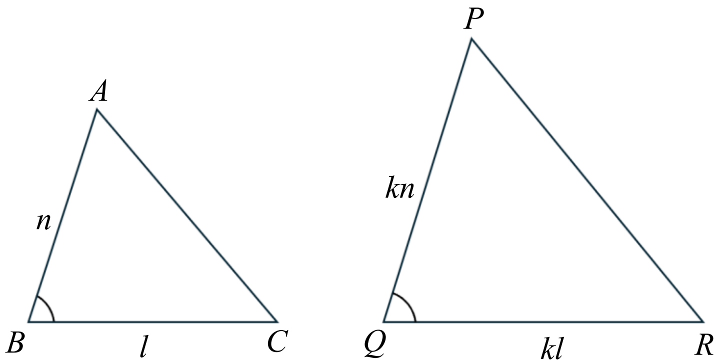
$\therefore \Delta ABC \equiv \Delta PQR$ (RHS Congruence Test)
</td>
<td>


</td>
</tr>
</table>

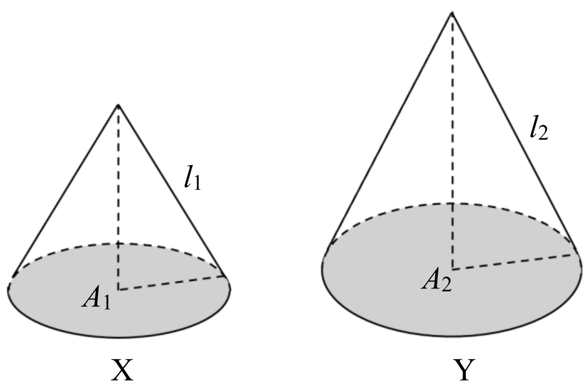
<table>
<tr><th colspan="2">Similarity Tests for triangles</th></tr>
<tr><td>Angle – angle
AA Similarity Test
➢ $\angle BAC = \angle QPR$
➢ $\angle ABC = \angle PQR$

$\therefore$ ΔABC and ΔPQR are similar. (AA Similarity Test)</td><td>
</td></tr>
<tr><td>Side – side – side
SSS Similarity Test
➢ Ratio of corresponding sides are equal.
$\frac{PQ}{AB} = \frac{QR}{BC} = \frac{PR}{AC}$

$\therefore$ ΔABC and ΔPQR are similar. (SSS Similarity Test)</td><td>
</td></tr>
<tr><td>Side – angle – side
SAS Similarity Test
➢ Ratio of corresponding sides are equal, and ‘corner’ angles are equal.
➢ $\frac{PQ}{AB} = \frac{QR}{BC}$ and $\angle ABC = \angle PQR$

$\therefore$ ΔABC and ΔPQR are similar. (SAS Similarity Test)</td><td>
</td></tr>
</table>

Area and volumes of similar figures and solids

<table>
<tr><td>If X and Y are two similar solids, then
➢ ratio of their corresponding lengths is $\frac{l_1}{l_2}$,
➢ ratio of their corresponding areas, $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$,
and
ratio of their corresponding volumes, $\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$.</td><td>
</td></tr>
</table>

Vectors: a quantity that has **magnitude** and **direction** and that is commonly represented by a directed line segment.

Let's say point A has coordinates (2, −3). It can be represented as a **column vector** in the form of $\begin{pmatrix} x \\ y \end{pmatrix}$. This is how it's written:

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

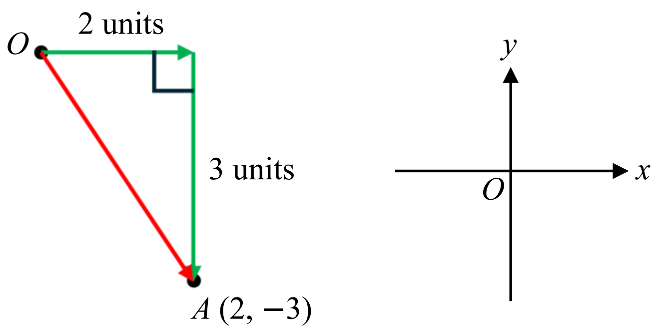
where

O is the starting point, and

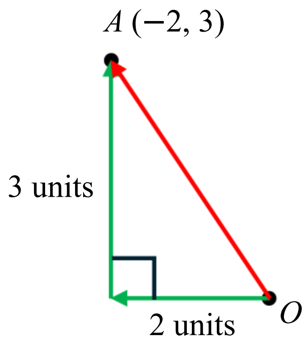
A is the ending point.

Column vector: a vector whose components are listed in a single column.

$\overrightarrow{OA} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ is a positive column vector. This means that a point starts to move 2 units to the right (positive x-axis) from the origin, O and 3 units down (negative y-axis) to point A. The line connecting from the origin to point A is called the **displacement** (distance travelled).



The negative column vector is $\overrightarrow{AO} = -\begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$. This means that the point travels the opposite direction. Instead of going South-East direction, the point travelled North-West direction. Now, the new coordinates of point A is (−2, 3).



We can use Pythagoras' Theorem to find the magnitude of vector $\overrightarrow{OA} = \sqrt{2^2 - (-3)^2}$

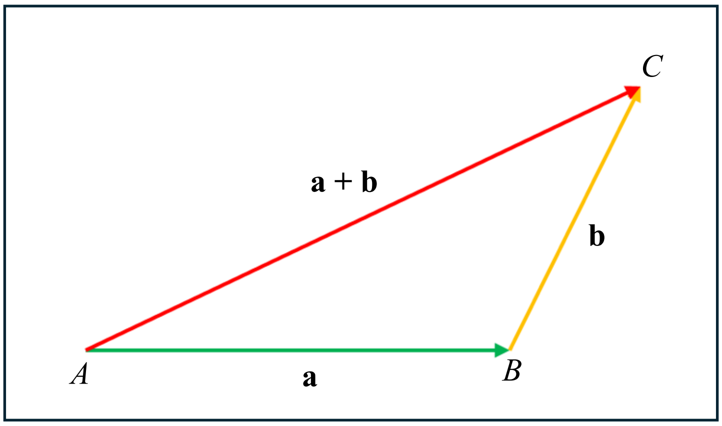
$= 5$ units.

> The magnitude of a column vector a $= \begin{pmatrix} x \\ y \end{pmatrix}$ is given by $|a| = \sqrt{x^2 - y^2}$.

Equal vectors: If two vectors **a** and **b** are the same, both vectors travel at the **same direction** and have the **same magnitude**.

<u>Addition of vectors</u>

Triangle Law of Vector Addition

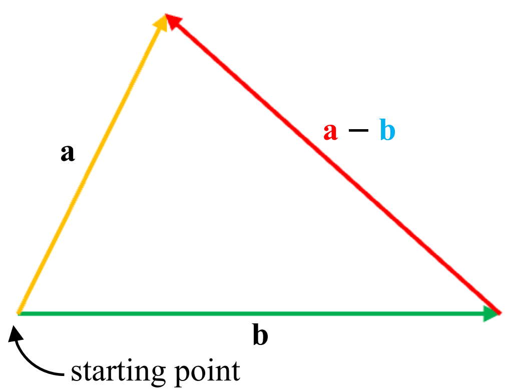
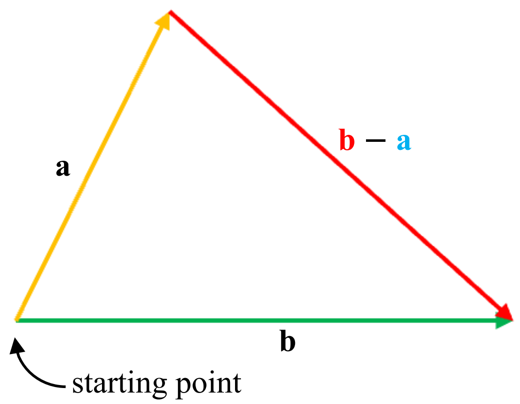


From the diagram above, we write the addition of vectors like this:

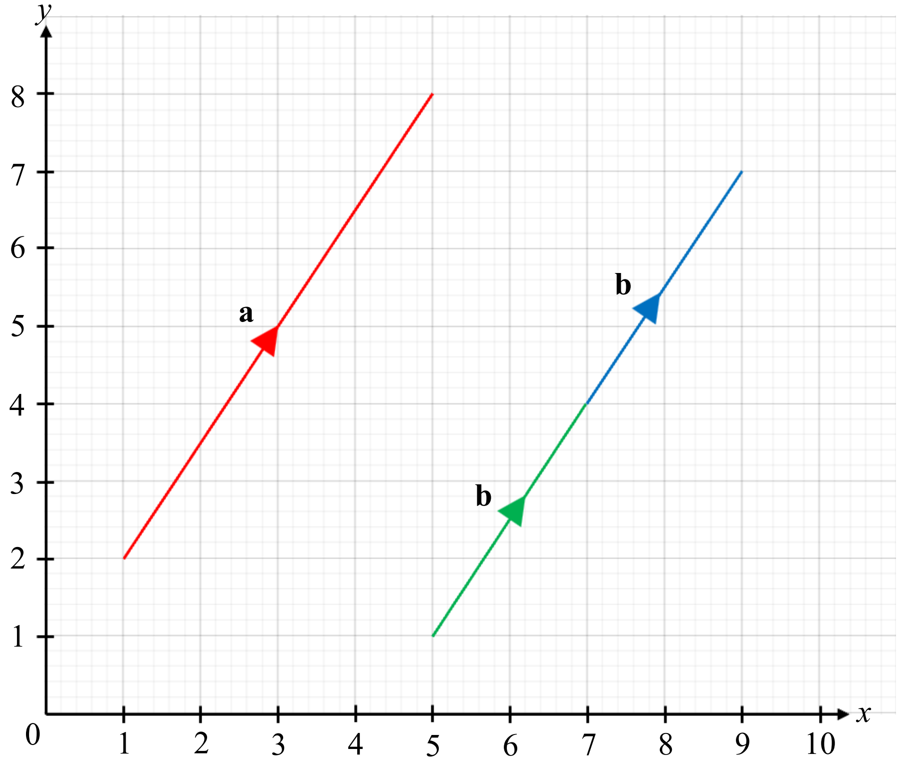
$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} = \mathbf{a} + \mathbf{b}$$

<u>Subtraction of vectors</u>

Triangle Law of Vector Subtraction

Vectors **a** and **b** start from the same point.	
 a: end point b: start point	 a: start point b: end point

Scalar multiple of a vector



We observe that $\mathbf{a} = \mathbf{b} + \mathbf{b} = 2\mathbf{b}$.

$2\mathbf{b}$ is a scalar multiply of $\mathbf{b}$.

In general,

if $\mathbf{a}$ and $\mathbf{b}$ are **parallel** vectors, then $\mathbf{a} = k\mathbf{b}$ where $k \neq 0$.

This also means that

if $\mathbf{a} = \begin{pmatrix} x \\ y \end{pmatrix}$, then $k\mathbf{a} = \begin{pmatrix} kx \\ ky \end{pmatrix}$ and $\lvert k\mathbf{a} \rvert = \lvert k \rvert \lvert \mathbf{a} \rvert$ for any real number k.

Collinear vectors: Vectors are parallel to each other, have the same gradient and the points all lie on a straight line.

For example, $\overrightarrow{PQ} = 2\mathbf{a} + 3\mathbf{b}$ and $\overrightarrow{QR} = 4\mathbf{a} + 6\mathbf{b}$. Show that points P, Q and R are collinear.

$\overrightarrow{PQ} = 2\mathbf{a} + 3\mathbf{b}$ and $\overrightarrow{QR} = 4\mathbf{a} + 6\mathbf{b}$

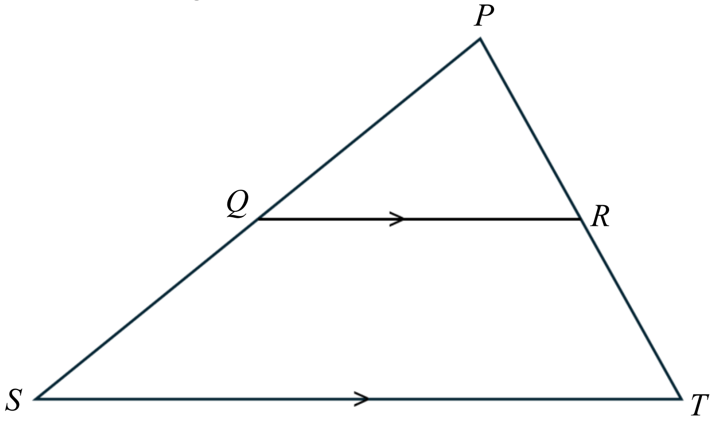
$= 2(2\mathbf{a} + 3\mathbf{b})$

$= 2\overrightarrow{PQ}$

Since $\overrightarrow{QR} = 2\overrightarrow{PQ}$ and point Q is the common point, then points P, Q and R are collinear. (shown)

Ratio of the area of three types of triangles

Similar triangles



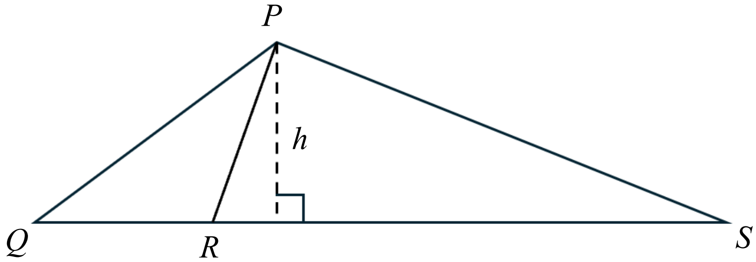
$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

$$\frac{\text{Area of } \Delta PQR}{\text{Area of } \Delta PST} = \left(\frac{QR}{ST}\right)^2$$

$$= \left(\frac{PQ}{PS}\right)^2$$

$$= \left(\frac{PR}{PT}\right)^2$$

Triangles with common heights

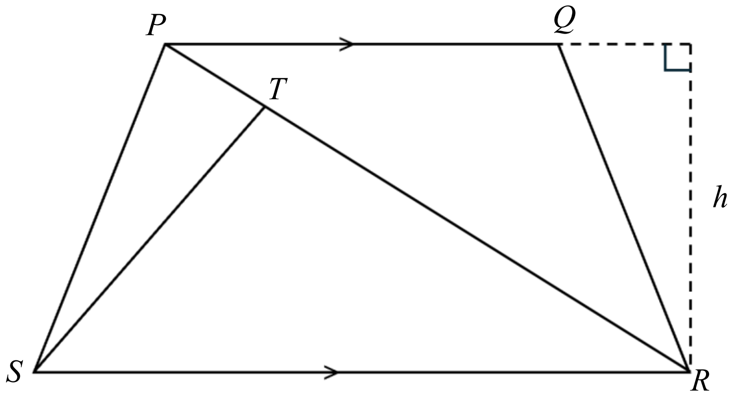


$$\frac{\text{Area of } \Delta PQR}{\text{Area of } \Delta PQS} = \frac{\frac{1}{2} \times b_1 \times h}{\frac{1}{2} \times b_2 \times h}$$

$$= \frac{\frac{1}{2} \times QR \times h}{\frac{1}{2} \times QS \times h}$$

$$= \frac{QR}{QS}$$

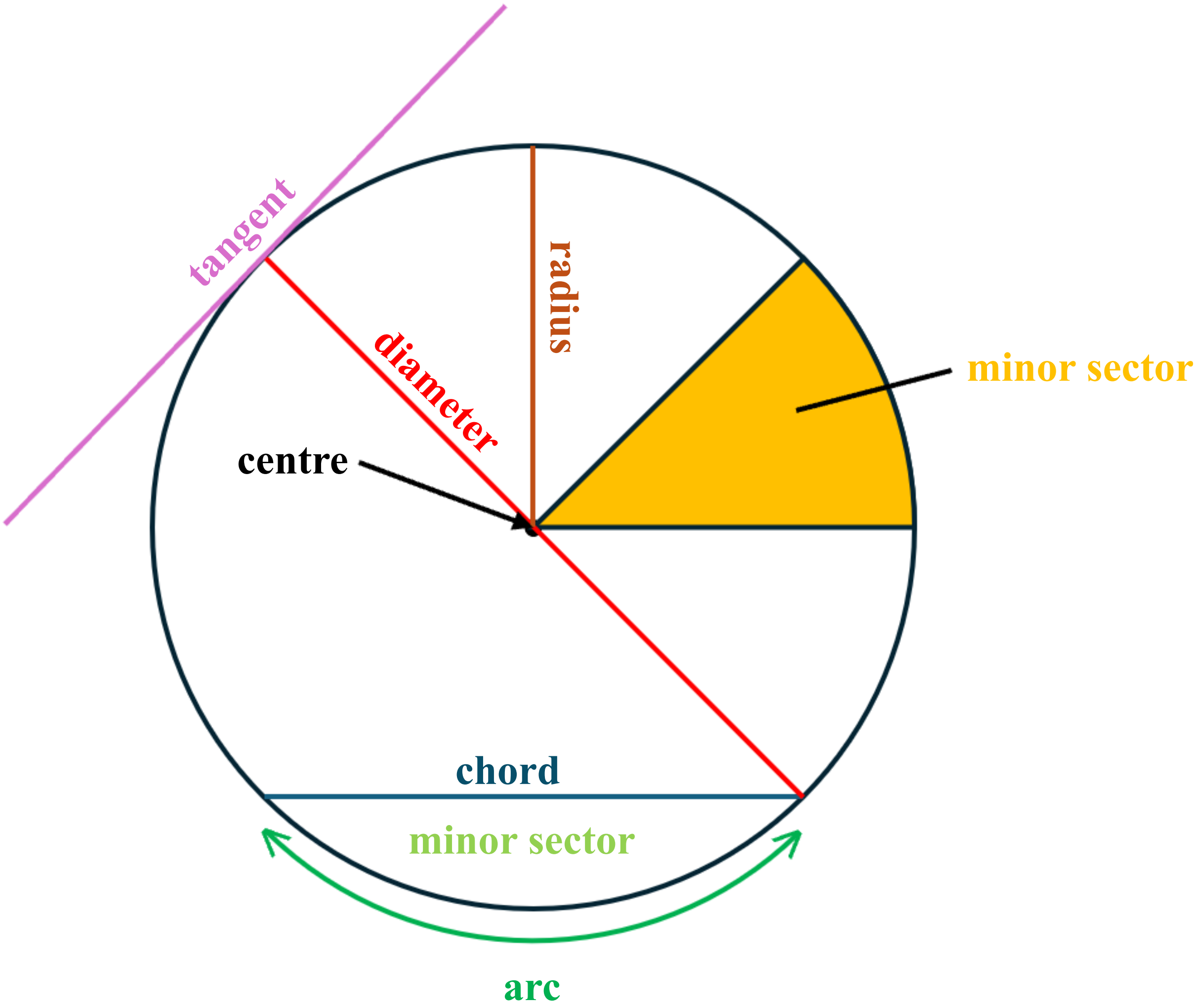
<u>Overlapping triangles</u>



$$\frac{\text{Area of } \Delta RST}{\text{Area of } \Delta PQR} = \frac{\text{Area of } \Delta RST}{\cancel{\text{Area of } \Delta PSR}} \times \frac{\cancel{\text{Area of } \Delta PSR}}{\text{Area of } \Delta PQR}$$

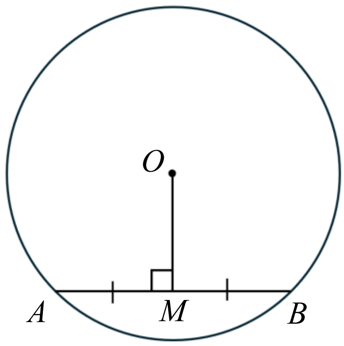
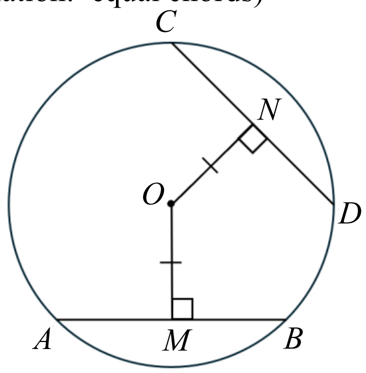
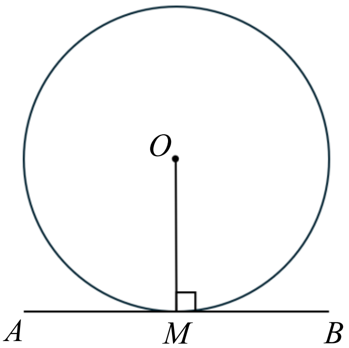
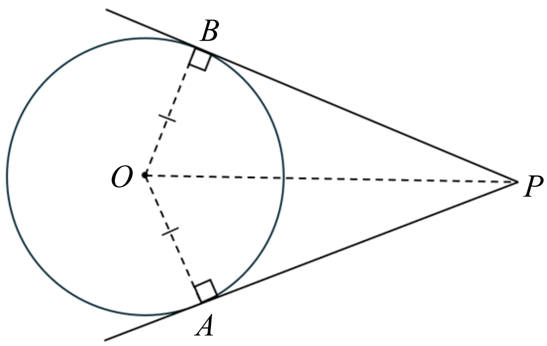
ΔPSR is the **common triangle** between the two ratios of areas.

Facts about circles



- **Centre:** A point in the middle of the circle.
- **Diameter:** A line that touches two points on the circumference of a circle and passing through the centre.
- **Radius:** A line that touches from the centre to one point on the circumference of a circle. It is also half the length of a diameter.
- **Minor sector:** A small fraction that make up a circle including a centre and two radii.
- **Minor segment:** A small fraction that make up a circle including a chord and an arc.
- **Chord:** A line that touches two points on the circumference ***without*** passing through the centre.
- **Tangent:** A line that touches the circle at only one point and is perpendicular to the diameter.

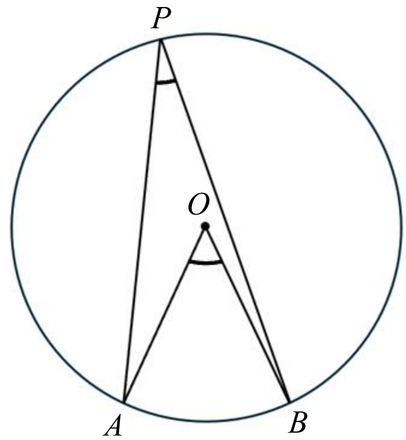
Symmetric properties of circle

	Perpendicular properties	**Equal length properties**
Properties of chords	Property 1: Perpendicular Bisector of Chord (abbreviation: $\perp$ bisector of chord)  (i) $OM \perp AB$ means OM bisects chord AB. (ii) The perpendicular bisector of a chord will pass through the centre of the *centre* of the circle.	Property 2: Equal Chords (abbreviation: equal chords)  (i) Equal chords are equidistant from the centre of the circle, if $AB = CD$, then $OP = OQ$. (ii) If two chords are equidistant from the centre of the circle, then they are equal (in length), if $OM = ON$, then $AB = CD$.
Properties of tangents	Property 3: Tangent Perpendicular to Radius (abbreviation: tangent $\perp$ radius)  The tangent to a circle is perpendicular to its radius at the point in contact, $AB \perp OM$.	Property 4: Tangents from External Point (abbreviation: tangents from ext. pt.)  (i) Tangents from an external point are equal. (ii) OP bisects $\angle APB$ and $\angle AOB$.

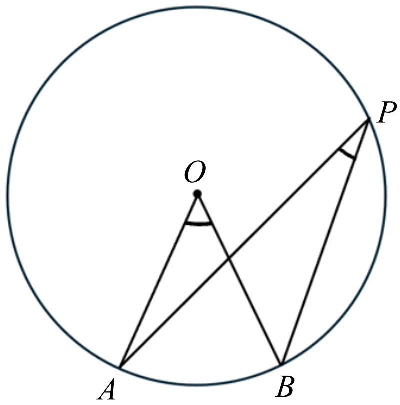
By u/One_Wishbone_4439

Angle properties of circle

Property 1: Angle at Centre
(abbreviation: ∠ at centre = 2 ∠ at circumference)

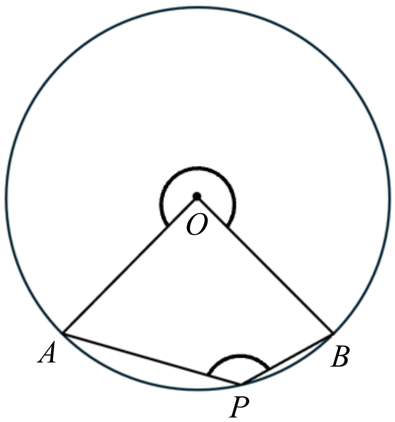


OR



$\angle AOB = 2\ \angle APB$

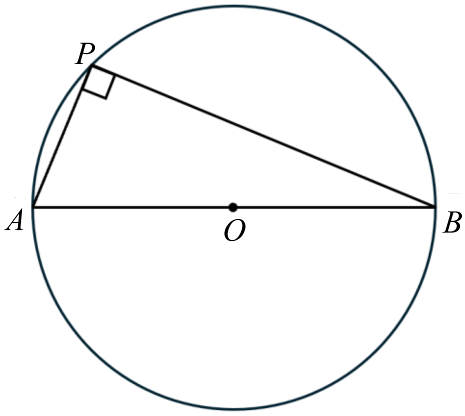
OR



reflex $\angle AOB = 2\ \angle APB$

An angle at the centre of the circle is **twice** that of any angle that lies at the circumference by the *same arc*.

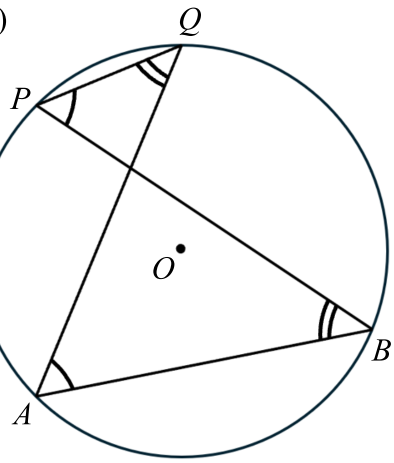
Property 2: Angle in Semicircle
(abbreviation: rt. ∠ in semicircle)



$\angle APB = 90°$

An angle in a semicircle is always a right angle, provided there is a diameter, *AOB*.

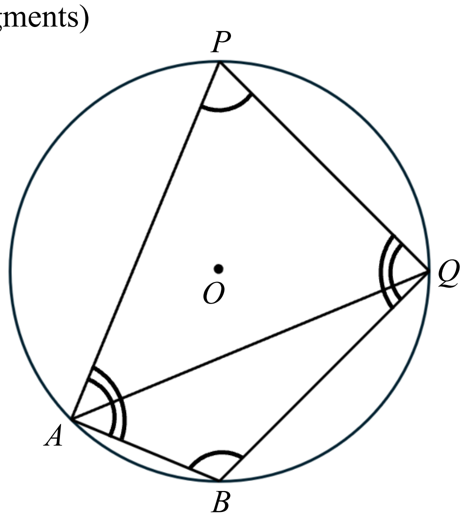
Property 3: Angles in Same segment
(abbreviation: ∠s in same segment)



$\angle QPB = \angle QAB$
$\angle PQA = \angle PBA$

Note: Just remember it looks like a shape of a 'butterfly' or a 'ribbon'. Angles in the same segment are *equal*. All *four* sides must lie on the circumference of a circle.

Property 4: Angles in Opposite Segments
(abbreviation: ∠s in opp. segments)



$$\angle APQ + \angle ABQ = 180°$$
$$\angle BAP + \angle BQP = 180°$$

The angles facing opposite each other sum up to 180°.

By u/One_Wishbone_4439

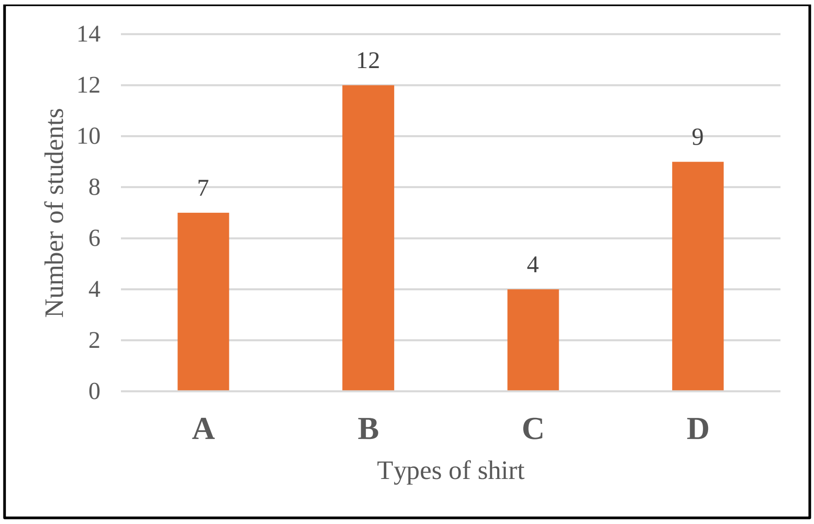
Statistic and probability

Pictogram/picture graph

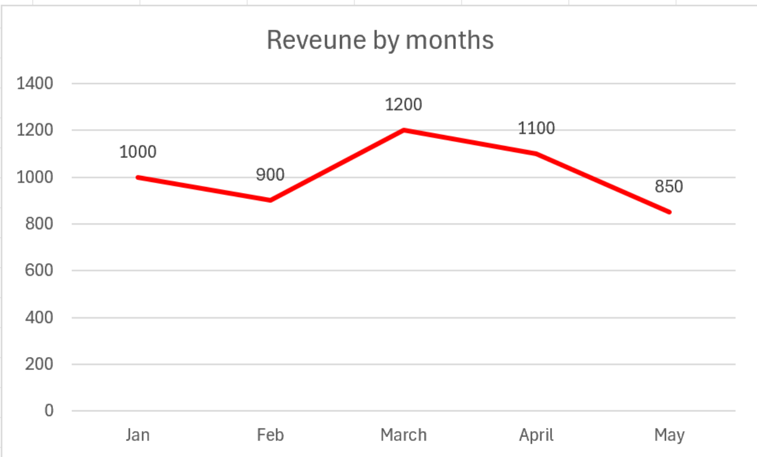
A	☼☼☼☼☼
B	☼☼☼
C	☼☼☼☼☼☼
D	☼☼☼☼☼☼☼☼☼☼

Key: ☼ represents one like

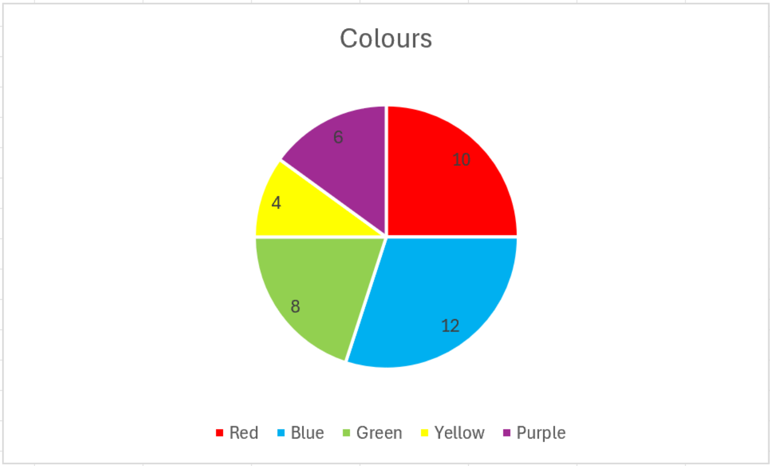
Bar graph



Line graph



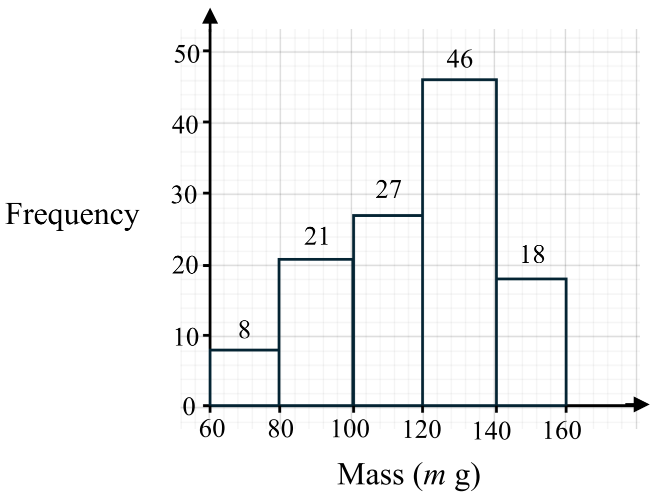
Pie Chart



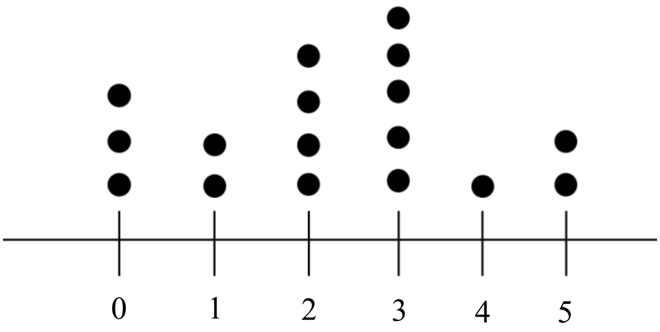
Tally and frequency tables

Height (h cm)	Tally	Frequency
$140 < h < 150$	||||	4
$150 < h < 160$	𝍸 𝍸 𝍸	15
$160 < h < 170$	𝍸 𝍸 𝍸 𝍸 |	21
$170 < h < 180$	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |||	38
$180 < h < 190$	𝍸 𝍸 ||	12

Histogram



Dot diagram



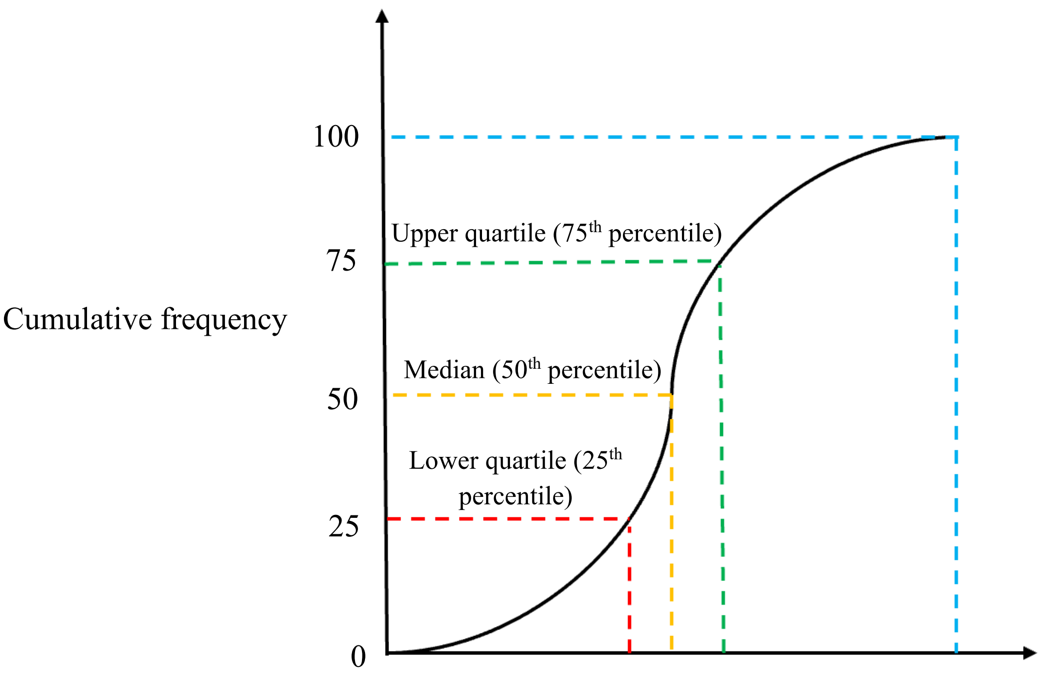
Stem-leaf-stem diagram

Boys					Stem	Girls				
		5	4	3	4	1	2	2	5	
	7	3	1	1	5	0	1	2	2	6
9	8	3	3	2	6	4	5			
			2	0	7	0				

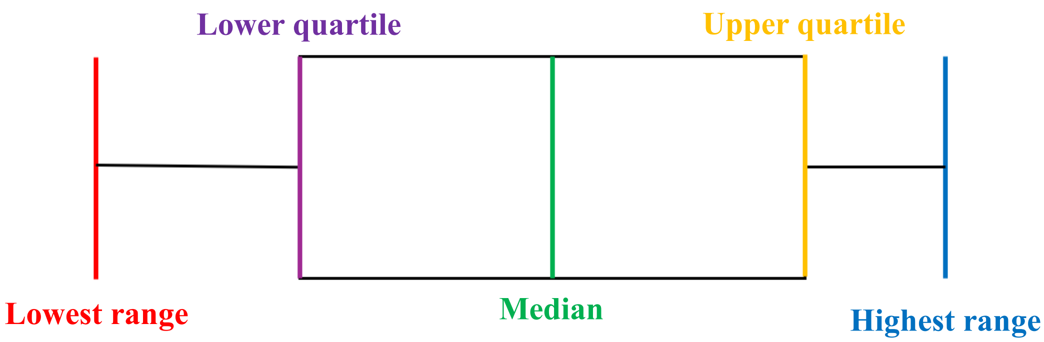
Key: 3 | 4 represents 43 kg

Key: 4 | 1 represents 41 kg

Cumulative frequency graph



Box-and-whisker diagram



Mean: It is the average of a set of data. It is calculated by dividing the total by the number of data. $\bar{x} = \frac{\sum fx}{\sum f}$

Median: It is the middle position of a series of data.

Mode: It is about most frequency or is the value that happens the most.

Interquartile range: It is a measure of how the middle 50% of the data are spread around the median. It is an appropriate measure of the spread of distribution when there are outliners.

Standard deviation: It is a measure of how the data are spread around the mean. It is an appropriate measure of the spread of distribution when there are **no** outliners.

$$\text{s.d.} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

When the **standard deviation** or the **interquartile range** of A is higher than B, it means that A is **less consistent** than B.

Probability: It is simply how likely something or an event will happen.

For example, if we say that the likelihood of getting a head after tossing a coin **once** is $\frac{1}{2}$, meaning the chances of getting a tail will also be $1 - \frac{1}{2} = \frac{1}{2}$ or 50%.

$P(x) = \frac{\text{number of outcomes}}{\text{total number of events}}$

The probability of events that **unlikely** to happen, $P'(x) = 1 - P(x)$.

Tree diagram: It helps to visualize the outcomes and how high or low the chances are.

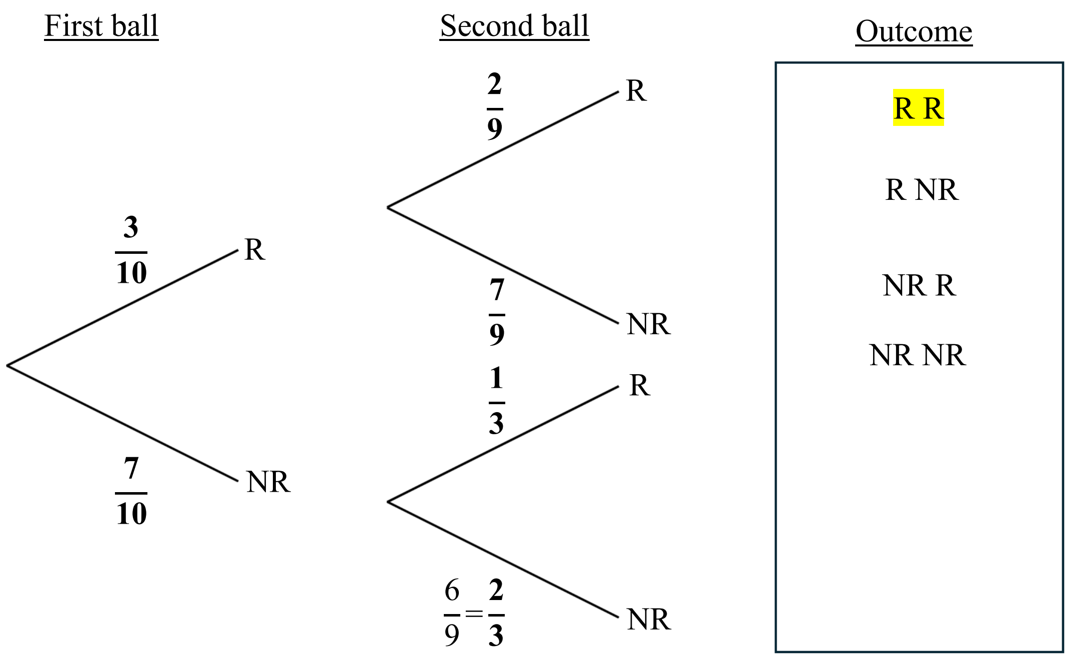
Now, if there are 10 balls in a bag, and a red ball is chosen at random once, with replacement. Given that there are 3 balls in the bag, what will be the possibility of getting a red ball?

$P(\text{red}) = \frac{3}{10}$

However, if a red ball is chosen at random twice, **without replacement**, what will be the probability of getting a red ball?

We draw a **tree diagram** for this problem.

Let R be red and NR be not red.



Hence, the probability of getting a red ball twice will be $\frac{3}{10} \times \frac{2}{9} = \frac{6}{90} = \mathbf{\frac{1}{15}}$