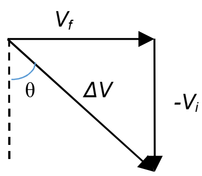
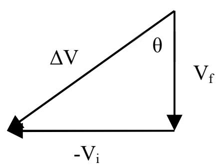
Solutions to 2023 Measurement Discussion Questions

	<u>Physical Quantities and Units</u>
D1.	(A)
	Unit of v = m s^{-1}
	Unit of $\sqrt{g\lambda} = \left[\left(\text{m s}^{-2}\right)\left(\text{m}\right)\right]^{1/2} = \text{m s}^{-1}$
	Note the use of the word "could" in the question. We do not know whether the expression is indeed correct, but to stand a chance of being correct, it has to be of the correct unit! Note also the presentation: "Unit of … = …", and not "v = m s^{-1}".
D2.	(D). Unit of C = unit of αT = unit of βT^3 = J K^{-1} Unit of α = unit of C / unit of T = = J K^{-1} / K = J K^{-2} Unit of β = unit of C / unit of T^3 = J K^{-1} / K^3 = J K^{-4}.
	This is on homogeneity. Take note of the working presentation of such questions: "Unit of …"
D3.	(C). Homogeneous implies that the units of all the terms separated by "+", "-" or "=" are the same.
D4.	(D). Since we can only add or subtract quantities of the same units, b should have the same unit as v, hence the unit of b is m s^{-1}. Rearrange the equation and make k the subject, $k = \frac{2P}{d(v-b)^3}$ Unit of k = (kg m^2 s^{-3})(kg^{-1} m^3) / (m^3 s^{-3}) = m^2
D5.	(a) ampere, mole, kelvin, candela (any two)
	(b) The unit of energy, the joule, can be expressed in terms of base units.
	(c) (i) (1) $\text{density} = \frac{\text{mass}}{\text{volume}}$ Unit of density, $\rho = \text{kg m}^{-3}$ (2) $\text{pressure} = \frac{\text{force}}{\text{area}}$ Unit of pressure, p = $\frac{kg\ m\ s^{-2}}{m^2}$ = kg m^{-1}s^{-2} (shown)
	(ii) unit of c = $\left(\frac{\text{kg m}^{-1}\text{s}^{-2}}{\text{kg m}^{-3}}\right)^{1/2} = \text{m s}^{-1}$ (iii) It might be the speed of the gas molecules.

D6.	(D) • Mass of a typical watermelon is about 4 to 6 kg. • Power output of a domestic electric kettle is normally from 500 to 1000 W. • Human reaction time is about 0.2 to 0.5 s. • Weight of a typical one year old baby is about 60 to 100 N (remember to multiply by *g*!) • Height of the overhead bridge outside Hwa Chong Institution (College) from the road surface is about 6 to 8 m
	What is order of magnitude?
D7.	(C) A smartphone is about 150 g, which corresponds to a weight of 0.15 × 9.81 ≈ 1.5 N
	centi: 10^{-2} and deci: 10^{-1}.
	<u>Errors and Uncertainties</u>
D8.	(B). Because the student did not correct for zero error, his reading is off the mark by four divisions, or $\frac{0.08}{2.16} \times 100\% = 3.7\%$. With the help of the markings on the instrument, the uncertainty of his reading must be smaller than one division. Taking one division, $\frac{0.02}{2.16} \times 100\% = 0.93\%$ He is precise but not accurate.
D9.	(B). The mean is close to the true value (small systematic error). The spread of readings is quite large (not precise).
D10.	(D). ΔV = 0.01 × 4.072 + 0.010 = 0.05 (1sf). Value should be expressed to the tenths place, same as the uncertainty.
	An unusual way to present the uncertainty. Option C can be eliminated straightaway since its uncertainty was expressed to 2 sf.

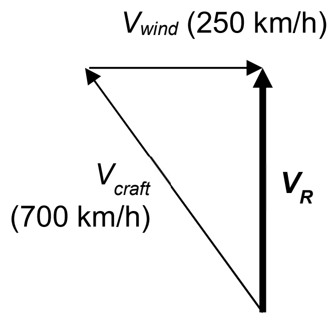
D11.	(D) $L = \frac{y^2}{x}$ Percentage uncertainty, $\frac{\Delta L}{L} \times 100\% = \left[2\left(\frac{\Delta y}{y}\right) + \left(\frac{\Delta x}{x}\right) \right] \times 100\%$ $= 2 \times 3\% + 1\%$ $= 7\%$
	Make L the subject first.
D12.	Making k the subject: $k = 4\pi^2 \frac{m}{T^2}$. Express k in terms of the given variables: $k = 4\pi^2 \frac{m}{\left(\frac{t}{10}\right)^2} = 400\pi^2 \frac{m}{t^2}$ $k = 400\pi^2 \frac{0.150}{6.2^2} = 15.405 \text{ N m}^{-1}$ $\frac{\Delta k}{k} = \frac{\Delta m}{m} + 2\frac{\Delta t}{t}$ $\frac{\Delta k}{15.405} = 0.01 + 2\frac{0.2}{6.2}$ $\Delta k = 1.148 \text{ N m}^{-1}$ $k = 15 \pm 1 \text{ N m}^{-1}$
	(1) Make the term of interest the subject and express it in terms of the given variables. *(2) Note that* $\frac{\Delta T}{T} = \frac{\Delta t}{t}$*, since T = t/10. See that the* $\frac{\Delta k}{k}$ *expression stays the same, even if one were to express k in terms of m and T.*
D13.	$\frac{\Delta \rho}{\rho} = 2\frac{\Delta d}{d} + \frac{\Delta V}{V} + \frac{\Delta L}{L} + \frac{\Delta I}{I}$ (a) $2\frac{\Delta d}{d} = 0.017;\ \frac{\Delta I}{I} = 0.033;\ \frac{\Delta L}{L} = 0.010;\ \frac{\Delta V}{V} = 0.020$ L has the smallest contribution to the uncertainty. (b) $\rho = \frac{\pi d^2 V}{4LI} = \frac{\pi (1.20)^2 (5.0)}{4(100)(1.50)} = 0.037699\ \Omega \text{ cm}$ $\frac{\Delta \rho}{0.037699} = 2\frac{0.01}{1.20} + \frac{0.1}{5.0} + \frac{1}{100} + \frac{0.05}{1.50}$

	$\Delta\rho = 3\times10^{-3}\ \Omega\ \text{cm}$ $\rho = (38\pm3)\times10^{-3}\ \Omega\ \text{cm}$
	One could obviously express it in Ω m too, in which case the answer would be (38 ± 3) × 10^{-5} Ω m.
D14.	(A). <u>Best Method</u> $f=\left(\frac{1}{u}+\frac{1}{v}\right)^{-1}$ Smallest value of $f=\left(\frac{1}{47}+\frac{1}{195}\right)^{-1}=37.87$ mm Largest value of $f=\left(\frac{1}{53}+\frac{1}{205}\right)^{-1}=42.11$ mm $\Delta f=\frac{42.11-37.87}{2}=2.1$ mm <u>Alternative method:</u> $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$. Hence, $\Delta\left(\frac{1}{f}\right)=\Delta\left(\frac{1}{u}\right)+\Delta\left(\frac{1}{v}\right)$. Analyse the term in u: $\frac{\Delta\left(\frac{1}{u}\right)}{\left(\frac{1}{u}\right)}=\frac{\Delta u}{u}$ thus $\Delta\left(\frac{1}{u}\right)=\frac{\Delta u}{u^2}$ --------(1) Similar to (1): $\Delta\left(\frac{1}{v}\right)=\frac{\Delta v}{v^2}$ and $\Delta\left(\frac{1}{f}\right)=\frac{\Delta f}{f^2}$ Thus $\frac{\Delta f}{f^2}=\frac{\Delta u}{u^2}+\frac{\Delta v}{v^2}$ Re-arrange: $\Delta f=\left(\frac{\Delta u}{u^2}+\frac{\Delta v}{v^2}\right)f^2=\left(\frac{3}{50^2}+\frac{5}{200^2}\right)(40)^2=2.1$ mm
	If f is made the subject, f= uv/(u+v), this form would not be appropriate for the normal error calculation, since u and v appear in both *the numerator and the denominator. One cannot in a single physical situation maximise the numerator and minimise the denominator (or minimise the numerator and maximise the denominator) at the same time.*

	<u>Scalars and Vectors</u>
D15.	(C). By definition, $\Delta v = v_f - v_i$  Change in velocity = $\sqrt{(8^2 + 6^2)} = 10$ m s^{-1} tan θ = 6/8 = 37°
	One could also say 10 m s^{-1} 53° south of east. In the exam, please provide a sketch as well to make yourself clear.
D16.	(i) By definition, $\Delta v = v_f - v_i$ 
	(ii) Change in speed = 25 – 30 = - 5 m s^{-1}
	(iii) Change in velocity = $\sqrt{(30^2 + 25^2)} = 39$ m s^{-1}, 50° West of South or a bearing of 230°. Note that θ = tan^{-1}(30/25) = 50°
	Note the difference in the computations for the change in speed (a scalar) and the change in velocity (a vector). The negative sign for change in speed does not indicate direction; it merely means the speed decreases. In the exam, please provide a sketch as well to make yourself clear.

<table>
<tr><td>D17.</td><td>(B). The plane flies slightly into the wind so that its resultant velocity is directed northward.

$V_R = V_{craft} + V_{wind}$



$v_R = \sqrt{700^2 - 250^2} = 654$ km/h</td></tr>
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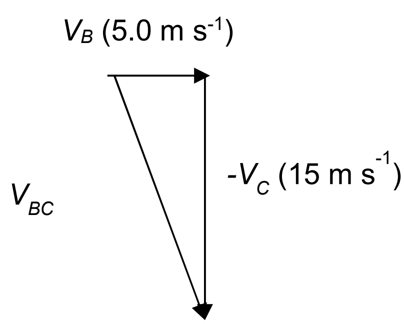
<tr><td>D18.</td><td>(C). $V_{BC} = V_B - V_C$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<tr><td></td><td>From the car's perspective, the bicycle is moving eastward at 5.0 m s^{-1} and moving southward at 15 m s^{-1}.</td></tr>
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