HILLGROVE SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2022
SECONDARY FOUR (EXPRESS) / 5 (NORMAL ACADEMIC)

CANDIDATE NAME	( )	CLASS	-
CENTRE NUMBER	S	INDEX NUMBER	

MATHEMATICS **4048/02**

Paper 2 **18 Aug 2022**

2 hours 30 minutes

8.10 AM to 10.40 AM

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your index number, class and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

For Examiner's Use	
TOTAL	100

Parent's/ Guardian's Signature: _____

Setter: Mrs Karen Ee

This document consists of **22** printed pages.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle ABC = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f}-\left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1 (a) Simplify $\frac{15x^3}{4-25x^2} \times \frac{10x-4}{3x^2}$.

Answer ..... [3]

(b) Given that $\sqrt{x-3y} : \sqrt{2x+y} = 2:3$, find the value of $\frac{2x}{y}$.

Answer ..... [3]

(c) It is given that $2x = \frac{4y - z}{3z + 1}$.

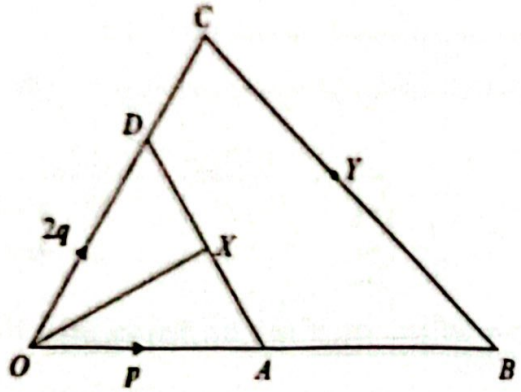
(i) Find y when $x = -1$ and $z = 8$.

Answer $y =$ .....[2]

(ii) Express z in terms of x and y.

Answer $z =$ .....[3]

2



In the diagram, $\overrightarrow{OA} = p$ and $\overrightarrow{OD} = 2q$.
$OD = 2\,DC$, $4\,CY = 3\,YB$, A is the midpoint of OB and X is the midpoint of AD.

(a) Express, as simply as possible, in terms of p and/or q,

(i) $\overrightarrow{DA}$,

Answer ..... [1]

(ii) $\overrightarrow{OX}$.

Answer ..... [1]

(b) Show that OX produced passes through Y.

Answer

Answer .....

..... [3]

3 Mr Tan opens two bubble tea shops, NiceTea and JustTea.

The matrix **T** shows the number of cups of each type of bubble tea that are sold on a particular day.

$$\mathbf{T} = \begin{matrix} \\ \text{NiceTea} \\ \text{JustTea} \end{matrix} \begin{matrix} \begin{matrix} \text{milk tea} & \text{fruit tea} & \text{oolong tea} \end{matrix} \\ \begin{pmatrix} 45 & 25 & 20 \\ 36 & 30 & 28 \end{pmatrix} \end{matrix}$$

(a) The selling price of each cup of milk tea, fruit tea and oolong tea is \$4.50, \$3 and \$3.50 respectively.
Represent these amounts in a 3×1 column matrix **C**.

Answer **C** = [1]

(b) Evaluate the matrix **S** = **TC**.

Answer **S** = [2]

(c) State what the second element of matrix **S** represents.

Answer .....

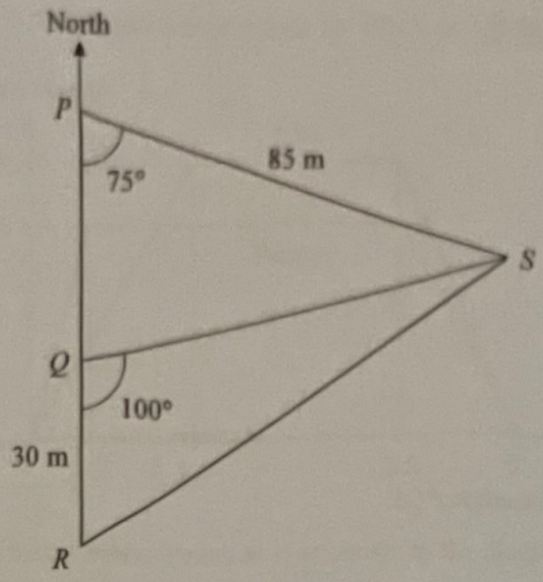
..... [1]

(d) Due to inflation, Mr Tan decides to increase the selling price of all three types of beverages. The selling price of the beverages sold in NiceTea and JustTea increases by 30% and 16% respectively.
Write down another matrix **Q** such that **QS** gives the total sales revenue of both bubble tea shops after the price increase.

Answer **Q** =

QS = [2]

4



The diagram shows a triangular laser tag enclosure $PQRS$ on horizontal ground.
P is due north of Q and R.
$PS = 85$ m and $QR = 30$ m.
Angle $SPQ = 75°$ and angle $SQR = 100°$.

(a) Show that the length of $QS = 83.37$ m, correct to 2 decimal places.

Answer

[2]

(b) Calculate

(i) the bearing of Q from S,

Answer ..... ° [1]

(ii) the length of RS.

Answer ..... m [2]

(iii) the area of triangle PQS.

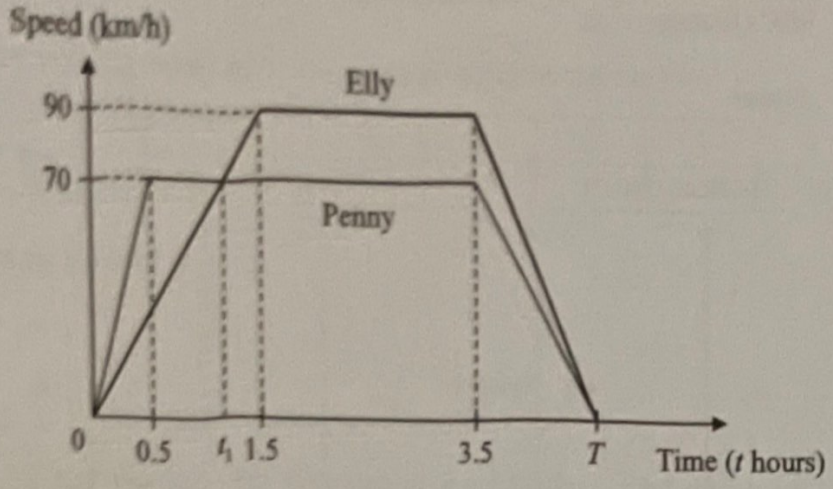
Answer ..... m^2 [2]

(c) In a laser tag game, Pete ran along the path SP at a speed of 28.8 km/h. Sammie, who was hiding at the top of a guard tower at point Q, spotted Pete running from point S towards point P. She fired a shot that hit Pete when he was closest to the guard tower.

Find the time, in seconds, that elapsed from the instant Sammie spotted Pete at point S to the instant Sammie fired the shot.

Answer ..... s [3]

5 The diagram shows the speed-time graphs for Elly's and Penny's journey.



(a) Elly will first overtake Penny at t_1 as shown in the diagram above.
Do you agree with the above statement? Give a reason for your answer.

Answer .....

.....

..... [2]

(b) Calculate the time, in hours, in which Elly will overtake Penny.

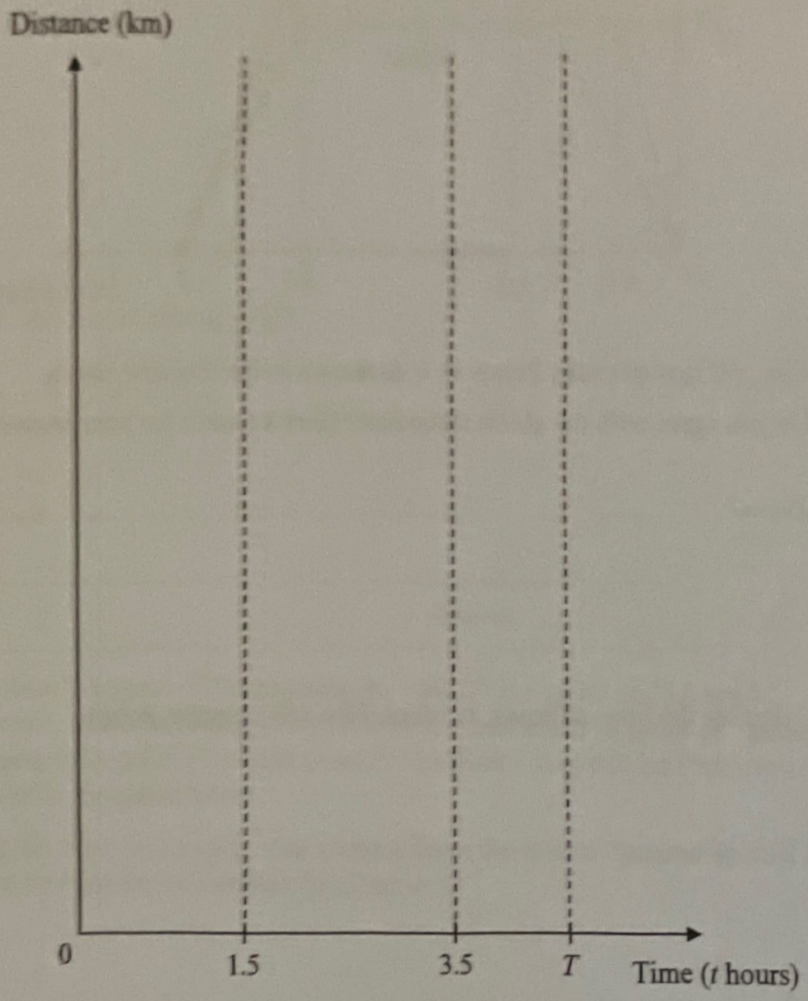
Answer ..... hours [3]

(c) At $t = 3.5$, both Elly and Penny begin to decelerate uniformly at 90 km/h^2 and 70 km/h^2 respectively before coming to rest at T. Find the value of T.

Answer $T =$ ..... [1]

(d) On the axes in the answer space, sketch the distance-time graph for the whole of Elly's journey. [3]

Answer



6 The variables x and y are connected by the equation $y=\frac{1}{2}x(10-x^2)$.

Some corresponding values of x and y are given in the table below.

x	−3	−2	−1	0	1	2	3	4
y	−1.5	−6	−4.5	0	4.5	p	1.5	−12

(a) Find the value of p.

Answer $p=$ ..... [1]

(b) On the grid opposite, draw the graph of $y=\frac{1}{2}x(10-x^2)$ for $-3\le x\le 4$. [3]

(c) By drawing a tangent, find the gradient of the curve at $(-1,-4.5)$.

Answer ..... [2]

(d) **(i)** On the same grid, draw the line $y=5-3x$. [2]

(ii) Write down the $x-$coordinate of the points where this line intersects the curve.

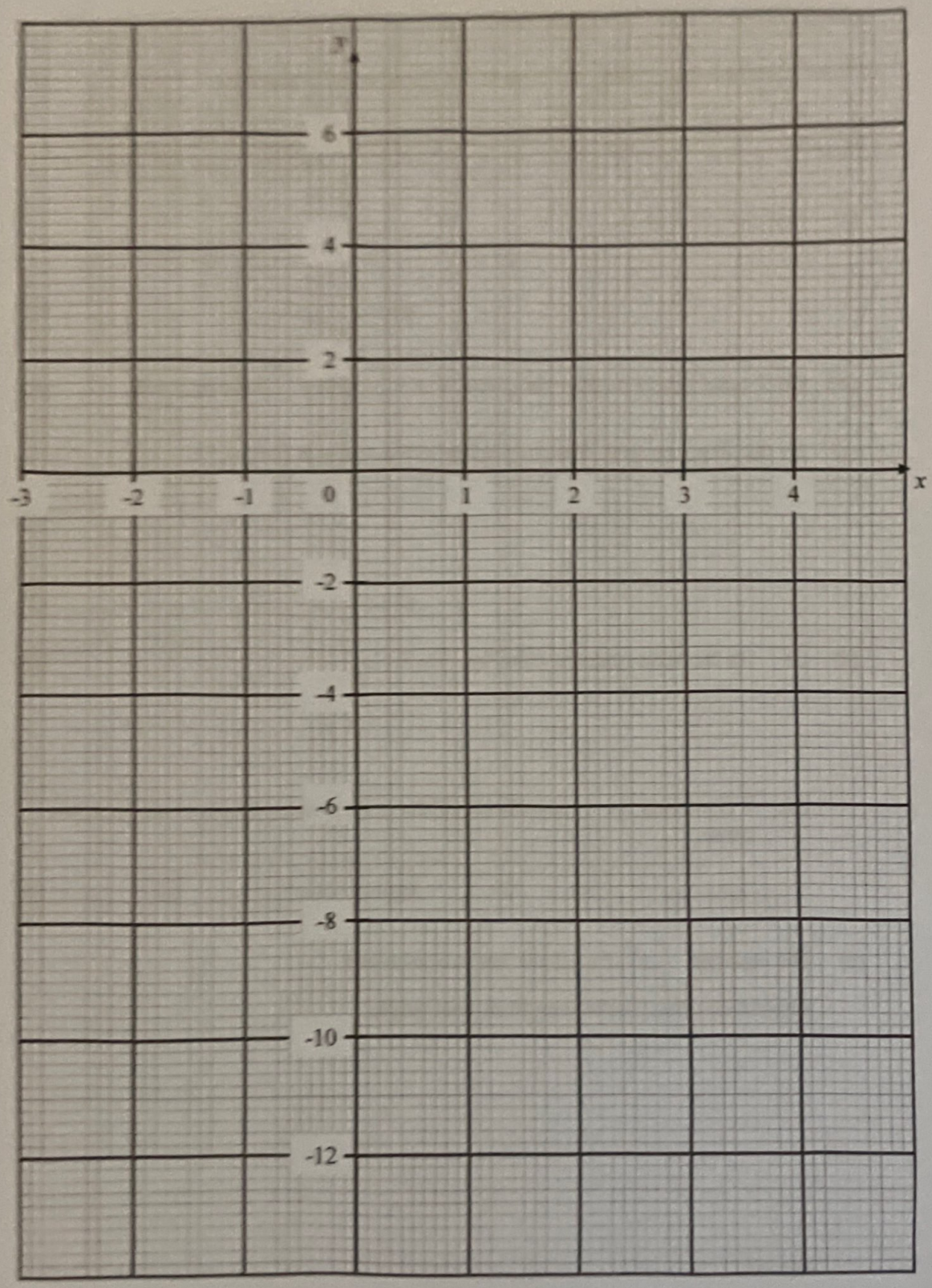
Answer $x=$ ....., $x=$ ..... [2]

(e) The equation $\frac{1}{2}x^3-5x+2=0$ has two solutions.

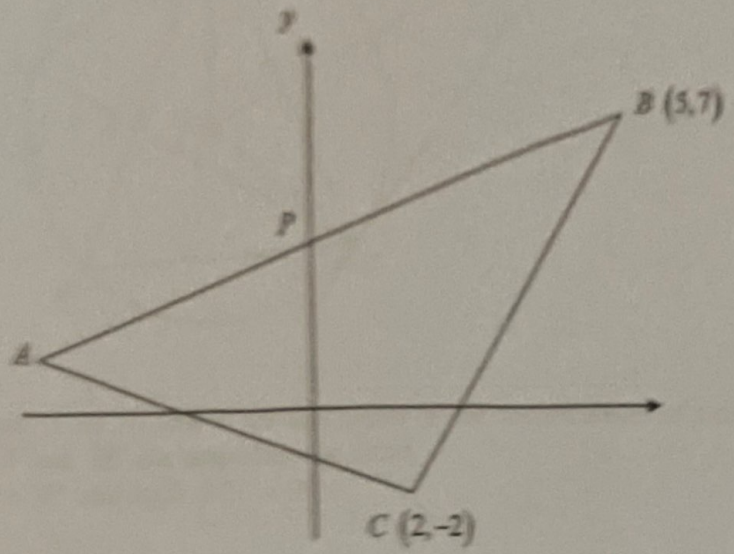
Explain how this can be seen from your graph.

Answer .....

..... [2]



7 The diagram shown a triangle ABC, where the coordinates of B and C are $(5,7)$ and $(2,-2)$ respectively. Line AB intersects the y-axis at point P.



(a) Given that the gradient of AB is $\frac{2}{3}$, find the equation of the line AB.

Answer .....[2]

(b) State the coordinates of P.

Answer P = ...[1]

(c) Given that the y-coordinate of A is 1, find the coordinates of A.

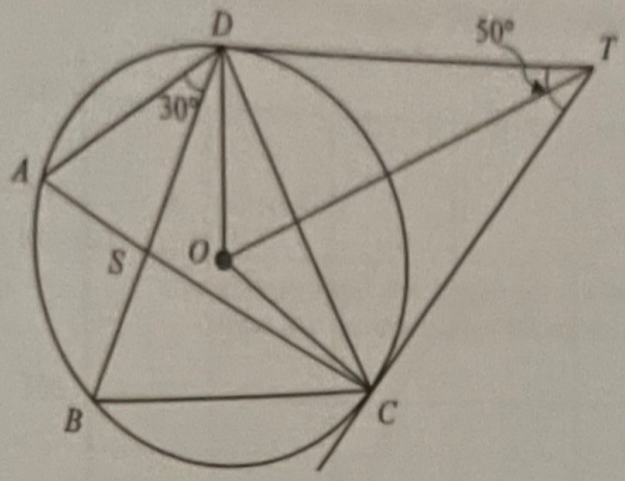
Answer A = ...[2]

(d) Find $\frac{\text{area of triangle } APC}{\text{area of triangle } BPC}$.

Answer ..... [3]

(e) Find the coordinates of Q such that $ABQC$ forms a parallelogram.

Answer $Q =$ ..... [1]



In the diagram, A, B, C and D are four points on the circumference of a circle with centre O. CT and DT are tangents to the circle.
Angle $ADB = 30°$ and angle $DTC = 50°$.

(a) Find angle BCS.
Give a reason for each step of your working.

Answer Angle BCS = ... ° [1]

(b) Find angle DAC.
Give a reason for each step of your working.

Answer Angle DAC = ... ° [2]

(c) **(i)** Name a pair of similar triangles.

Answer Δ ..... and Δ ..... [1]

(ii) Explain why the triangles in **(c)(i)** are similar.
Give a reason for each statement you make.

Answer .....

.....

.....

.....

..... [2]

(d) **(i)** State a triangle that is congruent to triangle *DOT* .

Answer Δ .....[1]

(ii) Explain why the triangles in **(d)(i)** are congruent.
Give a reason for each statement you make.

Answer .....

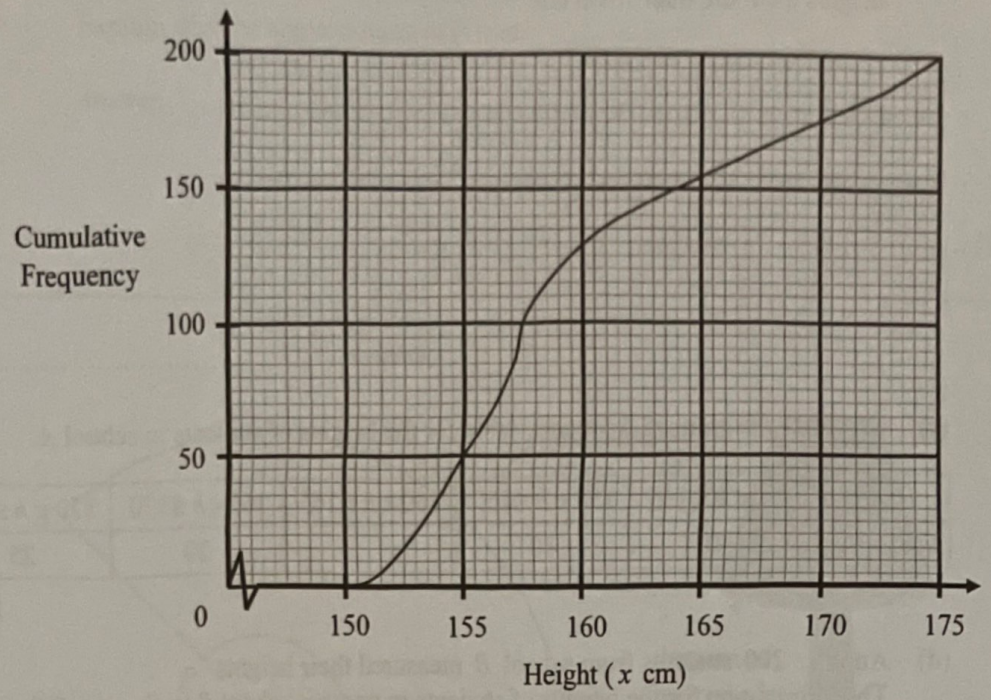
.....

.....

.....

..... [2]

9 The cumulative frequency curve illustrates the heights of 200 students in school A.



(a) Use the curve to find

(i) the median height,

Answer ..... cm [1]

(ii) the interquartile range of the heights,

Answer ..... cm [1]

(iii) the sixtieth percentile height.

Answer ..... cm [1]

(b) Two students are selected at random. Find the probability that both of their heights are more than 167.5 cm.

Answer ..... [2]

(c) Complete the grouped frequency table for the heights of students in school A.

Height	$150 < h \leq 155$	$155 < h \leq 160$	$160 < h \leq 165$	$165 < h \leq 170$	$170 < h \leq 175$
Frequency	50	80		20	25

[1]

(d) Another 200 students from school B measured their heights.
The information for the heights of students in another school B is given as follows:

School *B*
Mean = 166 cm
Standard Deviation = 5.25 cm

Make two comparisons between the heights of the students in schools A and B.

Answer .....

.....

.....

..... [4]

(e) Mrs Ee commented, "Mean is not a good gauge of the distribution of the students' heights in school *A* as compared to the median."

Explain why the above comment is true.

Answer .....

.....

..... [1]

10

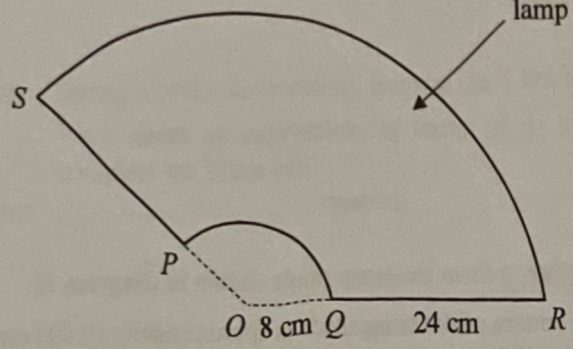


Diagram I

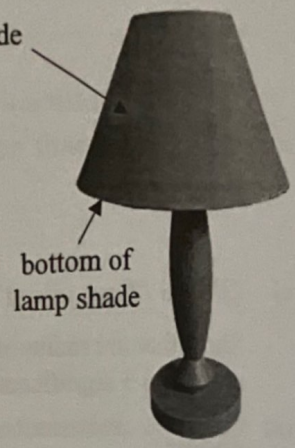


Diagram II

Diagram I shows the arc PQ and arc SR of two circles with centre O.
$OQ = 8$ cm and $QR = 24$ cm. Perimeter of $PQRS$ is 132 cm.

(a) Show that angle POQ is 2.1 radians.

Answer

[2]

(b) Find the area of $PQRS$.

Answer ..... cm^2 [2]

(c) QR and PS are joined together to form the lamp shade shown in **diagram II**.

Show that the radius of the bottom of the lamp shade is approximately 10.695 cm, correct to 5 significant figures.

Answer

[2]

11 The Standard Chartered Singapore Marathon (SCSM) is an annual international marathon race and the only World Athletics Gold Label road race in Southeast Asia. It is held annually on the first Sunday of December in Singapore. Due to Covid, the race was suspended for two years.

Joshua and Benson are planning to participate in the half-marathon race which is 21km. They hope to beat the 2019 record time of 2 hours and 19 minutes.

(a) During a particular training, Joshua ran at an average speed x km/h.
Write down an expression, in terms of x, for the time taken by Joshua to complete his 21km run.

Answer ..... [1]

(b) During a particular training, Benson ran 1 km/h slower than Joshua.
Write down an expression, in terms of x, for the time taken by Benson to complete his 21km run.

Answer ..... [1]

(c) The difference in both their timings was 16 minutes.
Write down an equation in x to represent this information, and show that it reduces to

$$4x^2 - 4x - 315 = 0.$$

Answer

[3]

(d) Solve the equation $4x^3 - 4x - 315 = 0$.
Explain why one of the solutions is rejected.

Answer $x =$ ..... or $x =$ .....

..... [4]

(e) From the answer obtained in **(d)**, do you think Joshua and Benson can beat the 2019 record timing of 2 hours and 19 minutes?
Justify your answer by showing your calculations clearly.

Answer .....

..... [3]