Chapter 19 QUANTUM PHYSICS

H2 PHYSICS 9749

Content

- Energy of a photon
- The photoelectric effect
- Wave-particle duality
- Energy levels in atoms
- Line spectra
- X-ray spectra
- The uncertainty principle

Learning Outcomes

Candidates should be able to:

(a) show an appreciation of the particulate nature of electromagnetic radiation.

(b) recall and use $E = hf$.

(c) show an understanding that the photoelectric effect provides evidence for a particulate nature of electromagnetic radiation while phenomena such as interference and diffraction provide evidence for a wave nature.

(d) recall the significance of threshold frequency.

(e) recall and use the equation $\frac{1}{2} mv_{max}^2 = eV_s$, where V_s is the stopping potential.

(f) explain photoelectric phenomena in terms of photon energy and work function energy.

(g) explain why the stopping potential is independent of intensity whereas the photoelectric current is proportional to intensity at constant frequency.

(h) recall, use and explain the significance of $hf = \Phi + \frac{1}{2} mv_{max}^2$.

(i) describe and interpret qualitatively the evidence provided by electron diffraction for the wave nature of particles.

(j) recall and use the relation for the de Broglie wavelength $\lambda = h/p$.

(k) show an understanding of the existence of discrete electron energy levels in isolated atoms (e.g. atomic hydrogen) and deduce how this leads to spectral lines.

(l) distinguish between emission and absorption line spectra.

(m) recall and solve problems using the relation $hf = E_1 - E_2$.

(n) explain the origins of the features of a typical X-ray spectrum.

(o) show an understanding of and apply $\Delta p \Delta x \gtrsim h$ as a form of the Heisenberg position-momentum uncertainty principle to new situations or to solve related problems.

19 Introduction

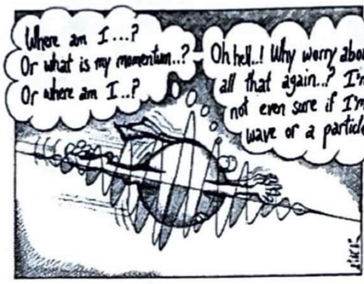
Modern Physics

EM radiation – Wave or Particle?

Wave theory of light has been highly successful in describing phenomena such as diffraction, interference and polarization, leading to the creation of radio waves, radar speed detectors and optical filters, just to name a few. However, wave theory of light is unable to explain phenomena such as blackbody radiation, **photoelectric effect** and **atomic line spectra**.

The search for an explanation for these experiments led to major breakthrough at the beginning of the 20th century, resulting in the birth of modern physics – **quantum mechanics** and **Einsteinian relativity**. While quantum mechanics deals with the "lows" (low temperature, small sizes), Einsteinian relativity deals with the "highs" (high velocities, large distances).

In this chapter, we will discuss important aspects of quantum theory – **quantisation, duality,** and **uncertainty**. The concept of quantisation was first suggested by Max Planck during his study of blackbody radiation in 1900. Inspired by Planck's work, Einstein theorised that EM radiation itself is quantised – that EM radiation transport energy in quanta called **photons** – and successfully explained the photoelectric effect in 1905. James Franck and Gustav Hertz then showed conclusively that energy levels in atoms are quantised in 1914, which then provided an explanation for atomic line spectra and the production of x - ray.



The bizarre behaviour of EM radiation – that it has both wave and particle properties – led de Broglie to propose the **wave-particle duality** for EM radiation and matter in his doctoral dissertation in 1924. His idea was way ahead of its time and there was a real possibility that de Broglie failed his doctoral interview, if not for the intervention by Einstein. Experiments conducted by Clinton Davisson and Lester Germer in 1927 showed that electrons, indeed, have wave-like properties.

The duality in behaviour of matter renders the classical "deterministic" picture – that a particle follows a well-defined trajectory – moot. Instead, quantum theory describes our universe as inherently **probabilistic**. The uncertain behaviour in the microscopic world is encoded in the Heisenberg Uncertainty Principle, first introduced by Werner Heisenberg in 1927.

Nonetheless, the unity of physics as a discipline only grew stronger through this quantum revolution – the deep ideas of classical physics provided some basis for the radical new ideas, and the quantum formalism naturally agreed with the classical one when applied in the realm of macroscopic systems.

Today, quantum mechanics is essential to the understanding in many fields of physics and chemistry, including condensed matter physics, solid-state physics, atomic physics, molecular physics, computational physics and chemistry, quantum chemistry, particle physics, nuclear physics, and chemistry.

These understanding, in turn, spurred many technological inventions that are essential in modern day living. The quantum computer, superconducting magnetic, light-emitting diode, laser, the transistor, electron microscope and magnetic resonance imaging would not be possible without quantum mechanics.

19.1 The Photoelectric Effect

The Photoelectric Effect

Evidence of the photoelectric effect was first discovered in 1887 by Heinrich Hertz (German, 1857-1894). In his experiments to prove the existence of electromagnetic radiation, he discovered that electrodes illuminated with ultraviolet light create electric sparks more easily.

This phenomenon, known as the **photoelectric effect**, can be easily demonstrated with the gold-leaf electroscope shown in **Fig. 19.1**.

First, the electroscope is given a positive charge. When ultraviolet (UV) light of wavelength 254 nm is incident on the zinc plate, the leaf divergence remains the same. This process is repeated with a negatively-charged electroscope and the leaf divergence fell rapidly. Lastly, no change in the leaf divergence is observed when UV light of wavelength 365 nm or a much brighter (higher intensity) white light source is used on a negatively charged electroscope.

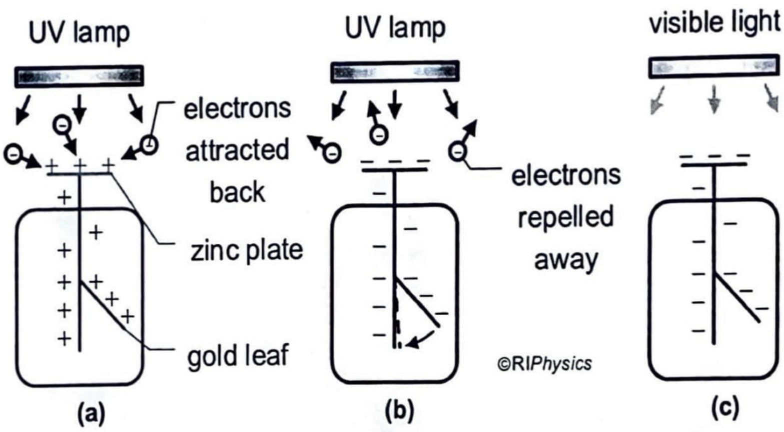


Fig. 19.1 (a) No change in leaf divergence as electrons are attracted back to zinc plate. **(b)** Leaf divergence decreases rapidly as electrons are repelled away. **(c)** No change in leaf divergence since no electrons are emitted.

Based on these observations, we can conclude that negatively-charged particles escaped from the surface of the zinc plate when light is incident on it but this effect is **dependent on frequency / wavelength** but **not on intensity** of light.

When the specific charge (q/m) of these ejected particles were measured, it was found that they were indeed electrons. There electrons are commonly referred to as **photoelectrons.**

Definition:

The **photoelectric effect** *is a process in which <u>electrons are emitted</u> from a metal surface when <u>electromagnetic radiation of sufficiently high frequency</u> is incident on the surface.*

Definition:

A ***photoelectron*** is an electron emitted from the surface of a material due to the incident electromagnetic radiation.

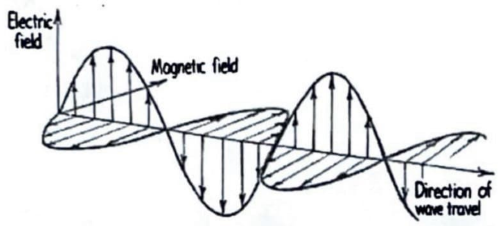
EM radiation – Wave or Particle?

Theories of Light

Two apparently contradictory theories of the nature of light were competing in the seventeenth century.

The **corpuscular theory** proposed by Sir Issac Newton (English, 1643-1727) regarded light as stream of tiny particles travelling at high speed in a straight line. The corpuscular theory did account for rectilinear propagation, reflection and refraction – the latter by assuming that on entering an optically denser medium the corpuscles are attracted, thereby causing bending towards the normal. On the other hand, the theory proposed by Christiaan Huygens (Dutch, 1629-1695) in 1678 considered light as a wave. His **wave model** can also account for reflection and refraction.

Opponents of the wave theory argued that waves require a medium for transmission and there did not appear to be one for light which was able to travel in vacuum. Subsequently a medium, called the ether, was invented but it defied all attempts at detection. An apparently crucial difference between the two theories was that whereas the corpuscular theory predicted light to have a greater speed in any medium denser than air while the wave theory predicted the opposite. The wave theory prediction was confirmed in 1862 when Jean Bernard Léon Foucault (French, 1819-1868) found the speed of light in water was less than that in air. In the meantime, Thomas Young (English, 1773-1829) demonstrated his famous Young's double-slit experiment in 1803 which were readily explicable in terms of waves.



The need for the ether disappeared when James Clerk Maxwell (Scottish, 1831-1879) suggested in 1864 that light, as an electromagnetic (EM) radiation, can travel through empty space at the speed of light. Maxwell's equations also predicted the existence of very low frequency radio waves (discovered in 1886). Since then, the entire EM spectrum also includes microwaves (1888), infra-red (1800), ultraviolet (1801), X-rays (1895) and gamma rays (1900). By the end of the nineteenth century Maxwell's **electromagnetic wave theory of light** had established itself as one of the great unifying principle of electricity, magnetism and light.

One plausible explanation of the photoelectric effect, according to wave theory, is that when EM radiation is incident on a surface of a material, the oscillating electric field causes the electrons to oscillate. Over time, these electrons may acquire sufficient energies to escape from the surface.

Furthermore, the wave theory made the following predictions:

Note

1. There should be a **time delay** (see example 1) between the incidence of EM radiation on the surface and emission of electrons from the surface. This is because energy of the EM radiation is spread uniformly over its entire wave front and each electron absorbs only a small fraction of this energy. *

2. Using EM radiation of **higher intensities**, electrons should be **emitted earlier** at a **higher rate** and **with greater energies**. This is because the energy per unit time incident on the surface by the EM radiation is directly proportional to its intensity. *

*see further elaboration in later section.

Photoelectric effect:

The Photoelectric Experiment

The experimental set up in **Fig. 19.2** is used to study the photoelectric effect.

The potential of **collector C** w.r.t. **emitter E**, **V_{CE}**, can be varied (positive or negative) by adjusting the d.c. supply while the rate with which photoelectrons reach collector C from emitter E can be measured by the sensitive ammeter.

Schematic of photoelectric experiment setup

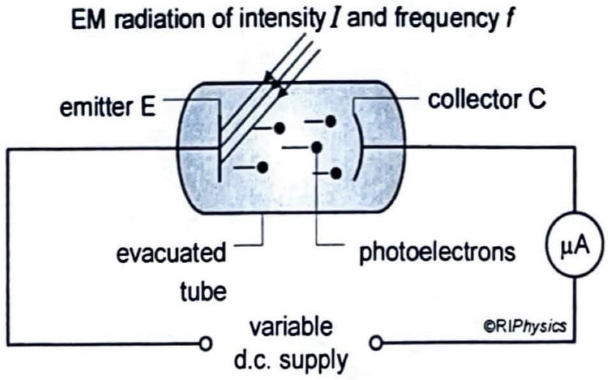


Fig. 19.2 The sensitive ammeter measures the rate at which electrons reaches the collector at a fixed potential between collector and emitter.
Java applet: https://phet.colorado.edu/en/simulation/photoelectric

When no EM radiation is incident on emitter E, no photoelectrons are liberated and therefore none arrives at collector C. So, the ammeter reads **zero photoelectric current.**

When photoemission occurs:

However, when EM radiation such as UV light is incident on emitter E and

Case 1	collector C is sufficiently positive w.r.t. emitter E (V_{CE} = +ve)	• Every single photoelectron is attracted to C • Rate of emission of photoelectrons = rate with which photoelectrons reach collector C • Ammeter reads a **constant saturated** (or **maximum**) photocurrent i_0. rate of emission of $e^- = \frac{dN_e}{dt} = \frac{i_0}{e}$
Case 2	collector C is sufficiently negative w.r.t. emitter E (V_{CE} = −ve)	• Most energetic photoelectrons do not have sufficient energy to reach collector C • Photocurrent i is **zero** max. K.E. of $e^- = \text{K.E.}_{max} = eV_s$

Note

Stopping Potential V_s

Definition:

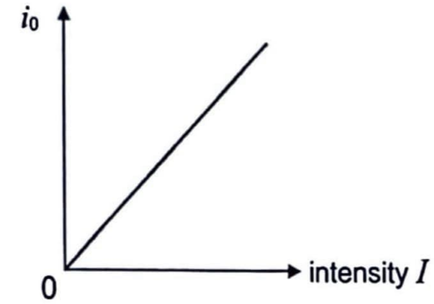
> The magnitude of the negative potential of collector with respect to emitter which prevents the most energetic photoelectrons from reaching the collector and hence resulting in zero photocurrent.

Experimental Observations

An experimental study of the photoelectric effect yields some surprising results which may be summarised as follows:

Observations:

Concurs with wave theory?



1. The **saturated photocurrent** (i.e. rate at which photoelectrons are emitted) is **proportional to the intensity** of the monochromatic EM radiation at a constant frequency.

F

2. <u>Existence of the Threshold Frequency</u>

   For a given material, there is a certain minimum frequency of EM radiation, called the **threshold frequency**, below which no emission of photoelectrons occurs irrespective of the intensity of the EM radiation

I

3. <u>Max. K.E. Independent of Intensity</u>

   The photoelectrons are emitted with a range of energies from zero up to a **maximum** which **increases linearly with frequency** but is **independent of intensity** of the EM radiation.

T

4. <u>Insignificant Time Lag</u>

   The time delay between the start of EM radiation to the emission of photoelectrons is **nearly zero** (~ 20 atto-sec or 2.0×10^{-17} s). (i.e. almost **immediate emission** of photoelectrons)

Contradictions with Classical Wave Theory

According to the classical wave theory, the intensity of a wave is defined as the energy incident per unit area per unit time at the point of observation, and the energy carried by an electromagnetic wave is proportional to the square of the amplitude of the wave.

$$\text{Intensity, } I = \frac{E}{tA} \propto (\text{Amplitude of wave})^2$$

where E is the energy incident perpendicularly to an area A over a time interval t.

Physicists realised that the classical wave theory cannot explain the last three experimental observations above.

F

1. **Existence of the threshold frequency**

   Since energy of the wave is dependent on the square of its amplitude, the classical wave theory predicts that if sufficiently intense light is used, the electrons would absorb enough energy to escape. There should not be any threshold frequency.

I

2. **Maximum K.E. of photoelectron (and hence stopping potential) is independent of intensity.**

   According to classical wave theory, by using light with higher intensity, the stopping potential V_s should increase as the incident light of higher intensity should eject photoelectrons with greater K.E. The classical wave theory cannot explain why the stopping potential is independent of intensity (but dependent on the frequency).

T

3. **Insignificant time lag**

   Based on classical wave theory, electrons should absorb energy over a period of time before it gains enough energy to escape the metal. Accordingly, a dim light would transfer sufficient energy to the electrons for ejection after some delay, whereas a very bright light would eject electrons after a short while. However, photoelectrons were emitted almost immediately even with low intensity radiation that was above the threshold frequency.

The Development of Quantum Theory

Quantum theory of Light

Einstein's theory of photoelectric emission was developed in 1905 from ideas first introduced by Max Planck to explain the laws of blackbody radiation.

Planck showed that the laws of blackbody radiation could be explained by assuming that light and all other forms of electromagnetic radiation is emitted in discrete 'packets' of energy called ***quanta***, and that the energy E carried in each quantum emitted was proportional to its frequency f given by:

Formula

$$\boxed{E = hf}$$

where $h = 6.63 \times 10^{-34}$ J s is known as the Planck's constant.

"$E = hf$" gives the smallest amount of energy (which corresponds to one quantum, also known as a photon) that is associated with light of frequency f.

Definition

> *A **photon** is a quantum of electromagnetic energy.*

By applying the relation $c = f\lambda$, we can then derive the energy of one photon to be

Formula

$$\boxed{E = hf = h\frac{c}{\lambda}}$$

where $c = 3.00 \times 10^{8}$ m s^{-1} is the speed of light.

Thus, in a monochromatic beam of light containing N photons, the total energy is

Formula

$$\boxed{E_{total} = Nhf = Nh\frac{c}{\lambda}}$$

Einstein made use of Planck's concept of quantisation to explain the photoelectric effect. Consider what happens when light is directed at a metal surface as shown in Fig. 19.3.

- A stream of photons bombards the surface of the metal.
- Any *free electron* near the *surface* could be struck by a photon and gain energy.
- If the gain in energy is sufficient, the electron can leave the plate (such an electron is known as a photoelectron).
- Each photoelectron from the metal plate gains the ***whole*** amount of energy of a single photon. If the photon is not absorbed, it will be reflected or transmitted. The photon's energy cannot be shared amongst electrons – it must give up **all or none** of its energy to a single electron.

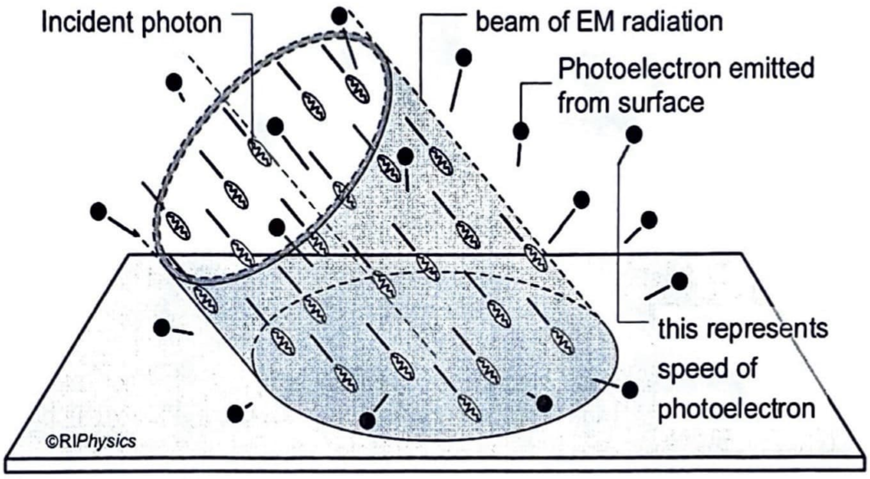


Fig. 19.3 When photons of sufficient energy are incident on a surface; photoelectrons are emitted with a range of energies up to a maximum.

Einstein's Photoelectric Equation

Einstein proposed in 1905 that electromagnetic radiation is emitted in quanta and also absorbed in discrete units.

The electrons in a metal are held to the atoms by attractive electrostatic forces. An electron near the surface can only escape if it gains ***enough energy from a single photon***. Some of the photon energy will be used to provide the work function energy to the electron (energy required to overcome electrostatic attraction and eject the electron from the surface) and, for an electron at the surface of the metal, the remainder of the energy will be seen as kinetic energy.

Definition

> The **work function energy** Φ of a metal is defined as the *minimum amount of energy* necessary for an electron to escape from the *surface* of a material.

The work function energy Φ is a constant for a given metal.

Einstein's photoelectric equation is another statement of the principle of conservation of energy where:

photon energy	=	work function energy	+	maximum kinetic energy of photoelectron

Einstein's photoelectric equation is written as:

Formula

$$\boxed{hf = \Phi + \frac{1}{2}m_e v_{max}^2}$$

- The maximum kinetic energy $\frac{1}{2}m_e v_{max}^2$ of a photoelectron is equal to the energy gained by absorbing a photon (*hf*), minus the work function energy Φ (i.e. work done to escape from the metal).

Applying Einstein's Ideas to the Observed Phenomenon

Max. KE of photoelectrons increases linearly with frequency but is independent of intensity.

- The photoelectrons with the maximum kinetic energy come from the surface of the metal. Due to collisions with other atoms, those below the surface emerge with a smaller kinetic energy.
- Hence electrons are emitted from the surface with a range of energies up to $K.E._{max}$.

$$K.E._{max} = hf - \Phi$$

I

- This explains why **K.E. of the most energetic photoelectrons increases linearly with frequency** but is **independent of intensity.**
- When K.E. = 0, the work function energy Φ is equal to hf_0, where f_0 is the threshold frequency.

Formula

$$\boxed{\Phi = hf_0}$$

F

Existence of threshold frequency f_0

The threshold frequency can now be explained in terms of the photoelectric equation. Electrons can only escape if the maximum kinetic energy is greater than zero.

$$\text{Hence, } E_{max} > 0 \qquad \therefore f > \frac{\Phi}{h}$$
$$\Rightarrow hf - \Phi > 0$$

If we let the threshold frequency be $f_0 = \frac{\Phi}{h}$, then $f > f_0$ for photoelectric effect to take place.

Hence, if photoelectric emission is to occur,

- the frequency of the incident electromagnetic radiation must be greater than the threshold frequency f_0, or
- the wavelength of light must be shorter than the threshold wavelength λ_0.

Definition

Threshold frequency is the minimum frequency of electromagnetic radiation below which no emission of photoelectrons occurs.

T

Instantaneous emission of electrons (insignificant time lag)

- A photoelectron is emitted if it gains enough energy from the photon.
- All the photon energy is delivered immediately to the electron in a single collision.

The emission has no time delay and it is independent of the intensity of the incident radiation.

Relating Maximum KE of photoelectrons with Stopping Potential V_s.

For a metal illuminated by electromagnetic radiation, electron will be emitted from the surface with a range of energies, and the electrons that expend the least amount of energy (i.e. work function Φ) will have the maximum kinetic energy $E_{k,\,max}$ given by:

Decrease in KE = Increase in EPE

$$\frac{1}{2}m_e v_{max}^2 - 0 = eV_S$$

Formula

$$\boxed{E_{k,\,max} = eV_S} \qquad V_S : \text{stopping potential}$$

Re-writing Einstein's photoelectric equation,

$$E_{k,\,max} = hf - \Phi$$

$$eV_S = hf - hf_0$$

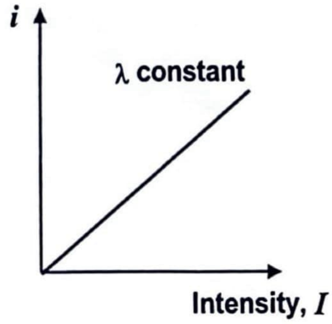
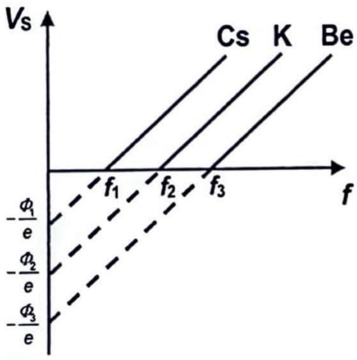
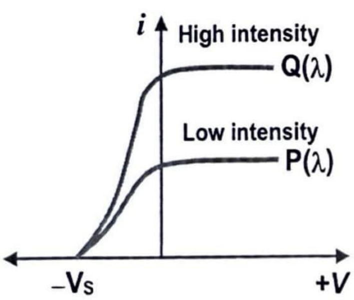
Formula

$$\boxed{V_S = \frac{h}{e}f - \frac{\Phi}{e}}$$

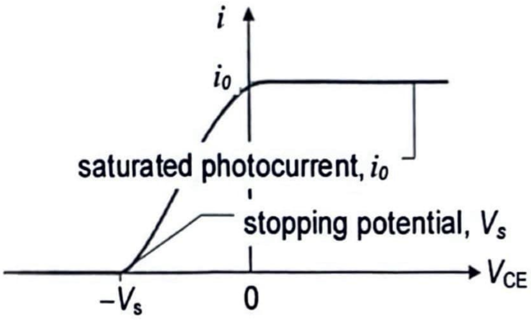
In Einstein's words, 'If the formula is correct, then V_S must be a straight-line function of the frequency f of the incident light.' Einstein predicted that the gradient did not depend on the material since h and e are fundamental constants, the same for any material. However, the vertical-intercept does depend on the material used since its value is $-\frac{\Phi}{e}$.

Graphical Representations of the Photoelectric Results

A metal is illuminated by monochromatic electromagnetic radiation of wavelength λ, which is below the threshold wavelength for the metal (i.e. frequency of radiation is above threshold frequency for the metal).

Graphical Representation	Observations
Variation of current i with intensity for a given wavelength λ below the threshold value (i.e. above threshold frequency). 	• The rate of emission of electrons $\propto$ rate of incident photons (intensity) i.e. $\frac{dN_e}{dt} \propto \frac{dN_p}{dt}$. Hence current i is proportional to the intensity and a straight line graph is obtained. • Since intensity of a beam of photons is the energy transmitted per unit area per unit time, $$Intensity = \frac{P}{A} = \frac{E}{tA} = nhf$$ where n is the number of photons passing per unit area per unit time. Note that intensity is measured in J s^{-1} m^{-2} or W m^{-2}. • An increase in intensity of incident radiation means a greater number of photons striking the metal surface per second and therefore a greater number of photoelectrons can be emitted per unit time.
Stopping potential V_S plotted against frequency f for three different metals. 	• The respective threshold frequencies are different because the metals have different work functions. But, the slope of the three lines is the same since the slope is given by $\frac{h}{e}$. (Recall that $V_S = \frac{h}{e}f - \frac{\Phi}{e}$) • Experimental confirmation of Einstein's theory was obtained in 1916 by Robert Millikan. The results obtained by Millikan fitted Einstein's predictions exactly, and gave a value for $h = 6.63 \times 10^{-34}$ J s. • A similar graph of $K.E._{max}$ against f can be plotted, with h as the gradient and Φ the y-intercept.
Variation of current i with applied potential difference V_{CE} for 2 light intensities for a given wavelength λ (i.e. constant frequency) 	• When the retarding potential V is increased negatively (i.e. V_{CE} negative), the stopping potential V_S is the same for a beam of low intensity P and one of high intensity Q (at constant frequency) • When V is positive all the photoelectrons are collected so that the current is constant. • A beam of high intensity Q produces more electrons per unit time than one of low intensity P. If the intensity of Q is twice that of P, the current i is twice as much. • Photoelectric current is proportional to intensity at constant frequency.

Graph of photocurrent *i* vs. potential of collector w.r.t. emitter V_{CE} (further elaboration)



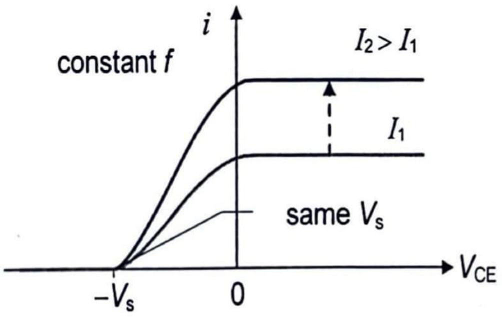
Observation:

Photocurrent i **decreases gradually** from **max. value of i_0** when V_{CE} is sufficiently positive, to **zero** when $V_{CE} = -V_s$.

Explanation:

- When V_{CE} is sufficiently positive, rate of emission of electrons = rate with which electrons reach collector. Hence photocurrent is **maximum.**
- As V_{CE} is more negative, an increasingly greater fraction of electrons is **repelled** from collector. Only electrons with sufficient energies can reach collector. Photocurrent i decreases.
- When $V_{CE} = -V_s$, not even the most energetic photoelectrons can reach the collector. Hence photocurrent $i = 0$. V_s is the stopping potential.

$$eV_S = K.E._{max}$$



Observation:

Saturated photocurrent i_0 increases with intensity I.

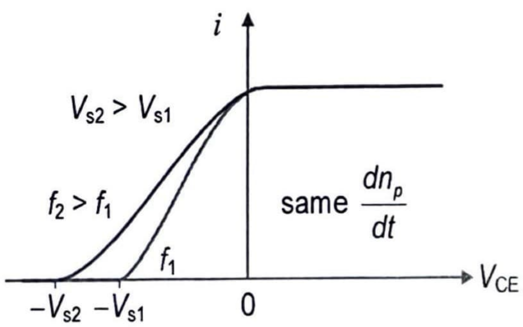
Stopping potential V_s **independent** of intensity I.

Explanation:

- See explanation for i_0 vs. I graph.
- K.E.$_{max}$ of electrons depends only on frequency f and work function Φ but not intensity I.

$$K.E._{max} = hf - \Phi$$

No change in stopping potential V_s as K.E.$_{max}$ remains the same.



Observation:

Stopping potential V_s **increases** with frequency f.

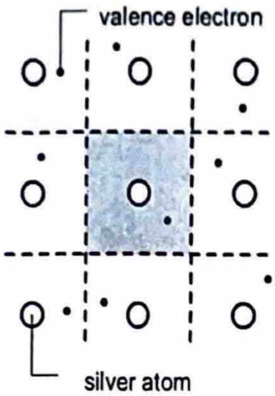
Explanation:

- More energetic photons liberate electrons with higher K.E.$_{max}$.
- **Greater** stopping potential V_s is needed to stop the more energetic electrons.

$$V_s = \frac{hf}{e} - \frac{\Phi}{e}$$

Example 1

Light of intensity 10 W m^{-2} shines on a silver surface that has one valence electron per atom. The atoms are approximately 2.6×10^{-10} m apart. According to wave theory of light, its energy is distributed uniformly over the surface. Assuming that this energy is absorbed entirely by the surface electrons, how long would it take for an electron to gain sufficient energy to be emitted from the surface given that a minimum energy of 4.7 eV is needed.



Solution

Area occupied by each electron $= (2.6 \times 10^{-10})^2 \text{ m}^2$

Power incident on each valence electron $= (10)(2.6 \times 10^{-10})^2 \text{ m}^2$

Time needed to gain sufficient energy $= \dfrac{4.7 \times 1.60 \times 10^{-19}}{10 \times \left(2.6 \times 10^{-10}\right)^2} = 1.1 \text{ s}$

Note: Frequency or wavelength of light is irrelevant in this calculation.

Example 2

When ultraviolet radiation of wavelength 319 nm falls on a metal surface, photoelectrons are emitted. The stopping potential was found to be 0.22 V. The same surface is then illuminated with electromagnetic radiation of wavelength 238 nm. Calculate

(a) the work function of the metal,
(b) the threshold frequency for the metal,
(c) the stopping potential when wavelength is 238 nm,
(d) the maximum speed of the photoelectrons for each wavelength.

($c = 3.00 \times 10^8$ m s^{-1}, $e = 1.60 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg,
$h = 6.63 \times 10^{-34}$ J s)

Solution

(a) $hf = \Phi + E_{K,max}$

$$(6.63 \times 10^{-34})(\frac{3.00 \times 10^8}{319 \times 10^{-9}}) = \Phi + \left(1.60 \times 10^{-19}\right)(0.22)$$

$$\Phi = 5.88 \times 10^{-19} \text{ J}$$

(b) $f_0 = \dfrac{\Phi}{h} = \dfrac{5.8831 \times 10^{-19}}{6.63 \times 10^{-34}} = 8.87 \times 10^{14} \text{ Hz}$

(c) $hf = \Phi + E_{K,max}$

$$(6.63\times10^{-34})(\frac{3.00\times10^{8}}{238\times10^{-9}}) = 5.8831\times10^{-19} + (1.60\times10^{-19})(V_s)$$

$$V_s = 1.55\ \text{V}$$

(d) For λ_1 = 319 nm, $\frac{1}{2}mv_1^2 = eV_S$

$$(\frac{1}{2})(9.11\times10^{-31})(v_1^{2}) = (1.60\times10^{-19})(0.22)$$

$$v_1 = 2.78\times10^{5}\ \text{m s}^{-1}$$

For λ_2 = 238 nm,

$$(\frac{1}{2})(9.11\times10^{-31})(v_2^{2}) = (1.60\times10^{-19})(1.55)$$

$$v_2 = 7.38\times10^{5}\ \text{m s}^{-1}$$

Example 3 A metal surface is illuminated with a 0.100 W ultraviolet laser of wavelength 254 nm. The work function of the metal is 4.20 eV.

Explain whether photoemission is possible, and calculate

(a) the rate of photons incident on the surface,

(b) the maximum kinetic energy of the emitted photoelectrons,

(c) the minimum retarding or "stopping" potential for these photoelectrons.

Solution

Energy of a photon

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34})(3.00\times10^{8})}{254\times10^{-9}} = 7.83\times10^{-19} = 4.89\ \text{eV}$$

Since E > Φ, photoemission is possible.

a) the rate of photons incident on the surface,

$$\frac{dN_p}{dt} = \frac{P}{E} = \frac{0.100}{7.83\times10^{-19}} = 1.28\times10^{17}\ \text{s}^{-1}$$

b) the maximum kinetic energy of the emitted photoelectrons,

$$\begin{aligned} KE_{max} &= \frac{hc}{\lambda} - \Phi \\ &= \frac{(6.63\times10^{-34})(3.00\times10^{8})}{254\times10^{-9}} - (4.20)(1.60\times10^{-19}) \\ &= 1.11\times10^{-19}\ \text{J} \end{aligned}$$

c) the minimum retarding or "stopping" potential for these photoelectrons.

$$V_s = \frac{KE_{max}}{e} = \frac{1.11\times10^{-19}}{1.60\times10^{-19}} = 0.694\ \text{V}$$

Example 4 A metal surface when illuminated by light of frequency 1.00×10^{15} Hz, emits electrons which can be stopped by a potential of 0.600 V. When light of frequency 1.36×10^{15} Hz is used, the required stopping potential is 2.10 V.

(a) Explain why a higher stopping potential is required in the latter case.
(b) Determine
 (i) the Planck's constant, and
 (ii) the work function of the metal.

Solution

(a) Photoelectrons are emitted with greater maximum kinetic energy ($K.E._{max}$) when incident light has a high frequency, according to Einstein's photoelectric equation $K.E._{max} = hf - \Phi$, where hf is the energy of an incident photon. Work function Φ is a constant for the metal.

Hence a higher stopping potential V_s is required to prevent electrons (emitted by the incident light of higher frequency), from reaching the collector.

$$K.E._{max} = eV_s$$

Let $V_{s1} = 2.10$ V and $V_{s2} = 0.600$ V

b) Determine

i. the Planck's constant,

$$eV_{S1} = hf_1 - \Phi \quad ; \quad eV_{S2} = hf_2 - \Phi$$

$$\begin{aligned} h &= \frac{eV_{s2} - eV_{s1}}{f_2 - f_1} = \frac{e(V_{s2} - V_{s1})}{f_2 - f_1} = \frac{(1.60\times10^{-19})(2.10 - 0.600)}{(1.36 - 1.00)\times10^{15}} \\ &= 6.67\times10^{-34}\ \text{J s} \end{aligned}$$

ii. the work function of the metal.

$$\begin{aligned} \Phi &= hf_1 - eV_{s1} \\ &= (6.67\times10^{-34})(1.00\times10^{15}) - (1.60\times10^{-19})(0.600) \\ &= 5.70\times10^{-19}\ \text{J} \end{aligned}$$

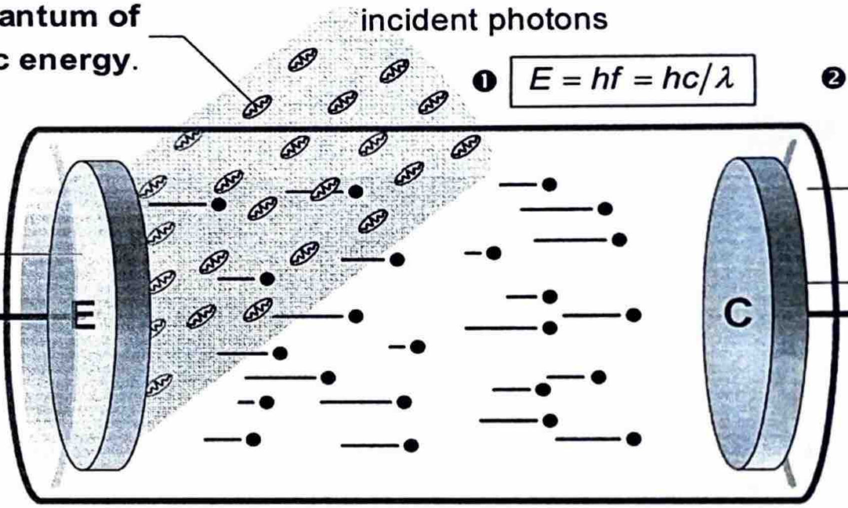
Photoelectric Experiment By Quantum Theory

A **photon** is a **quantum of electromagnetic energy**.

incident photons

❶ $E = hf = hc/\lambda$

❷ $\boxed{\dfrac{dN_p}{dt}} = \dfrac{P}{hf}$ — rate of incident photons



emitter E

work function Φ

evacuated tube

collector C

μA

Work function Φ of a material is the **minimum amount of energy** needed for an electron to **escape from the surface** of the material.

$\Phi = h\boxed{f_0}$ — threshold frequency

- Photoelectrons are emitted when $hf > \Phi$
- Photoelectrons are emitted with a **range of energies**
- K.E. of the **most energetic** photoelectrons

$$\text{K.E.}_{max} = \tfrac{1}{2}mv_{max}^2 = hf - \Phi$$

variable power supply

With C **sufficiently negative** w.r.t. E, even the most energetic photoelectrons cannot reach the C. The photocurrent is **zero**.

$$\text{K.E.}_{max} = \tfrac{1}{2}mv_{max}^2 = e\boxed{V_S}$$

stopping potential

Quantum yield

$$Q = \frac{dN_e}{dt} \div \frac{dN_p}{dt}$$

With C **sufficiently positive** w.r.t. E, all photoelectrons are attracted to C ⇒ **constant saturated photocurrent**.

$$\boxed{\frac{dN_e}{dt}} = \frac{i_0}{e}$$

rate of emission of photoelectrons

19.2 Wave-Particle Duality

Introduction

Particulate Nature of Light

We found that light can behave as a particle (photoelectric effect) or as a wave (reflection, refraction, diffraction, interference. This shows the wave-particle duality of electromagnetic energy.

In 1925, Louis de Broglie suggested that particles might also exhibit this duality and have wave properties.

He suggested that a particle with momentum ***p* = *mv*** that exhibits wave behaviour has an associated wavelength λ (de Broglie wavelength) given by

Formula

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}}$$

Electromagnetic radiation of wavelength λ that exhibits particle behaviour has an associated momentum *p* given by

Formula

$$\boxed{p = \frac{h}{\lambda} = \frac{hf}{c}}$$

Electron Diffraction

Diffraction of electrons was discovered two years later, by Davisson & Germer, when it was shown that a beam of electrons directed at a single crystal produced a diffraction pattern like an X-ray diffraction pattern.

About the same time, George Thomson (son of J.J. Thomson) tested de Broglie's theory by directing electrons at a thin metal foil. Thin metal foils contain lots of tiny crystals called grains. Thomson showed that the electrons were diffracted to form a pattern of rings. The same pattern is obtained when X-rays are directed at the foil. Such a result shows that particles do have a wave-like nature. In other words, matter has a dual nature which is wave-like or particle-like.

Fig. 19.4 shows the pattern captured on a coated fluorescent screen when a beam of electron falls on a thin layer of graphite atom and is dispersed. The pattern is identical to that obtained due to interference of electromagnetic radiation. This is known as electron diffraction phenomenon.

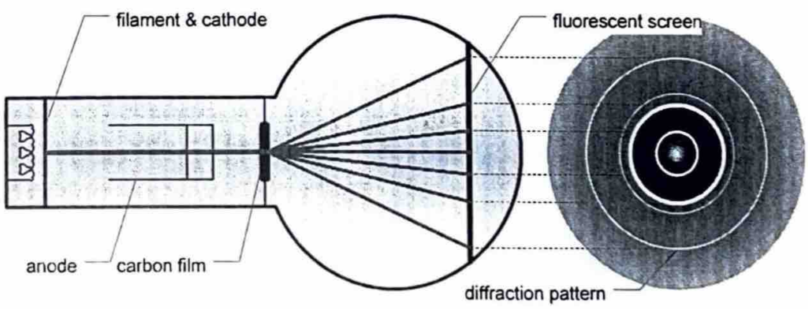


Fig. 19.4

Example 5 Calculate the associated de Broglie wavelength of

(a) a bullet of mass 30 g moving at a velocity of 250 m s^{-1},
(b) electrons in an electron beam which has been accelerated through a potential difference of 4000 V.

Solution

(a) $\lambda = \dfrac{h}{p} = \dfrac{6.63\times10^{-34}}{(30\times10^{-3})(250)} = 8.84\times10^{-35}$ m

(b) $\dfrac{p^2}{2m} = eV \Rightarrow p = \sqrt{2meV}$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}} = \frac{6.63\times10^{-34}}{\sqrt{2(1.60\times10^{-19})(9.11\times10^{-31})(4000)}} = 1.94\times10^{-11}\text{ m}$$

For a particle to have an observable wave nature, the de Broglie wavelength should not be too small. As the value of *h* is very small, this implies that the mass *m* of the particle is also very small in order for the wavelength to be noticeable. That is the reason why only electrons and sub-atomic particles exhibit observable wavelike properties.

Wave-Particle Duality

The theory where matter and waves have particle-like and wave-like characteristics.

De Broglie wavelength

The wavelength associated with wave-like properties of a particle.

Summary: Wave-Particle Duality

	Evidence
Light behaves as a wave	Interference / Diffraction of light
Light behaves as particles	Photoelectric effect
Electrons behave as particles	Electrons undergo collision, have mass and charge
Electrons behave as a wave	Electron diffraction

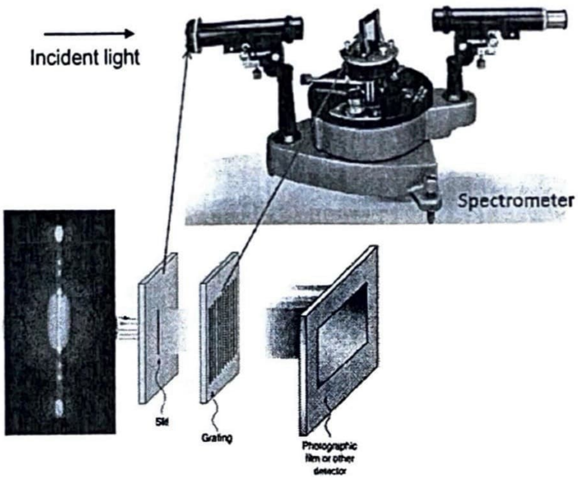
19.3 Energy Levels and Line Spectra

Introduction

In the early 1900s, scientists were fascinated with the spectra obtained from gaseous elements and were carrying out investigations on the properties of these spectra. However, during that time, one could neither explain why the spectral lines form such distinctive patterns nor how they came about.

Spectral lines are often used to identify atoms and molecules based on their electromagnetic radiation. These spectral lines are unique to each type of atoms and can serve as unique "fingerprints" for its identification via comparison to previously collected "fingerprints" of atoms and molecules. This method enables one to identify the atomic and molecular components of stars and planets, which otherwise would be impossible.

Fig. 19.5 shows the typical setup (simplified) to view spectral likes, and Fig. 19.6 shows and different types of spectra that can be observed.



(left) Fig. 19.5 Typical setup for the observation of spectral lines. Sources are viewed though a spectrometer.

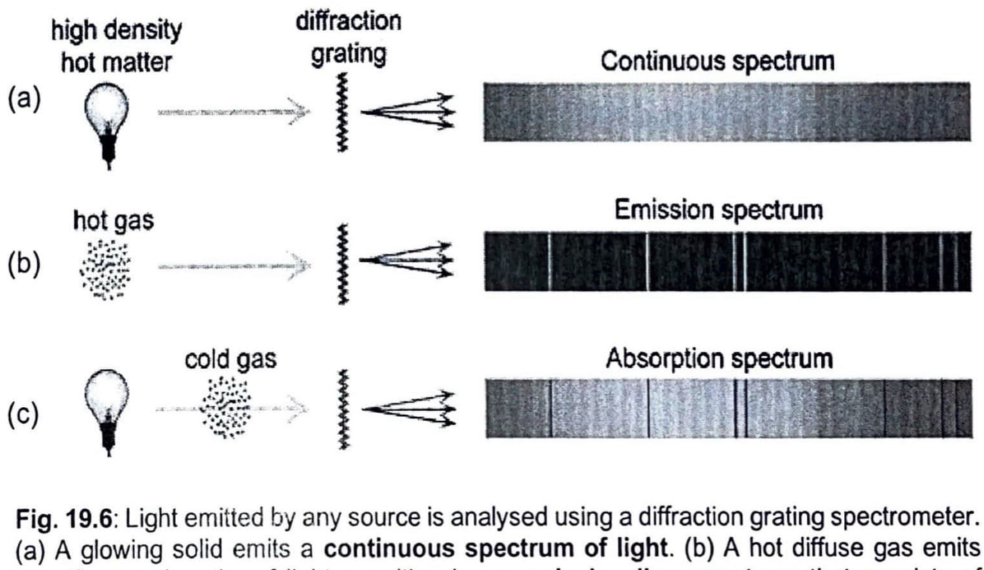


Fig. 19.6: Light emitted by any source is analysed using a diffraction grating spectrometer. (a) A glowing solid emits a **continuous spectrum of light**. (b) A hot diffuse gas emits specific wavelengths of light, resulting in an **emission line spectrum** that consists of **bright lines on a dark background**. (c) White light after passing though cool diffuse gas, results in an **absorption spectrum** that consists of **dark lines on a bright background.**

Quantisation of Energy Levels in Atoms - Bohr's Atomic Model

In 1913, Niels Bohr produced the answer by considering the spectral lines of the hydrogen atom and applying Planck's quantum theory to this atom. Bohr's model of the atom is like a miniature solar system. He imagined the electron of the hydrogen atom whirling round the nucleus, like a planet revolving around the Sun (Fig. 19.7).

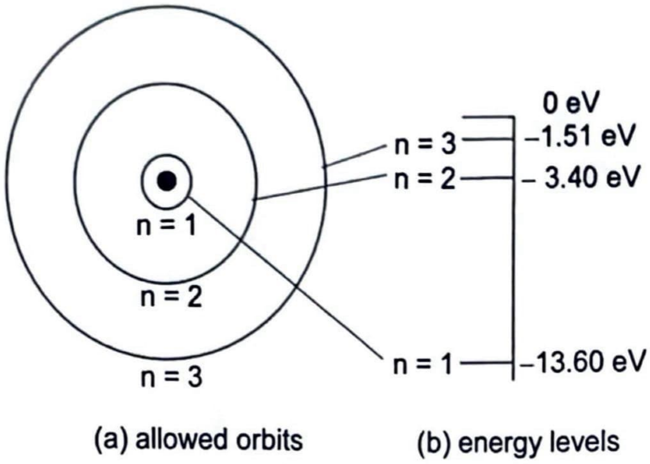


Fig. 19.7

Bohr postulated that for the hydrogen atom or for any other ***isolated atom***:

1. There are only certain allowed orbits in the atom, and each correspond to a fixed energy state. Electrons are only allowed to revolve about the nucleus in these orbits.
2. Electrons are in a stable state when they occupy orbits corresponding to the lowest energy levels. An atom in its lowest energy level is said to be in the **ground state.**

Formula

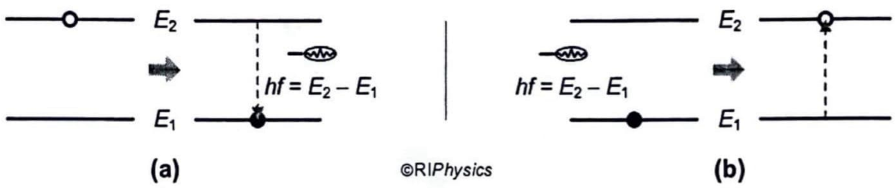


Fig 19.8: (a) An electron in a unstable higher energy state de-excites to a lower energy state by emitting a photon. (b) An electron in a lower energy state can excite to a higher energy state by absorbing a photon of same energy. The photon ceases to exist. However, no excitation of electron is possible if the energy of photon $hf \neq E_2 - E_1$

3. An atom can only radiate energy when an electron transits from a more energetic state to a lower energy state. The energy is emitted as one quantum (or photon) of radiation of frequency f given by:

$$\boxed{E_i - E_f = hf}$$

where E_i, E_f = the energies of the initial and final states respectively,
hf = the energy of the emitted photon

By using these postulates, Bohr was able to work out the energy levels of the hydrogen atom and account for the frequencies observed in the hydrogen spectrum. Fig. 19.8 shows the energy level diagram for the hydrogen atom.

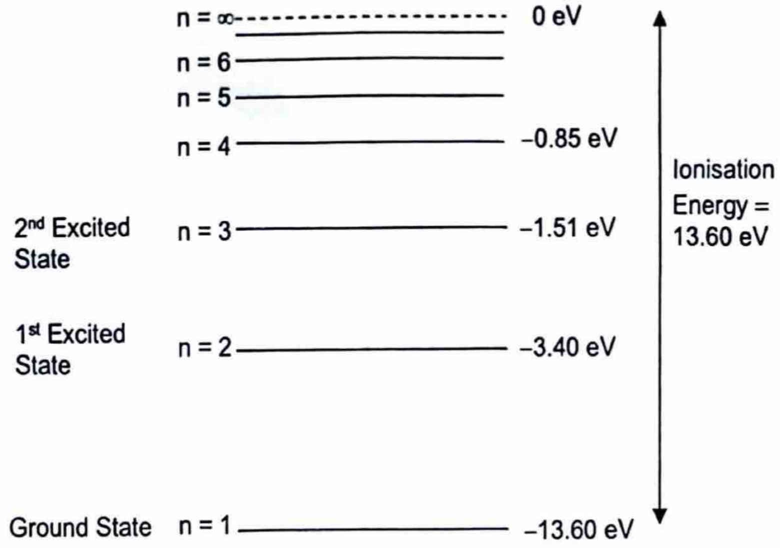


Fig. 19.8

Key Features of the Energy-Level Diagram

1. The number n is known as the principal quantum number and can only take integral values from 1 to ∞.

2. The ground state refers to the lowest energy level ($n = 1$) in which the atom is the most stable. Normally the atom is in the ground state. When it absorbs energy from external sources, it goes to a higher energy state.

3. An atom is said to be in an excited state when its electrons are found in the higher energy levels.

4. The highest energy level $n = \infty$ corresponds to an energy state whereby the electron is no longer bound to the atom, i.e. the electron has escaped from the atom. By convention, it is usually assigned an energy value of 0 eV.

5. The lower the energy level, the more negative the energy value associated with that level. The lower energy states correspond to more stable states.

6. The energy difference between any two adjacent levels gets smaller as n increases, which results in the higher energy levels converging to a continuum.

7. The **ionisation energy** of an atom is the minimum energy required to remove an electron completely from the atom. This corresponds to a transition of the atom from the ground state $n = 1$ to the infinite level $n = \infty$. For hydrogen, the ionisation energy is 13.60 eV.

Note

If the ground state is assigned an energy value of 0 eV (i.e. reference level), then the higher levels will have energy values more positive than the ground state.

Electronic Transitions in a Bohr Atom: Ionisation and Excitation

Ionisation is the process of creating charged particles. A positive ion is produced when electrons are removed from a neutral atom.

Excitation is the process whereby atoms absorb energy without ionisation.

There are two main sources of energy that can cause the electron to be ionised or excited to a higher energy state:

1. **Particle collision** (for example, atoms, ions, electrons)

   A high-speed particle can be used to collide with and impart its energy to an atom to cause an electron to be excited to a higher energy level. The colliding particle can transfer part or whole of its energy to the electron.

   The amount of energy transferred must be sufficient for the orbital electron to transit up to a higher energy level. The energy of the colliding electron **need not match** the difference in energy levels of the atom.

2. **Photons**

   For an orbital electron to gain energy from an incident photon, the photon energy must be **exactly equal** to the energy difference between 2 levels in the atom. In this case, the photon will be absorbed by the electron. If the photon energy is not exactly equal to the energy difference between 2 levels in the atom, then the photon will **not** be absorbed at all.

De-Excitation of an Atom

What happens to the orbital electrons that are now excited by the absorption of energy? They will return to the ground state (de-excite) and give up the excess energy in the form of **photons.**

As seen in Fig. 19.10, the excited orbital electron does not necessarily jump directly to the ground state, but can make a series of jumps. The electron can either jump directly from n = 3 to n = 1, or take two jumps from n = 3 to n = 2 followed by n = 2 to n = 1. As a result, several photons of different frequencies can be emitted.

When the emitted light is passed through a diffraction grating, it gives rise to the emission spectrum as shown in Fig. 19.9.

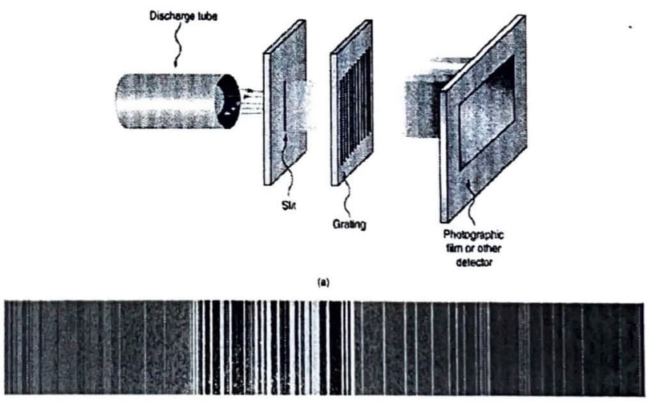


Fig. 19.9

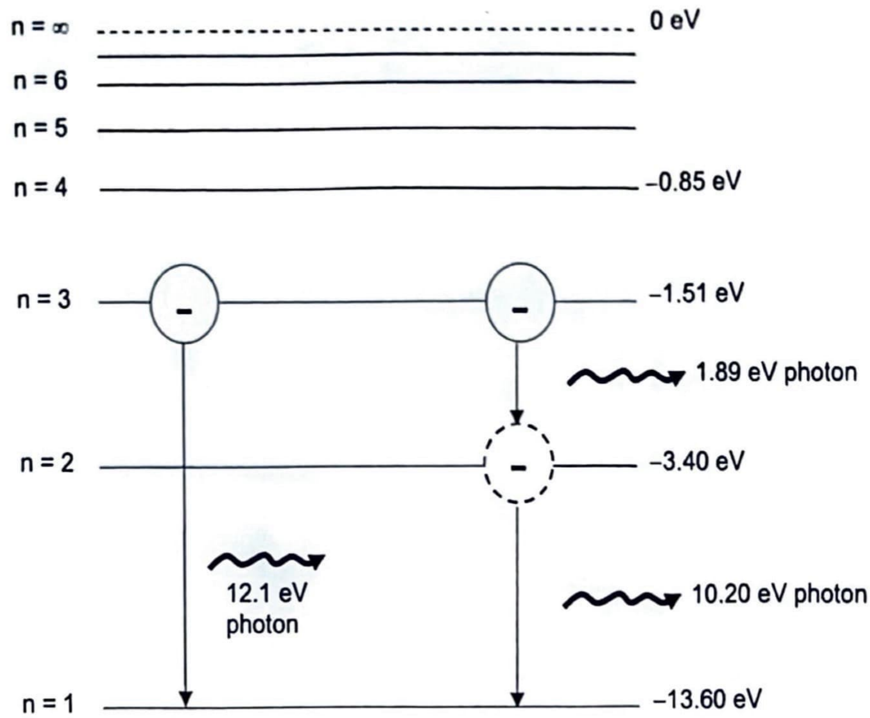


Fig. 19.10

Energy Levels and Line Spectra of Hydrogen

The hydrogen spectrum can be observed using a spectrometer/prism to view light from a hydrogen-filled discharge tube as in Fig. 19.11. and 19.12. The energy level diagram of hydrogen is shown in Fig. 19.13. As hydrogen atoms have just one electron each, Fig. 19.13 shows the energy levels of the single electron of the hydrogen atom.

When an excited electron de-excites to a lower level, it loses an exact amount of energy by emitting a photon. The photon carries away an amount of energy equal to the difference between the initial and final energy levels. ($hf = \Delta E$)

The Lyman (ultraviolet) series of lines corresponds to electron transitions (de-excitation) to the level n = 1, the Balmer (visible) series are produced by transitions to n = 2 and the Paschen (infrared) series are produced by transitions to n = 3; each series being named after its discoverer. (Fig. 19.14)

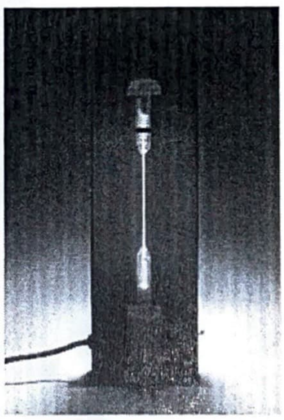


Fig. 19.11

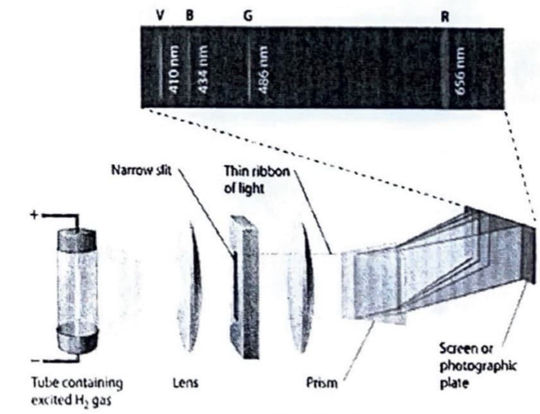


Fig 19.12

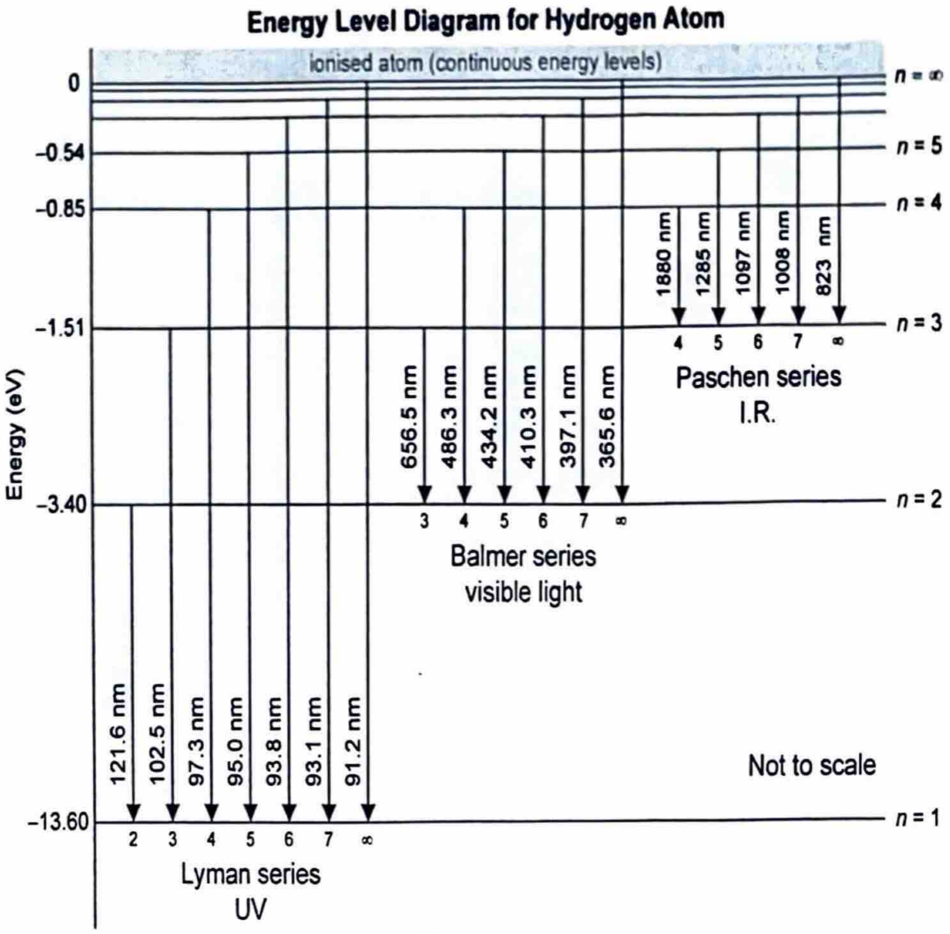


Fig. 19.13

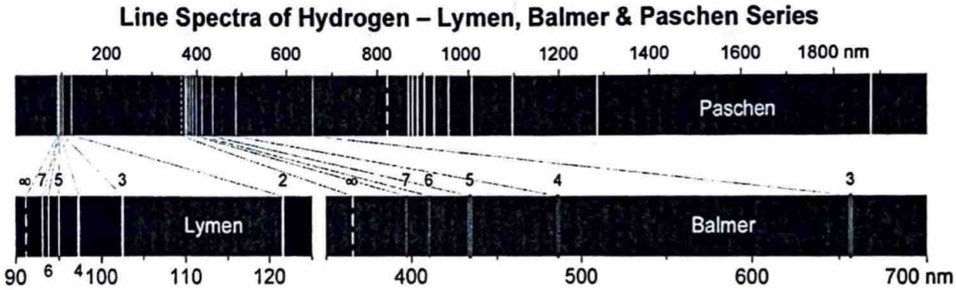


Fig. 19.14

Emission Line Spectrum

Emission Line Spectrum

When gases like hydrogen or neon are placed in a discharge tube at low pressure with a high voltage applied between the ends of the tube, the gas starts to glow and light is emitted from the electric discharge tube. If this light is examined through a diffraction grating (or prism) with a spectrometer, a spectrum consisting of well defined, distinct lines is observed (Fig.19.15). This type of spectrum is known as an **emission line spectrum**.

How this forms is when a gas is heated or bombarded by electrons, the electrons in the gas atoms are excited to higher energy levels. The excited electrons remain there momentarily before de-exciting to lower energy levels. Upon de-excitation, photons are emitted with energy corresponding to the energy difference between the two energy levels.

As the energy levels are discrete, the energy differences between the energy levels are also discrete. This emission of photons corresponds to the series of lines on the emission line spectrum.

Atoms emit characteristic electromagnetic radiations. For example, neon tubes glow orange-red while sodium lamps emit yellow.

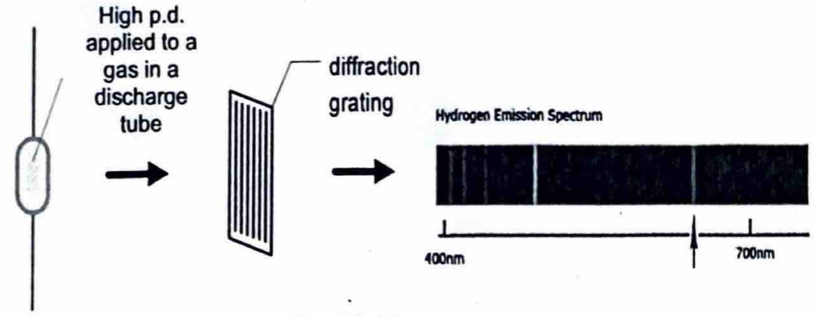


Fig. 19.15

- The emission line spectrum consists of **discrete bright lines** of definite wavelength (different colours) **on a dark background**.
- A series of lines is observed which form a definite pattern.
- Different gases will result in different patterns (unique to the atom).

Absorption Line Spectrum

Absorption Line Spectrum

Similarly, if **white light** (i.e. light containing visible frequencies) passes through a cool gas, the atoms of the cool gas absorb photons of certain frequencies to jump from a lower energy level to a higher one. The frequencies of the photons absorbed must satisfy hf = energy difference between the two energy levels.

After the absorption, the excited atom will eventually return to the lower energy state by emitting the same photons. However, these emissions occur in all directions and therefore have lower intensities. Hence, when the light emerging from the discharge tube is passed through a diffraction grating, light of these frequencies ***appear to be missing*** from the continuous spectrum.

This continuous bright spectrum crossed by dark lines is called the **absorption line spectrum**.

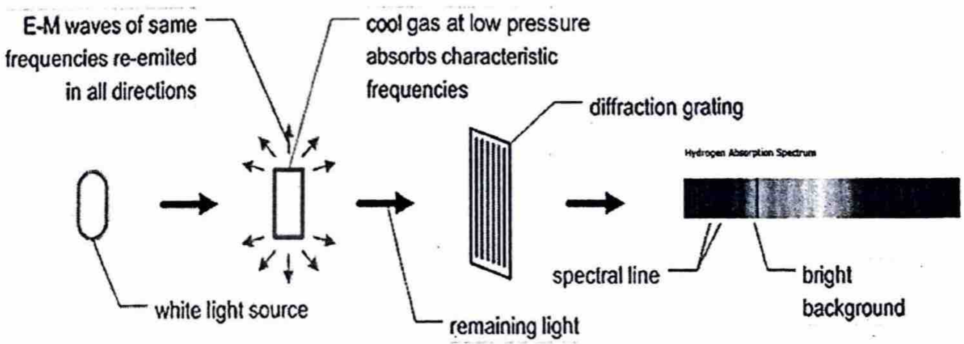


Fig. 19.16

- The absorption line spectrum **consists of dark lines on a bright background**.
- A series of lines is observed which form a definite pattern.
- Different gases will result in different patterns (unique to the atom).

Example 6

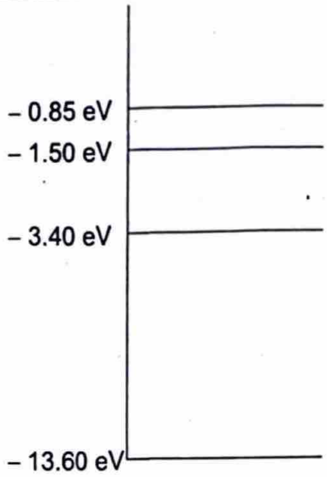
Explain how line spectra provided evidence that the energy levels of an atom is quantised.

Solution

- Each line in the (emission or absorption) line spectra corresponds to photons on one frequency (or wavelength) and hence one particular energy.
- One photon is emitted when an atom de-excite from a higher energy state to a lower energy state so only certain transitions are allowed.
- Therefore, energy levels in an atom must be quantised.

Example 7

The figure represents the energy levels of the four lowest states of a hydrogen atom. The energies are in units of electron volts.



(a) Calculate the longest wavelength which might be emitted by a spectral transition between any pair of these four levels.

(b) Calculate the excitation energy, in joules, for a transition from the ground state to the next higher level.

(c) State the ionisation energy.

(d) Determine the total number of spectral lines, which might be detected in the emission spectrum of atomic hydrogen, due to transitions between these four states.

Solution

(a) Longest wavelength → lowest frequency → smallest difference in energy levels.

$$E_f - E_i = h\frac{c}{\lambda}$$

$$[-0.85-(-1.50)](1.60\times10^{-19}) = (6.63\times10^{-34})(\frac{3.00\times10^{8}}{\lambda})$$

$$\lambda = 1.91\times10^{-6}\text{ m}$$

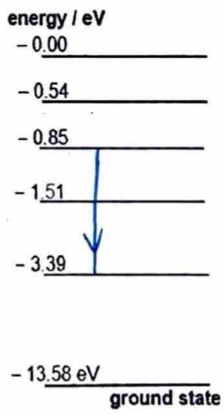
(b) Excitation energy
= [-40 - (-13.60)](1.60×10^{-19}) = 1.63×10^{-18} J

(c) 13.60 eV

(d) 6

Example 8

The figure shows some of the energy levels of the hydrogen atom.



(a) Mark on the figure a transition which will result in the emission of radiation of wavelength 489 nm. Show your calculation.

(b) What is likely to happen to a beam of photons of energy

(i) 12.07 eV and;

(ii) 5.25 eV when passed through a vapour of atomic hydrogen?

(c) Given these 6 energy levels, state the total number of possible transitions.

Example 8 (sol)

Solution

(a) $\Delta E = \dfrac{hc}{\lambda} = \dfrac{(6.63\times10^{-34})(3.00\times10^{8})}{489\times10^{-9}} = 4.07\times10^{-19}\text{ J} = 2.54\text{ eV}$

Since it is an emmision, the electron has to transit between 2 energy levels which has an energy difference of 2.54 eV. Hence, the electron transited from –0.85 eV to –3.39 eV. Draw a downward arrow from –0.85 eV to –3.39 eV.

(b) (i) Electrons at ground state will be excited to (–13.58 + 12.07) = –1.51 eV energy level.

(ii) -13.58 + 5.25 = -8.33 eV.

(c) There is no such corresponding energy level. Hence the photons will not be absorbed by the hydrogen atoms.

c) 1 + 2 + 3 + 4 + 5 = 15

19.4 X-ray Spectrum

Production of X-ray

X-rays are produced when high-speed electrons are suddenly slowed down, for example, when a metal target (tungsten or molybdenum) is struck by electrons that have been accelerated through a potential difference of several thousand volts. An X-ray tube is shown in Fig. 19.17.

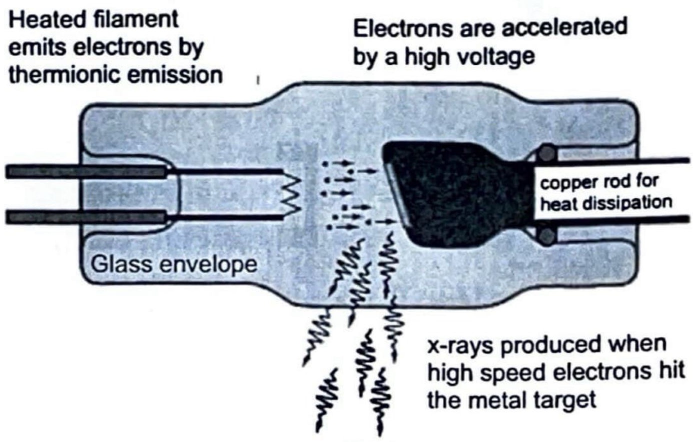


Fig. 19.17

A typical plot of X-ray intensity versus wavelength for the spectrum of radiation emitted by an X-ray tube is shown in Fig. 19.18.

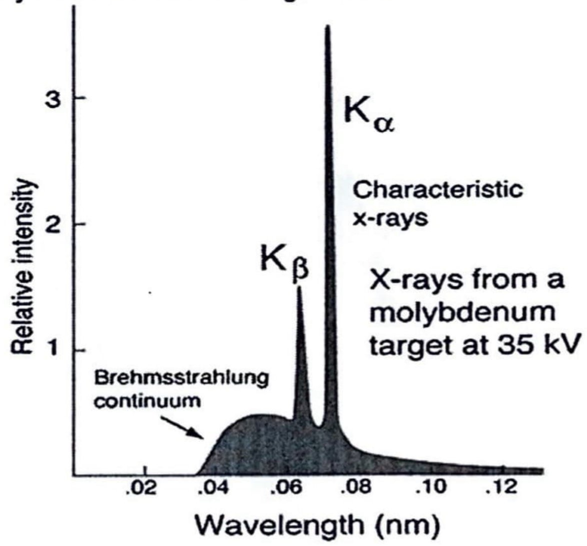


Fig. 19.18

The spectrum has **two** distinct components.

1. <u>The continuous spectrum</u>

   The continuous spectrum is sometimes called *bremsstrahlung*, a German word meaning "braking radiation".

2. <u>The characteristic X-rays</u>

   These are the peaks of the sharply defined wavelengths as shown in Fig. 19.18.

The Continuous Spectrum

In an X-ray tube, **electrons** emitted by the heated filament are made to accelerate through a high voltage before they collide with the metal target.

The continuous spectrum is due to electromagnetic radiation emitted by the high speed electrons when they are slowed down in the metal target due to interaction with the nuclei of the target atoms. When the electrons decelerate, the loss in kinetic energy emerges as the energy of the X-ray photons.

Not all electrons are stopped in a single collision. Since different electrons will be "slowed down to a different extent", the energy of the X-ray photons will cover a wide range of wavelengths, giving rise to the continuous spectrum.

An extreme example would be an electron losing all of its kinetic energy in a single collision. In this case, the energy of the electron (eV) is transformed completely into the energy of the X-ray photon (hf_{max}). In equation form,

Formula

$$\boxed{eV = \frac{1}{2}m_e v_e^2 = hf_{max} = h\frac{c}{\lambda_{min}}}$$

where eV is the energy of the electron after it has been accelerated through a potential difference of V volts and e is the charge of the electron.

Thus, the shortest wavelength of radiation that can be produced is

Formula

$$\boxed{\lambda_{min} = \frac{hc}{eV}}$$

It can be seen that the λ_{min} depends on the potential difference which the electrons are accelerated through, and not the type of metal target used.

The Characteristic X-rays

The second way an X-ray photon is produced is when an accelerated electron from the cathode collides into an electron of a target atom that is orbiting in the *K-shell*. If sufficient energy is transferred by the accelerated electron to the orbiting electron, the latter electron can be ejected from the target atom.

When the vacancy in the *K*-shell (n = 1) is filled by an electron from the *L*-shell (n = 2), an X-ray photon of the K_α characteristic X-ray is emitted (Fig. 19.19).

If the vacancy in the K-shell is filled by an electron dropping from the *M*-shell (n = 3), an X-ray photon of the K_β characteristic X-ray is emitted (Fig. 19.20).

The wavelengths of these X-rays produced can be determined by the following equation:

Formula

$$\boxed{hf = \frac{hc}{\lambda} = E_n - E_1 \text{ ; where } n = 2,3,...}$$

The rates of emission of the K_α and K_β characteristic X-rays are high. As a result, their intensities are high. However, the intensity of the K_α characteristic X-ray is typically greater than the K_β characteristic X-ray. This is because the electrons in the L - shell are nearer to the K-shell, hence there is a greater probability that the vacancy in the K-shell is filled by an electron from the L-shell than from the M - shell.

Note

(When you draw the X-ray spectrum, the K_α -line intensity should be drawn higher than that of the K_β -line.)

*Since the **energy differences** between electrons in the various energy levels are **characteristics of the target atom**, the wavelengths of the K_α and K_β characteristic X-rays are **unique** for each element.*

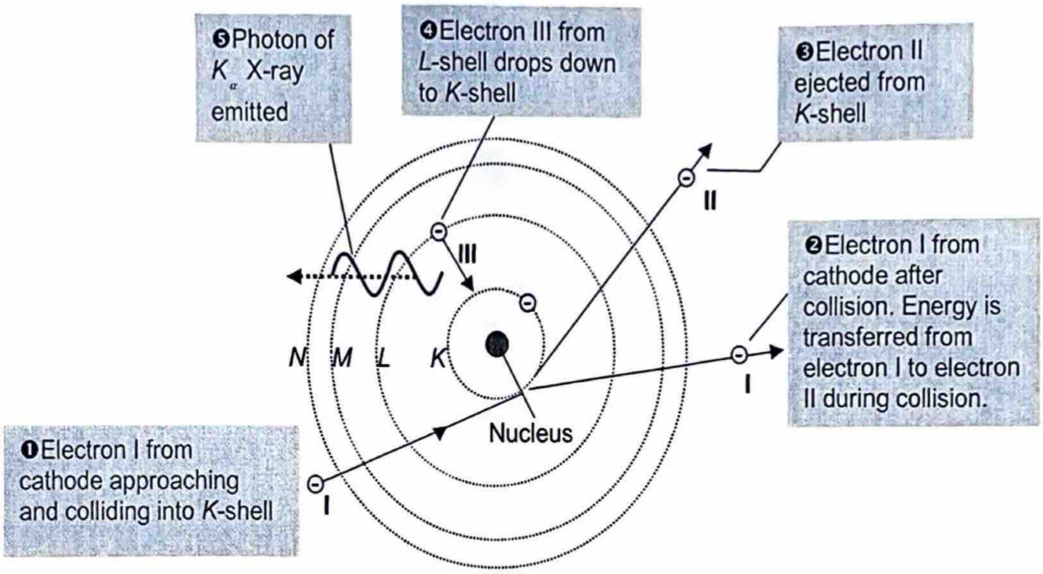


Fig. 19.19

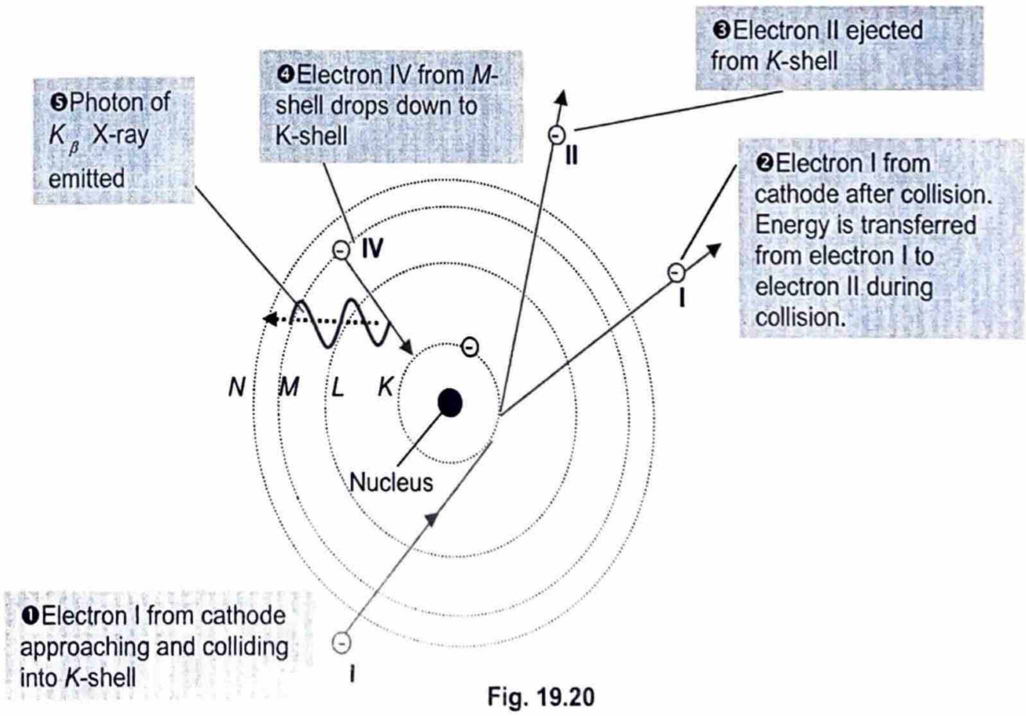
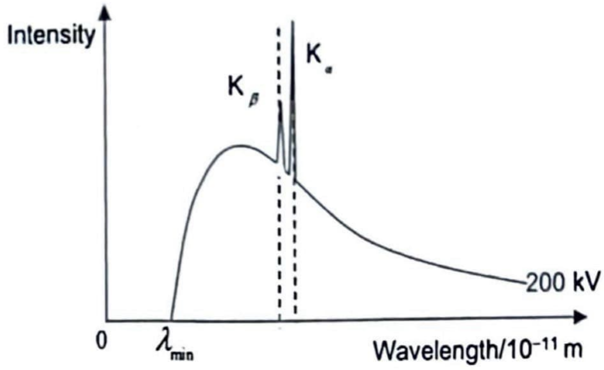


Fig. 19.20

Example 9

The X-ray spectrum produced by an X-ray tube with tungsten anode and maintained at a constant accelerating potential of 200 kV is as shown in the figure. Determine λ_{min}.



Solution

$$\lambda_{min} = \frac{hc}{eV} = \frac{(6.63\times10^{-34})(3.00\times10^{8})}{(1.60\times10^{-19})(200\times10^{3})} = 6.22\times10^{-12}\ \text{m}$$

Example 10

Calculate the maximum photon energy of the radiation from an X-ray tube operating at a peak voltage of 70 kV.

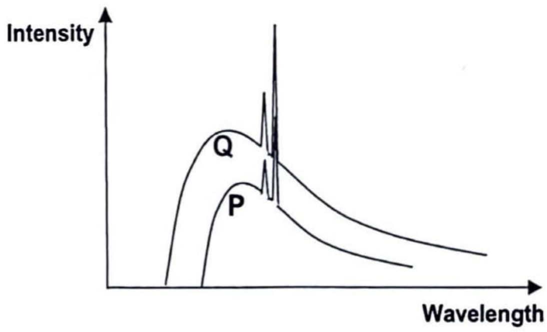
Solution

$$\text{Maximum photon energy} = hf_{max} = \frac{1}{2}m_e v_e^2 = eV$$
$$= (1.60\times10^{-19})(70\times10^{3}) = 1.12\times10^{-14}\ \text{J}$$

Example 11

X-ray spectra are taken from two X-ray tubes P and Q. The intensity of the X-rays is plotted against the wavelength in both cases and shown.

What deduction can be made from these plots?



A. X-ray tube Q has the higher voltage applied to it and the target material in both tubes is the same.

B. X-ray tube Q has the higher voltage applied to it and the target material in both tubes are different.

C. X-ray tube P has the higher voltage applied to it and the target material in both tubes is the same.

D. X-ray tube P has the higher voltage applied to it and the target material in both tubes are different.

Solution A.

Since the characteristic peaks for both P and Q are at the same wavelength, the target material is the same in both tubes.

Quantum Mechanics

At the end of the 19th Century, just as some scientists thought that they knew *everything* about the world; certain experiment observations jolted them out of their comfort zone. Classical physics theories, such as Newtonian mechanics, failed to explain certain phenomena such as the photoelectric effect and the observation of discrete line spectra. Moreover, the phenomenon of radioactivity appeared to be a completely random process, which was conflicting with the *deterministic* nature of classical physics.

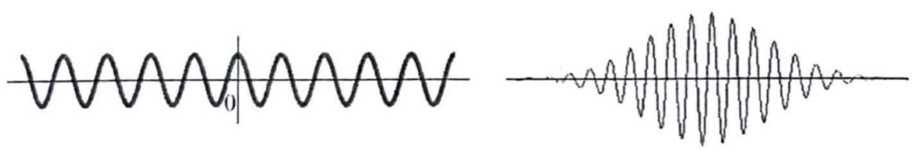
Hence by the turn of the 20th Century, scientists began to develop a new theory (now known as Quantum Physics) to explain these phenomena. Although concepts in Quantum Physics might seem counter-intuitive, their validity and soundness have been proven by experimental results. This however, does not mean that classical physics is incorrect, it is just incomplete.

In contrast to the deterministic viewpoint of classical physics, quantum physics on the other hand is inherently probabilistic, due to the **Heisenberg uncertainty principle**, which states that there is a fundamental limit to the precision with which certain pairs of physical properties of a particle can be measured. The **Heisenberg uncertainty principle** plays a huge role in quantum mechanics (QM), but as QM is out of the syllabus, only the **Heisenberg uncertainty principle** will be introduced and applied to order-of-magnitude calculations.

19.5 Heisenberg Uncertainty Principle

Background

Uncertainty is inherent in the nature of things, even with the use of perfect instruments and experimental techniques. To understand the uncertainty principle, first imagine that a particle has a single wavelength that is known *exactly*. According to the de Broglie relation $\lambda = \frac{h}{p}$, we will thus know the momentum of the particle to be precisely $p = \frac{h}{\lambda}$. In reality, a single-wavelength wave would exist throughout space; any region along this wave is hence the same as any other region, as illustrated in Fig. 19.21 (a).



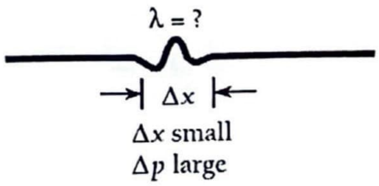
(a) (b)

Fig. 19.21

If we were to ask, *"Where is the particle that this wave represents?"*, there would be no special location in space along the wave that could be identified with the particle – all points along the wave are the same. Therefore, we have *infinite* uncertainty in the position of the particle and we know nothing about its location. *Perfect knowledge of the particle's momentum has cost us all information about its location*.

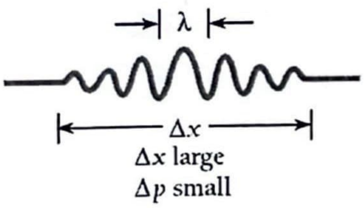
Conversely, we can construct a wave packet such as that shown in Fig. 19.21(b) by adding waves of different wavelengths together which will produce an interference pattern which will localise the wave. Since the wave packet has a finite spatial extent, we now have a better idea where the particle is located. However, we can no longer speak with certainty what the wavelength (and hence momentum) of the particle is, since the wave packet consists of waves of different wavelengths.

Another possible analogy would be to imagine you are holding one end of a rope and you generate a wave by shaking it up and down periodically. If someone asks, "Where *precisely* is that wave?", you would probably not be able to answer. The wave isn't precisely anywhere, but it is spread out over a length of say 10 metres. On the other hand, if someone asks "What is the wavelength", you could give a reasonable answer, say 30 centimetres. By contrast, if you gave the rope a sudden jerk, you would get a relatively narrow bump travelling down the line. This time, the first question ("Where *precisely* is that wave?") is a sensible one, while the second question ("What is the wavelength?") is a nutty one as the motion isn't even periodic, so how can one assign a wavelength?



A wave with well-defined position, but ill-defined wavelength.

momentum and position.



A wave with well-defined wavelength (and hence momentum), but with ill-defined position.

This forms the crux of the Heisenberg Uncertainty Principles. In 1927, the German physicist Werner Heisenberg suggested that nature imposes a limit to the precision with which the position and momentum of a particle can be measured simultaneously. However, Heisenberg was also careful to point out that these inevitable uncertainties arise from the *quantum structure of matter* instead of imperfections in practical measuring instruments.

For intermediate cases, you can say that it has a *fairly* defined wavelength and a *fairly* defined position, but there is an inescapable trade-off between the momentum and position.

Definition

Thus, the **Heisenberg Uncertainty Principle** states:

If a measurement of the position of a particle is made with uncertainty Δx and a simultaneous measurement of its x-component of momentum is made with uncertainty Δp_x, the product of the two uncertainties can never be smaller than $\frac{\hbar}{2}$, i.e.

$$\boxed{\Delta x \Delta p_x \geq \frac{\hbar}{2}}$$

where $\hbar = \frac{h}{2\pi}$ is the reduced Planck constant.

Formula

It can also be considered as an approximation that

$$\boxed{\Delta x \Delta p_x \gtrsim h}$$

Example 12

The speed of an electron is measured to be 5.00×10^3 m s^{-1} to an accuracy of 0.0020 %. Calculate the uncertainty in determining the position of this electron.

Solution

The momentum of the electron is

$p = mv = (9.11\times10^{-31})(5.00\times10^3) = 4.555\times10^{-27}$ kg m s^{-1}

The uncertainty in p is 0.0020% of this value, hence

$$\Delta p = \left(\frac{0.0020}{100}\right)(4.555\times10^{-27}) = 9.11\times10^{-32} \text{ kg m s}^{-1}$$

The minimum uncertainty in position Δx can thus be calculated using the uncertainty principle,

$$\Delta x \gtrsim \frac{h}{(\Delta p)} = \frac{6.63\times10^{-34}}{(9.11\times10^{-32})} = 7.28 \text{ mm}$$

Example 13

According to the Bohr model of the hydrogen atom, the electron in the ground state moves in a circular orbit of radius 0.529×10^{-10} m (Bohr's radius), and the speed of the electron in this state is 2.2×10^6 m s^{-1}. Estimate the minimum uncertainty in the radial position of the electron.

Solution

Assuming uncertainty in the electron's momentum in the radial direction is less than the momentum itself, we have:

$$\Delta p < mv = (9.11\times10^{-31})(2.2\times10^6) = 2.0042\times10^{-24} \text{ kg m s}^{-1}$$

Using the uncertainty principle, we can estimate the *approximate* uncertainty in the radial position of the electron Δr as

$$\Delta r \gtrsim \frac{h}{\Delta p} = \frac{6.63\times10^{-34}}{2.0042\times10^{-24}} = 3.31\times10^{-10} \text{ m}$$

Example 14

A bullet (m = 50 g) and an electron (m_e = 9.1×10^{-31} kg) are both moving at 300 m s^{-1}. The uncertainty in the speed measurement is 0.010 %. If position and speed are measured at the same time, with what accuracy can we measure the position of each object?

Solution

Electron:

$$\Delta p = m\Delta v = (9.1\times10^{-31})\left(\frac{0.010}{100}\right)(300) = 2.73\times10^{-32}\ \text{kg m s}^{-1}$$

$$\Delta x \gtrsim \frac{h}{(\Delta p)} = \frac{(6.63\times10^{-34})}{(2.73\times10^{-32})} = 0.0243\ \text{m}$$

Bullet:

$$\Delta p = m\Delta v = (50\times10^{-3})\left(\frac{0.010}{100}\right)(300) = 1.50\times10^{-3}\ \text{kg m s}^{-1}$$

$$\Delta x \gtrsim \frac{h}{(\Delta p)} = \frac{(6.63\times10^{-34})}{(1.50\times10^{-3})} = 4.42\times10^{-31}\ \text{m}$$

We see that these quantum principles are only significant or important for very Small particles.

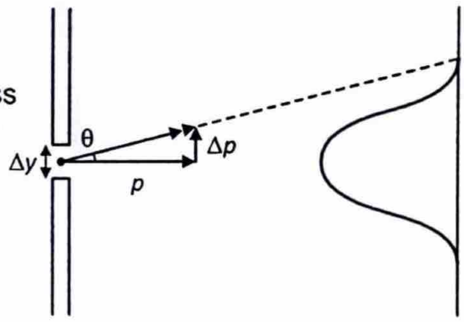
Example 15

A beam of electron of speed v (<< c) is incident on a narrow slit of width b. Show that the width of the pattern detected on a screen far from the slit obeys the Heisenberg Uncertainty Principle.

Solution

When the beam of electron is made to pass through the narrow slit, the uncertainty in the y-position

$$\Delta y = b$$



From Heisenberg Uncertainty Principle,

$$\Delta y \Delta p_y \approx h$$

So we can deduce that $\Delta p_y \approx \dfrac{h}{b}$

So the narrow beam is now spread over a small angular displacement

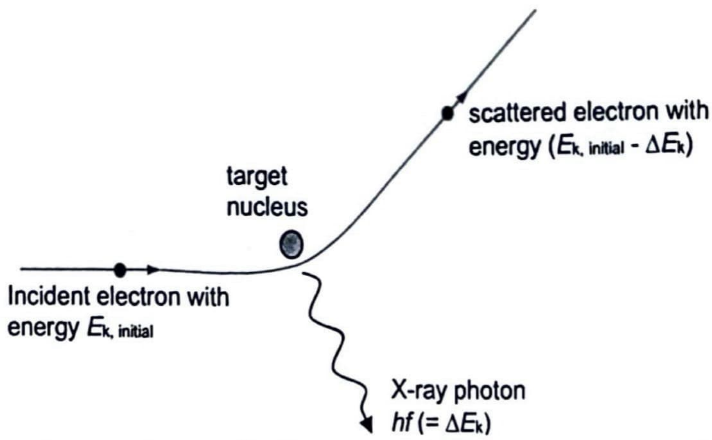
$$\theta \approx \frac{\Delta p_y}{p} = \frac{h/b}{h/\lambda} = \frac{\lambda}{b}$$

which agrees in general with diffraction of a wave.

Bremsstrahlung X-ray	The broad spectrum, high energy EM radiation produced when highly energetic electrons are decelerated; it is characterized by a minimum wavelength corresponding to photon emitted by the loss of maximum energy of an electron.
Characteristic X-ray	The sharp spectrum, high energy EM radiation produced when electrons from a higher shell de-excite to a vacancy in the lower shell, created by highly energetic electrons.

APPENDIX

The Continuous X-ray Spectrum



Consider an electron with an initial kinetic energy $E_{k,\ initial}$ that collides with a target atom as shown in the figure. The fast moving electron interacts with the nucleus of the target metal and as the electron approaches a nucleus it undergoes an acceleration due to the attractive force between the nucleus and the electron. As the electron accelerates (due to a change in direction), it will emit electromagnetic energy in the form of a photon (X-ray). Since it loses energy, the electron will leave the target nucleus with lesser kinetic energy.

The energy of the photon released depends on how close the negative electron comes into contact with the positive nucleus. The closer an electron approaches a nucleus, the higher the energy of the released photon.

This scattered electron which now has energy less than $E_{k,initial}$ may have a subsequent collision with another target atom, generating a second X-ray photon whose energy, in general, is different from the energy of the X-ray photon produced in the first collision.

This electron-scattering process continues until the electron is approximately stationary, i.e., it loses all its energy. All the X-ray photons generated from these collisions between electrons and target atoms form part of the continuous X-ray spectrum.

Glossory of Terms

Photoelectron – an electron emitted from surface of a material due to incident electromagnetic radiation.

Photoemission (Photoelectric effect) – the process whereby electrons is emitted from the surface of a material when electromagnetic radiation of sufficiently high frequency or short wavelength is incident on a surface.

Photon – a quantum of electromagnetic radiation or energy.

Stopping potential – the negative potential of collector w.r.t. emitter which prevents the most energetic photoelectrons from reaching the collector and hence results in zero photocurrent.

Threshold frequency – the min. frequency of EM radiation below which no emission of photoelectrons occurs.

Threshold wavelength – the max. wavelength of EM wave to cause photoemission.

Work function Φ – the minimum amount of energy needed to liberate an electron from the surface of a material.

Absorption line spectra – dark spectral lines on continuous bright background obtained when white light illuminating a cool vapour passes through a diffraction grating.

Emission line spectra – bright spectral lines on dark background obtained when EM radiations emitted by diffuse gas (excited by heat or electric discharges) passes through a diffraction grating.

Ground state – the lowest and most stable energy state of an atom.

Bremsstrahlung X-ray – the broad spectrum, high energy EM radiation produced when highly energetic electrons are decelerated; it is characterized by a minimum wavelength corresponding to photon emitted by the loss of maximum energy of an electron

Characteristic X-ray – the sharp spectrum, high energy EM radiation produced when electrons from a higher shell de-excite to a vacancy in the lower shell, created by highly energetic electrons

Wave-particle duality – the theory where matter and waves have particle-like and wave-like characteristics

de Broglie wavelength – the wavelength associated with wave-like properties of a particle

Uncertainty Principle – a mathematical relationship between a pair of physical quantities (position and momentum or energy and time) which states that fundamentally, no such pair of quantities can be measured with infinite precision simultaneously

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A. Beiser. Concepts of Modern Physics.

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http://en.wikipedia.org/wiki/Line_spectra

http://en.wikipedia.org/wiki/X-ray

http://en.wikipedia.org/wiki/Matter_wave

http://en.wikipedia.org/wiki/Wave-particle_duality

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Tutorial 19 QUANTUM PHYSICS

H2 PHYSICS 9749

Data: speed of light in free space: $c = 3.00 \times 10^8$ m s^{-1}
the Planck constant: $h = 6.63 \times 10^{-34}$ J s
elementary charge: $e = 1.60 \times 10^{-19}$ C
rest mass of electron: $m_e = 9.11 \times 10^{-31}$ kg
rest mass of proton: $m_p = 1.67 \times 10^{-27}$ kg

Photoelectric Effect

S1 What is a *photon*? Which photon has highest and lowest energy, red, green or blue light?

S2 Write down an expression for the *energy* of a photon, in terms of (a) its *frequency*, (b) its *wavelength*.

S3 What is the *photoelectric effect*?

S4 Describe how the *wave model of light* is unable to explain the experimental observations in the photoelectric effect? How does *quantum theory* address the same experimental observations?

S5 State *Einstein's photoelectric equation*.

S6 What is meant by (a) *threshold frequency* and (b) the *work function* of a surface, by reference to the photoelectric effect?

S7 What is meant by *stopping potential*? Why is stopping potential dependent on frequency of the light and work function of the metal but independent of the intensity of light?

S8 Sketch the variation of $K.E._{max}$ of emitted electrons with frequency for metals A and metal B, where metal A has a lower work function than metal B.

Wave-Particle Duality

S9 Explain what is meant by *wave-particle duality*.

S10 State the experiment(s)/phenomenon that shows that light and matter has (a) wave properties (b) particle properties.

Atomic Spectra & X-ray

S11 Distinguish between emission and absorption line spectra.

S12 Explain how the line spectra show that the energy in an atom exists in discrete levels.

S13 What is *ionisation energy* of an atom?

S14 Explain the *features* of the X-ray spectrum using quantum theory.

Heisenberg Uncertainty Principle

S15 State the expression for position-momentum uncertainty principle.

S16 Do the uncertainty principles arise because of imprecision in measuring instruments?

S17 Does the position-momentum uncertainty principle imply that a precise measurement of a particle's position is impossible?

Self – Practice Questions

1 Three energies are listed below.

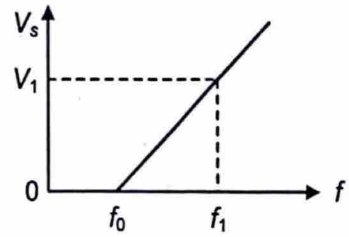
1 the energy of a photon of a 3 m wavelength radio wave
2 the energy of an X-ray photon
3 the energy of a photon of yellow light from a sodium lamp

Which of the following puts these energies in order of increasing magnitude?

A	1	2	3
B	1	3	2
C	2	1	3
D	2	3	1
E	3	2	1

[N92/1/27]

2 In a photoelectric experiment, the potential difference V that must be maintained between the emitter and the collector just to prevent any electrons from reaching the collector is determined for different frequencies f of the incident light. The graph on the right is obtained.



What is the maximum kinetic energy of the electrons emitted at frequency f_1?

A hf_1

B $V_1/(f_1 - f_0)$

C $h(f_1 - f_0)$

D V_1

E $eV_1/(f_1 - f_0)$

[J87/1/28]

3 A beam of monochromatic radiation falls on to a metal X and photoelectrons are emitted. The rate of emission of photoelectrons will be double if

A a beam of double the intensity is used.

B radiation of double the frequency is used.

C radiation of double the wavelength is used.

D the thermodynamic temperature of the metal is doubled.

E a metal with a work function half that of X is substituted for X. **[J82/1/14]**

4 Two beams, P and Q, of light of the same wavelength, fall upon the same metal surface causing photoemission of electrons. The photoelectric current produced by P is four times that produced by Q.

Which of the following gives the ratio

$$\frac{\text{wave amplitude of beam P}}{\text{wave amplitude of beam Q}}?$$

A 1/4 **B** 1/2 **C** 2 **D** 4 **E** 16

[J93/1/29]

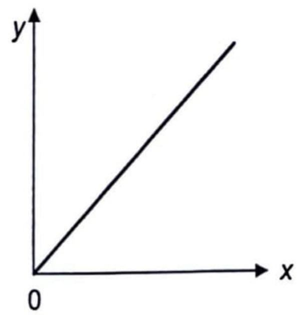
5 An ultraviolet radiation source causes emission of photoelectrons from a zinc plate.

How would the maximum kinetic energy E_k of the photoelectrons and the number of photoelectrons emitted per second n be affected by substituting a more intense source of the same wavelength?

	E_k	n
A	decreased	increased
B	unchanged	unchanged
C	unchanged	increased
D	increased	unchanged
E	increased	increased

[J92/1/30]

6 In a series of photoelectric emission experiments on a certain metal surface, possible relationships between the following quantities were investigated: threshold frequency f_0, frequency of incident light f, light intensity I, photocurrent i, maximum kinetic energy of photoelectrons T_{max}.



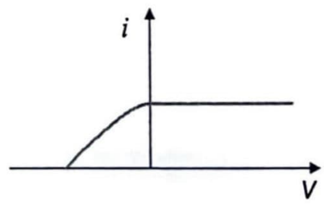
Two of these quantities, when plotted as a graph of y against x, give a straight line through the origin.

Which of the following correctly identifies x and y with the photoelectric quantities?

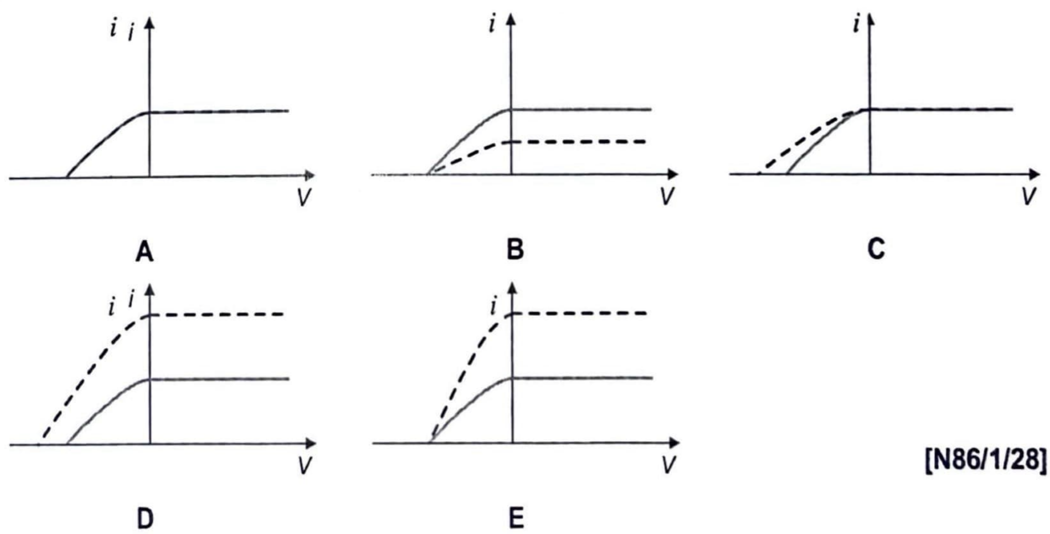
	x	y
A	i	f_0
B	f	T_{max}
C	I	i
D	I	T_{max}

[N96/1/28]

7 A metal surface in an evacuated tube is illuminated with monochromatic light causing the emission of photoelectrons which are collected at an adjacent electrode. For a given intensity of light, the way in which the photocurrent I depends on the potential difference V between the electrodes is as shown in the diagram below.



If the experiment were repeated with light of twice the intensity but the same wavelength, which of the graphs below would best represent the new relation between I and V? (Result of the new experiment is indicated by a broken line.)



[N86/1/28]

8 What is the de Broglie wavelength of a particle of mass m and kinetic energy E?

A $h\sqrt{2mE}$

B $\sqrt{2mE}\,/\,h$

C $h/\sqrt{mE}$

D $h/\sqrt{2mE}$

E $h\sqrt{2}/\sqrt{mE}$

[N82/2/33]

9 The intensity of a beam of monochromatic light is doubled. Which one of the following represents the corresponding change, if any, in the momentum of each photon of the radiation?

A increased to fourfold

B doubled

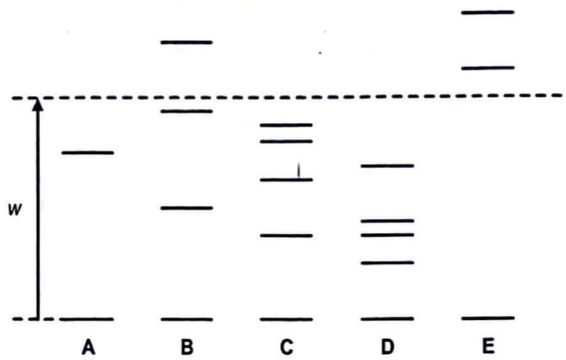
C the same

D halved

E reduced to a quarter

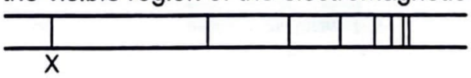
[J83/2/34]

10 The diagram shows the electron energy levels, referred to the ground state (the lowest possible energy) as zero, for five different isolated atoms.



Which atom can produce radiation of the shortest wavelength when atoms in the ground state are bombarded with electrons of energy W? **[J89/1/29]**

11 The diagram shows part of a typical line emission spectrum. This spectrum extends through the visible region of the electromagnetic spectrum into the ultraviolet region.



Which statement is true for emission line **X** of the spectrum?

A It has the longest wavelength and is at the ultraviolet end of the spectrum.

B It has the highest frequency and is at the ultraviolet end of the spectrum.

C It has the lowest frequency and is at the red end of the spectrum.

D It has the shortest wavelength and is at the red end of the spectrum. **[N95/1/27]**

12 Transitions between three energy levels in a particular atom give rise to three spectral lines of wavelengths, in order of increasing magnitude, λ_1, λ_2 and λ_3. Which of the following equations correctly relates λ_1, λ_2 and λ_3?

A $\dfrac{1}{\lambda_1}=\dfrac{1}{\lambda_2}+\dfrac{1}{\lambda_3}$

B $\dfrac{1}{\lambda_1}=\dfrac{1}{\lambda_3}-\dfrac{1}{\lambda_2}$

C $\dfrac{1}{\lambda_1}=\dfrac{1}{\lambda_2}-\dfrac{1}{\lambda_3}$

D $\lambda_1=\lambda_2-\lambda_3$

E $\lambda_1=\lambda_3-\lambda_2$

[N89/1/30]

13 In an X-ray tube, electrons, each with a charge e, are accelerated through a potential difference V and are then made to strike a metal target. If h is the Planck constant and c is the speed of light, the minimum wavelength of the emitted radiation is given by the formula

A $\dfrac{he}{cV}$ **B** $\dfrac{eV}{hc}$ **C** $\dfrac{eV}{h}$ **D** $\dfrac{hc}{eV}$ **E** $\dfrac{hcV}{e}$

[N76/2/34]

14 High energy electrons of energy E are used to strike a target.
Which one of the following statements is NOT true?

A The continuous X-ray spectrum is formed by the high energy incident electrons repeatedly colliding with different atoms, each time causing the atoms to emit photons of different wavelengths.

B The characteristic X-ray spectrum formed can be used to identify the element.

C The minimum wavelength of the continuous X-ray spectrum formed depends on the element used as the target.

D The characteristic X-ray spectrum is formed when electrons in the higher energy shells make transitions to the "holes" vacated by the inner shell electrons that are ejected by the high energy incident electrons.

15 An electron and a 140-g baseball are each travelling at a speed of 150 m s^{-1} with an uncertainty of 0.05%.
Calculate and compare the uncertainty in their positions.

16 Explain why it is that the more massive an object is, the easier it becomes to predict its future position.

Discussion Questions

Photoelectric Effect

1 In a photoelectric emission experiment, ultra-violet radiation of wavelength 254 nm and of intensity 210 W m^{-2} was incident on a silver surface in an evacuated tube, so that an area of 12 mm^2 was illuminated. A photocurrent of 4.8×10^{-10} A was collected at an adjacent electrode.

(a) What was the rate of incidence of photons on the silver surface?

(b) What was the rate of emission of electrons?

(c) The photoelectric quantum yield is defined as the ratio

$$\frac{\text{number of photoelectrons emitted per second}}{\text{number of photons incident per second}}$$

(i) Calculate the quantum yield of this silver surface at the wavelength of 254 nm.

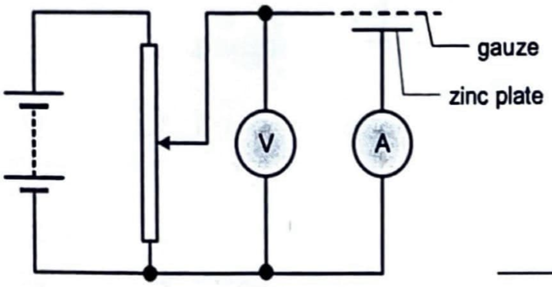
(ii) Give two reasons why this value might be expected to be much less than one.

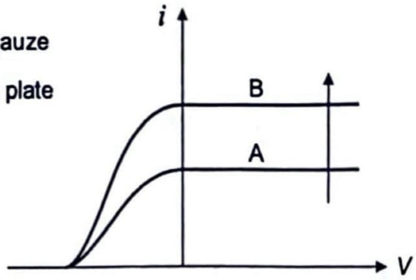
(d) When the experiment was repeated with ultra-violet radiation of wavelength 313 nm, no photoelectrons were emitted. Explain this observation.

[J83/1/16 part]

2 **(a)** A source of ultra-violet light has a wavelength 2.55×10^{-7} m. Calculate the energy of a photon of this wavelength.

(b) The source referred to in **(a)** illuminates a zinc plate which has been cleaned and placed a few millimeters beneath a piece of gauze. Photoelectrons are emitted from the plate and are attracted to the positive gauze because of the potential difference V between the plate and the gauze. When V is varied it is found that the photoelectric current i varies as shown in curve A.





(i) Explain why, for curve A, the photoelectric current reaches a maximum value no matter how large V is made.

(ii) The battery connections are reversed so that the potential difference V is made negative. Photoelectrons are now repelled, but some will still reach the gauze. Explain why some electrons still reach the gauze.

(c) The intensity of illumination is then increased and curve B is obtained.

(i) Explain why the maximum photoelectric current is increased.

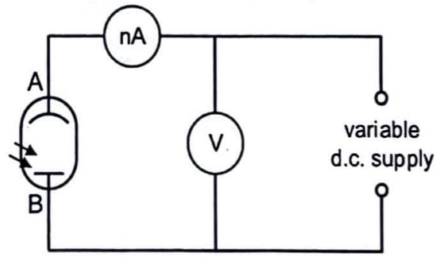
(ii) Suggest why the stopping potential remains constant.

(d) The zinc plate is replaced with a nickel plate which has a greater work function.

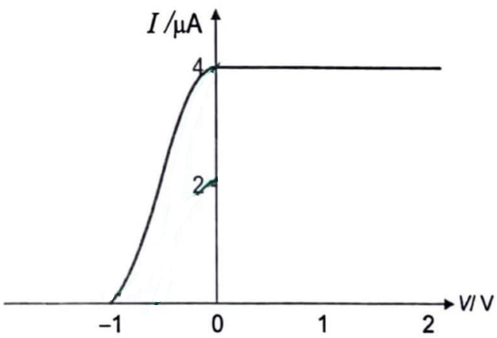
Sketch in the figure, the variation of photoelectric current i with potential difference V. Explain the changes.

[N97/2/8 modified]

3 Two metal electrodes A and B are sealed into an evacuated glass envelope and ~~a~~ potential difference V, measured using the voltmeter, is applied between them as shown in the figure.



B is then illuminated with monochromatic light of wavelength 365 nm, and the current I is measured for various values of V. The results are shown in the graph.



(a) From this graph, deduce the p.d. V required to stop the photoelectrons from reaching electrode A.

(b) Calculate the maximum kinetic energy of the photoelectrons.

(c) Deduce the work function energy of the material of B.

(d) If only one in three incident photons succeed in ejecting a photoelectron, which goes to form the saturated current, calculate rate of photons incident on the cathode.

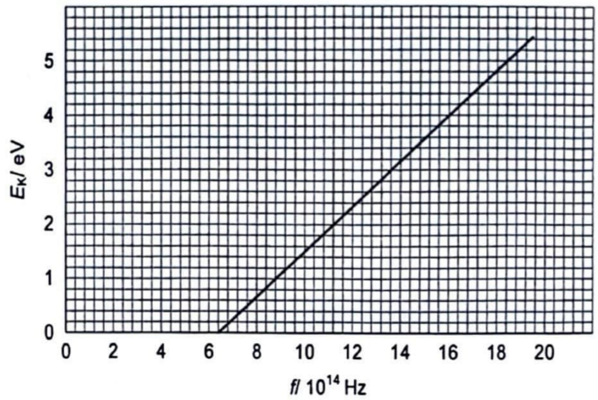
(e) Sketch on the same graph how current varies with potential

(i) if intensity of source is reduced, while keeping the rest constant;

(ii) if light of longer wavelength is used, while keeping the rest constant.

[J91/2/7 Part]

4 The following graph shows how the maximum kinetic energy E_K of a photoelectron from a particular material varies with the frequency f of the electromagnetic radiation that causes the emission of photoelectrons.



(a) Use the graph to determine

(i) the threshold frequency for this material,

(ii) the maximum kinetic energy of photoelectrons from this material when it is illuminated with electromagnetic radiation of frequency 18.0×10^{14} Hz. Give your answer in joules.

(b) Use the photoelectric equation and your answers in **(a)** to determine the Planck constant.

[N2000/3/6 Part]

Wave-Particle Duality

5 A parallel beam of violet light of wavelength 4.5×10^{-7} m and intensity 700 W m^{-2} is incident normally on a surface.

(a) Calculate

(i) the energy of a photon of violet light;

(ii) the number of photons incident per second on 1.0×10^{-4} m^2.

(b) (i) State the de Broglie relation for the momentum p of a particle in terms of its associated wavelength λ.

(ii) Use the equation in **(b)(i)** to calculate the momentum of a photon of the violet light.

(c) (i) Use your answers to **(a)** and **(b)** to calculate the change in momentum of photons incident on 1.0×10^{-4} m^{-2} of the surface in one second. Assume that all photons are reflected by the surface.

(ii) Suggest why the quantity you have calculated in **(c)(i)** is referred to as a 'radiation pressure'.

[N98/2/7]

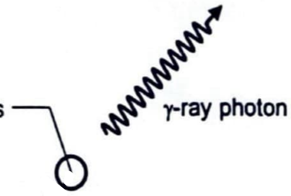
6 A stationary nickel nucleus of mass 9.95×10^{-26} kg emits a γ-ray photon of energy 1.17 MeV.

(a) For the γ-ray photon,

(i) show that its wavelength is approximately 1.06×10^{-12} m,

(ii) calculate its momentum.

(b) The direction in which the γ-ray photon is emitted is as shown.



(i) On the figure, draw an arrow to show the direction of motion of the nickel nucleus after the emission of the photon.

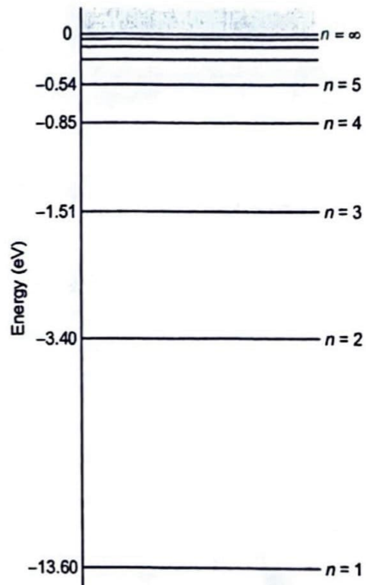
(ii) Calculate the speed of the nickel nucleus after the emission of the photon.

(c) A second nickel nucleus that is moving emits a γ-ray photon. Suggest, with a reason, whether angle between the final direction of motion of the nucleus and that of the emitted photon will be the same as that in the figure.

[N2005/2/2]

Line Spectra & X-Rays

7 The diagram below represents some of the energy levels in a hydrogen atom.



(a) Use the diagram to explain what is meant by the terms

(i) ground state,

(ii) excitation energies and

(iii) ionisation energy

(b) State what transitions may happen when a photon of light hits and interacts with the hydrogen atom (initially at its ground state), when its energy content is

(i) 6.5 eV

(ii) 10.2 eV

(iii) 11.4 eV

(iv) 12.~~1~~9 eV

(c) State what may happen if an electron with the same energies as the photon hits and interacts with the hydrogen atom (initially at its ground state).

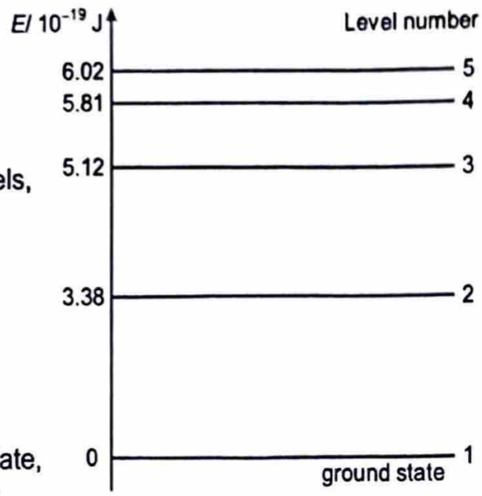
(i) 6.5 eV

(ii) 10.2 eV

(iii) 11.4 eV

(iv) 12.~~1~~9 eV

8 The diagram is a simplified representation of the 5 lowest energy levels of the outermost electron in the sodium atom.



(a) Considering transitions between only these levels,

(i) which spectral transition has the shortest wavelength (give your answer in terms of level numbers),

(ii) how many spectral emission lines might be produced by transitions among these levels?

(b) If the sodium atoms are initially in the ground state, how many absorption lines might be detected?

(c) Cool sodium vapour at low pressure is bombarded with electrons of kinetic energy E. Which transitions would you expect to observe if E has the value

(i) 3.0×10^{-19} J

(ii) 4.0×10^{-19} J

(iii) 5.5×10^{-19} J

(d) In practice, the highest level inferred from observation of single electron transitions in the sodium spectrum is 8.21×10^{-19} J.

(i) Explain the significance of this value.

(ii) Calculate the range of potential differences which would accelerate bombarding electrons to produce spectral line emission, but no free electrons.

(e) In fact, level 2 consists of a pair of closely spaced levels. Transitions from them give rise to sodium "D-lines" of wavelengths 589.0 nm and 589.6 nm.

(i) Calculate the energy difference between the two closely spaced levels.

(ii) Identify the other level involved from which the transitions took place.

[N84/III/6]

9 The measured wavelengths, λ_m, of selected lines in the hydrogen spectrum are given empirically by

$$\frac{1}{\lambda_m} = R\left(\frac{1}{4} - \frac{1}{m^2}\right)$$

where R is a constant and has the value 1.097×10^7 m^{-1} and m is an integer taking the values 3, 4, 5, ...etc.

(a) Calculate the value of the wavelength when $m = 4$.

(b) Calculate the minimum wavelength given by this equation.

(c) Draw a diagram showing the approximate positions of the lines on a horizontal axis of wavelength. Mark the two values you have already calculated and also mark the red and the violet ends of the spectrum.

(d) Explain why it is that although there is an infinite number of lines in this spectrum, the spectrum is nevertheless seen as a line spectrum.

[J88/2/12 part]

10 X-ray diffraction may be observed using a crystal as the diffraction grating. Electron diffraction may also be observed using a similar crystal if the de Broglie wavelength of the electron is appropriate.

The X-rays have a wavelength 2.4×10^{-10} m. For an electron to have a de Broglie wavelength of 2.4×10^{-10} m, determine

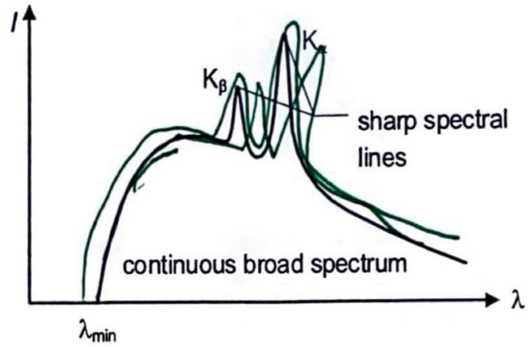
(a) the momentum of the electron,

(b) the potential difference through which the electron must be accelerated from rest.

[N2009/3/3]

11 **(a)** Why does the minimum cut-off wavelength in X-ray spectrum support a particulate nature of light?

(b) The X-ray spectrum produced by an X-ray tube with a tungsten anode and maintained at a constant accelerating potential of 200 kV is shown.



Describe and explain any changes to the graph if the following changes are made independently:

(i) The accelerating potential is increased to 250 kV.

(ii) An X-ray tube with a copper target is used.

Heisenberg Uncertainty Principle

12 An electron and a proton, each initially at rest, are accelerated across the same potential difference. Assuming that the uncertainty in their position is given by their de Broglie wavelength, calculate the ratio of the uncertainty in their momentum.

13 What is the uncertainty in the location of a photon of wavelength 300 nm if this wavelength is known to an accuracy of one part in a million?

14 A measurement establishes the uncertainty of the position of a proton to be 1.5×10^{-11} m. What is the minimum uncertainty in the proton's position after 1.00 s?

Challenging Questions

1 A light source emitting radiation at 7.00× 10^{14} Hz is incapable of ejecting photoelectrons from a certain metal. In an attempt to use this source to eject photoelectrons from the metal, the source is given a velocity towards the metal.

(a) Explain how this procedure can produce photoelectrons.

(b) When the speed of the light source is equal to 0.280*c*, photoelectrons just begin to be ejected from the metal. What is the work function of the metal?

(c) When the speed of the light source is increased to 0.900*c*, determine the maximum kinetic energy of the photoelectrons.

2 According to classical physics, a charge *e* moving with an accelerating *a* radiates at a rate

$$\frac{dE}{dt} = -\frac{1}{6\pi\varepsilon_0}\frac{e^2a^2}{c^3}$$

where *c* is the speed of light.

(a) Show that an electron in a classical hydrogen atom spirals into the nucleus at a rate

$$\frac{dr}{dt} = -\frac{e^4}{12\pi^2\varepsilon_0^2 r^2 m_e^2 c^3}$$

(b) Find the time interval over which the electron reaches $r = 0$ starting from r. $r_0 = 2.00 \times 10^{-10}$ m.

3 When an excited atom emits a photon, the linear momentum of the photon must be balanced by the recoil momentum of the atom. As a result, some of the excitation energy of the atom goes into the kinetic energy of its recoil.

(a) Write down an equation to include this effect.

(b) Calculate the ratio between the recoil energy and the photon energy for the n = 3 to n = 2 transition in hydrogen, for which $E_i - E_f = 1.9$ eV. Is this effect significant?

4 Use the uncertainty principle to make an order of magnitude estimate for the kinetic energy (in eV) of an electron in a hydrogen atom.

SELF-CHECK QUESTIONS SUGGESTED ANSWERS

S1 A photon is a packet or **quantum of electromagnetic energy.**
In order of increasing energy: red, green, blue.

S2 $E = hf = \dfrac{hc}{\lambda}$

S3 Photoelectric effect is the phenomenon in which **(photo)electrons are emitted from the surface of a metal** when **light of sufficiently high frequency is incident on it.**

S4 *Observation 1*

Maximum K.E. of photoelectrons depends only on frequency of light (and work function) but not on intensity.

Classical Prediction

Energy of light **depends only on intensity** and **not frequency**. Therefore **photoelectrons of greater K.E. are emitted using light of higher intensity.**

Quantum Explanation

Light energy is emitted, transmitted and absorbed in **discrete quanta** of $E = hf$ known as **photons**. An electron is emitted by **absorbing the entire energy of a single photon**. Electrons expend varying amount of energy W to escape from the surface. **Energy in excess of W becomes K.E. of photoelectron.**

$$\text{K.E.} = hf - W$$

Electrons that **expend the least amount of energy**, known as the **work function Φ**, escape with the **greatest K.E.**

Hence photoelectrons are emitted with a **range of energies** up to $\text{K.E.}_{max} = hf - \Phi$.

Observation 2

The existence of **threshold frequency** for photoemission.

Classical Prediction

Photoelectrons are emitted **independent of the wavelength** or frequency of light.

Quantum Explanation

The minimum energy to eject an electron from surface of metal is the **work function Φ**. Since it is **highly unlikely** for an electron to **absorb the energy of more than one photon,** photoemission takes place only if **energy of photon** $hf > \Phi$.

Observation 3

Photoemission is **instantaneous.**

Classical Prediction

Photoelectrons are emitted after an **appreciable time lag.**

Quantum Explanation

Energy of each photon is absorbed instantaneously so **no appreciable time lag is detected.**

S5 $\text{K.E.}_{max} = \frac{1}{2}mv_{max}^2 = hf - \Phi$

S6 The threshold frequency f_0 of a metal is the **minimum frequency of EM radiation to cause photoemission**. Every metal has a **characteristic threshold frequency.**

The work function Φ of a metal is the **minimum amount of the energy needed to remove an electron from the surface of the metal.**

Work function and threshold frequency are related by $\Phi = hf_0$.

S7 The stopping potential V_s is the **minimum value of the potential difference** between the collector and emitter (with collector at lower potential than emitter) **needed to stop the most energetic photoelectrons from reaching the collector** (zero photocurrent). So $eV_s = \text{K.E.}_{max}$.

According to quantum theory, light energy is emitted, transmitted and absorbed as quanta known as photons. The energy E of each photon depends only on its frequency f, where

$$E = hf$$

The energy needed to liberate the **most loosely bounded electrons** from the surface of the material is known as the **work function** Φ so photoemission occurs if $hf > \Phi$ (since each electron can only absorb the entire energy of at most only one photon). These electrons are emitted with the **greatest kinetic energy** that only **depends on the frequency *f* of light** and the **work function** Φ of the metal

$$\text{K.E.}_{max} = hf - \Phi$$

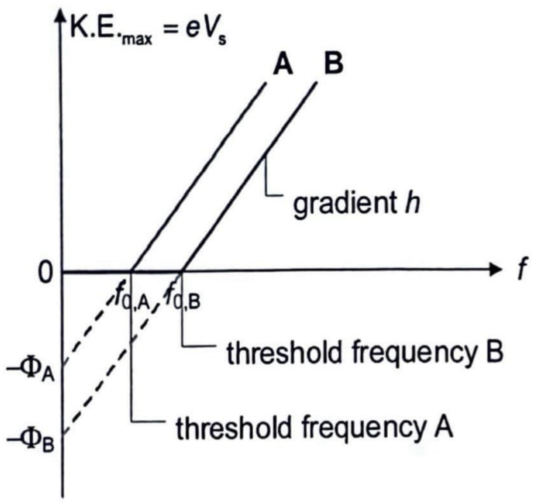
Therefore the stopping potential is

$$V_s = \frac{hf}{e} - \frac{\Phi}{e}$$

The intensity of light, on the other hand, determines the rate of emission of photoelectrons from the surface of the metal and hence the photocurrent i

$$i \propto \frac{dN_e}{dt} \propto \frac{dN_p}{dt} \propto I$$

S8



S9 Wave-particle duality states that EM radiation and matter have both **wave-like** and **particle-like characteristics.**

S10 Light
(a) diffraction, interference or polarisation of light
(b) photoelectric effect, Compton scattering, radiation pressure

Matter
(a) electron diffraction or interference
(b) deflection of electrons in electric or magnetic field, velocity selector

S11 Emission line spectrum consists of a series of **discrete bright lines** of definite wavelength on a **dark background**, whereas absorption line spectrum is a series of **discrete dark lines** on a **bright background** (colour spectrum for visible light).

S12 Each discrete line in (emission or absorption) line spectra corresponds to **photons of one frequency** (or wavelength) and hence **one particular energy. One photon** is emitted when an atom **de-excite from a higher energy level to a lower energy level** so **only certain amount of energy loss are allowed**. Therefore energy in an atom must be quantised.

S13 The **minimum amount of energy** needed to remove the **valence electron** when the atom is in the **ground state.**

S14 The X-ray spectrum has two distinct features: the **continuous spectrum** and the **sharp spectral lines.**

Continuous Broad Spectrum

Energetic bombarding electrons (energies ranging from 20 keV to 150 keV) experience **rapid deceleration** when they are scattered by strong electric field near the nuclei in the target metal. The amount of energy lost by an electron can **vary from zero up to its entire kinetic energy**. Some of these energies (approximately 1%; most of the energy are lost as heat so an efficient cooling system is needed to prevent overheating) are **carried away as X-ray photons** (*bremsstrahlung radiation*) with a **continuous range of energy up to the entire kinetic energy** of the bombarding electrons. The minimum wavelength of the continuous spectrum belongs to that of the most energetic X-ray photon.

For incident electrons accelerated by p.d. V, the **minimum wavelength** of the X-ray photon is

$$\lambda_{min} = \frac{hc}{eV}$$

Sharp Spectral Lines

Superimposed on the continuous broad spectrum is a series of sharp and intense spectral lines. Some bombarding electrons have sufficient energy to **knock out an electron in the inner shell** (shells are labelled starting from K, L, M, N, ...). X-ray photons are emitted when **orbital electrons from a higher shell de-excite to fill up the vacancy in the inner shell.** Since the energy of the electron shell are quantised, the **photons are emitted with discrete energies that are characteristics of the target metal.**

S15 $\Delta x \Delta p \gtrsim h$ or ($\Delta x \Delta p \geq \frac{1}{2}\hbar$; this is the correct equation, but don't use this)

S16 No. Heisenberg was careful to point out that the uncertainty is not due to the measuring instruments but is a **fundamental, intrinsic nature of the universe.**

S17 No. It just states that the more precisely the position of a particle is determined, the less precisely the momentum is known at the same instant, and vice versa.

SOLUTIONS TO SELF-PRACTICE QUESTIONS

1 Energy of photon

$$E = hf = \frac{hc}{\lambda}$$

Wavelength of radio wave 3 m

Wavelength of X-ray ~ 10^{-9} m

Wavelength of yellow light ~ 6×10^{-7} m

Answer: **B**

2 Einstein's photoelectric equation

$$\text{K.E.}_{max} = hf_1 - \Phi = h(f_1 - f_0)$$

Answer: **C**

3 Since rate of incident photons is directly proportional to the intensity of light

$$I \propto \frac{dN_p}{dt}$$

and quantum efficiency is constant

$$\frac{dN_e}{dt} \propto \frac{dN_p}{dt}$$

Therefore, rate of emission of photoelectrons will double when intensity of light is doubled.

Answer: **A**

4 Since the photocurrent

$$i \propto \frac{dN_e}{dt} \propto I$$

From wave theory of light,

$$I \propto A^2 \qquad \Rightarrow \qquad i \propto A^2$$

So 4 times the photocurrent ⇒ twice the amplitude.

Answer: **C**

5 From the earlier question,

$$I \propto \frac{dN_e}{dt}$$

On the other hand, max. K.E. of photoelectrons is dependent only on frequency of light and work function of metal. So max. K.E. remains unchanged.

Answer: **C**

6 Option A: Incorrect. Threshold frequency f_0 is a constant.

Option B: Incorrect. $T_{max} = hf - \Phi$. The graph of T_{max} vs. f is a straight line that does not pass through the origin.

Option D: Incorrect. T_{max} is only dependent on frequency of light and work function of metal. T_{max} is independent of intensity (or power) of light.

Option C: Correct.

Since $I \propto \frac{dN_p}{dt}$ and $\frac{dN_e}{dt} \propto \frac{dN_p}{dt}$ and $i \propto \frac{dN_e}{dt}$ $\Rightarrow$ $i \propto I$

So a graph of photocurrent vs intensity will yield a straight line graph though the origin.

Answer: **C**

7 From previous question, twice the light intensity ⇒ twice the saturated photocurrent.

Stopping potential depends only on the frequency (or wavelength) of light and the work function.

$$V_s = \frac{hf}{e} - \frac{\Phi}{e} = \frac{hc}{e\lambda} - \frac{\Phi}{e}$$

So stopping potential remains the same.

Answer: **E**

8 The momentum of a particle related to its kinetic energy by

$$p = \sqrt{2mE}$$

So the de Broglie wavelength of the particle is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

Answer: **D**

9 The wavelength of the light remains unchanged, Hence according to de Broglie equation, the momentum of each photon remains unchanged.

The intensity of the light is doubled due to the double rate of emission of photons by the source.

Answer: **C**

10 To produce photons of shortest wavelength, the energy difference between the excited state and the ground state must be greatest.

$$\Delta E = \frac{hc}{\lambda} \qquad \Rightarrow \qquad \lambda = \frac{hc}{\Delta E}$$

Atom B may be excited to the highest energy level compared to the other atoms so when it returns to the ground state directly from this energy level, it will emit a photon of shortest wavelength.

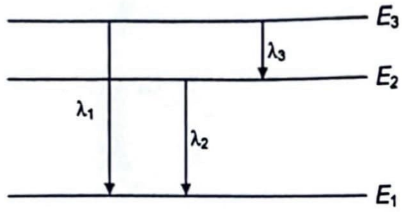
Answer: **B**

11 The lines in the emission spectrum are closer at higher frequency since the energy difference is smaller at higher energy levels in an atom. Therefore X is at lower frequency (or longer wavelength) and is at the red end of the visible spectrum.
Answer: **C**

12 The energies of the photons are related by

$$\frac{hc}{\lambda_1} = \frac{hc}{\lambda_2} + \frac{hc}{\lambda_3} \quad \Rightarrow \quad \frac{1}{\lambda_1} = \frac{1}{\lambda_2} + \frac{1}{\lambda_3}$$

Answer: **A**



13 The kinetic energy of electrons accelerated by a p.d. of V is eV.
A photon is emitted when the electron loses some or all of its kinetic energy.

$$E = \frac{hc}{\lambda} \le eV \quad \Rightarrow \quad \lambda \ge \frac{hc}{eV}$$

Answer: **D**

14 Option A: Not true.
Option B: True, since it is the characteristic of the target metal.
Option C: Not true. Minimum wavelength of the X-ray spectrum depends on the energy of the electron that strikes the target.
Option D: True. The electrons in the atoms of the target undergo transitions from higher energy state to lower energy state, in the process, emit a single photon of the same energy.
Answer: **A & C**

15 The uncertainty in the velocity is

$$\Delta v = \frac{0.05}{100} \times 150 = 0.075 \text{ m s}^{-1}$$

For the electron,

$$\Delta x \simeq \frac{h}{m\Delta v} = \frac{6.63 \times 10^{-34}}{(9.11 \times 10^{-31})(0.075)} = 9.7 \times 10^{-3} \text{ m}$$

For the baseball,

$$\Delta x \simeq \frac{h}{m\Delta v} = \frac{6.63 \times 10^{-34}}{(0.140)(0.075)} = 6.3 \times 10^{-32} \text{ m}$$

The uncertainty for the electron is greater than that of the baseball by a factor of 1.5×10^{-29}.

16 In general, for a given initially uncertainty in position, as a particle becomes more massive, the uncertainty of its velocity ($\Delta v = \Delta p / m$) becomes smaller. Thus the future uncertainty of its position smaller than a particle of smaller mass.

NUMERICAL ANSWERS TO DISCUSSION QUESTIONS

1 3.22×10^{15} s^{-1}, 3.00×10^{9} s^{-1}, 9.3×10^{-7}
2 7.8×10^{-19} J
3 – 1.0 V, 1.60×10^{-19} J, 3.85×10^{-19} J, 7.5×10^{13} s^{-1}
4 6.4×10^{14} Hz, 7.7×10^{-19} J, 6.62×10^{-34} J s
5 4.42×10^{-19} J, 1.58×10^{17} s^{-1}, 1.47×10^{-27} kg m s^{-1}, -4.66×10^{-10} N s
6 1.06×10^{-12} m, 6.24×10^{-22} kg m s^{-1}, 6.27×10^{3} m s^{-1}
8 10, 4, $2.11\text{ V} \le V < 5.13\text{ V}$, 3×10^{-22} J or 4×10^{-22} J
9 486 nm, 365 nm
10 2.76×10^{-24} N s, 26.1 V
12 42.8
13 0.300 m
14 2.65×10^{4} m