RAFFLES INSTITUTION
PHYSICS DEPARTMENT

Tutorial 13 ELECTRIC FIELD

Data

Permitivity of free space	$\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}
Elementary charge	$e = 1.60 \times 10^{-19}$ C
Rest mass of electron	$m_e = 9.11 \times 10^{-31}$ kg

Self-Check Questions

S1. Define electric field strength and electric potential at a point.

S2. Sketch the electric field lines for

(a) two point charges of equal magnitude and same polarity.
(b) two point charges of equal magnitude and opposite polarity.

S3. Describe the motion of a positively charged particle that enters a uniform electric field with a velocity that is perpendicular to the direction of the electric field

S4. What is the relationship between the electric field strength E at a point in an electric field and the potential V at that point? What is the electric field strength between charged parallel plates?

S5. What are the similarities and differences between electric force and gravitational force?

Self-Practice Questions

J85/I/18

P1 Two small conducting spheres A and B are hung by light, non-conducting threads from fixed points P_A and P_B, and are at the same level. A has mass M and carries charge Q; B has mass 2M and carries charge 2Q. The repulsion between them causes the threads to make small angles θ_A and θ_B with the vertical.

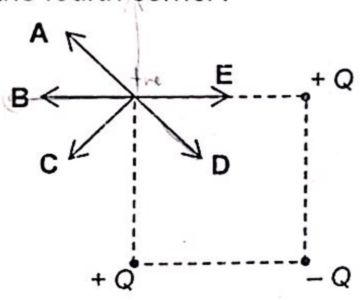
What is the approximate value of the ratio $\theta_A : \theta_B$?

A 4.0
B 2.0
C 0.5
D 0.25

N89/I/16

P2 Point charges, each of magnitude Q, are placed at three corners of a square as shown below.

What is the direction of the resultant electric field at the fourth corner?



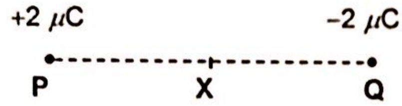
N87/I/19

P3 In the direction indicated by an electric field line,

A the electric field strength must increase.
B the electric field strength must decrease.
C the electric potential must remain constant.
D the electric potential must increase.
E the electric potential must decrease.

N97/I/16

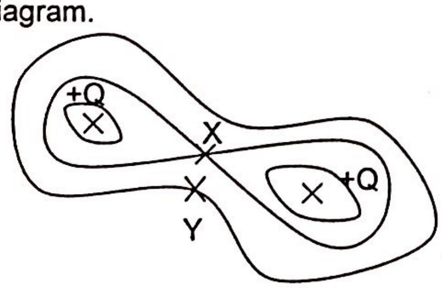
P4 Two charges of +2 μC and −2 μC are situated at points P and Q respectively. X is midway between P and Q.



Which of the following correctly describes the electric field and the electric potential at point X?

	electric field	electric potential
A	towards **Q**	zero
B	towards **Q**	negative
C	towards **P**	zero
D	towards **P**	positive

P5 The equipotential lines of a pair of point charges, each of charge +Q, are shown in the diagram.

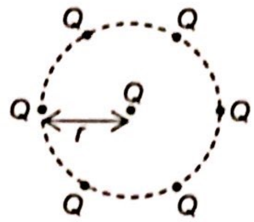


If an electron moves from X to Y,

A its electric potential energy falls.
B it experiences a resultant force pulling it back to X.
C it experiences a resultant force pushing it away from X.
D it does not experience a resultant force.

J89/I/16

P6 A point charge is surrounded symmetrically by six identical charges at distance *r* as shown in the diagram.



How much work is done by the forces of electrostatic repulsion when the point charge at the centre is removed to infinity?

A $\dfrac{6Q^2}{4\pi\varepsilon_0 r}$ **C** $\dfrac{6Q^2}{4\pi\varepsilon_0 r^2}$

B $\dfrac{6Q}{4\pi\varepsilon_0 r}$ **D** $\dfrac{6Q}{4\pi\varepsilon_0 r^2}$

J82/II/21

P7 A metal sphere of radius 0.1 m was insulated from its surroundings and given a large positive charge.

A small charge was brought from a distant point to a point 0.5 m from the sphere's center.

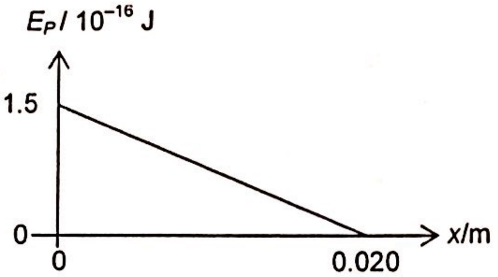
The work done against the electric field was *W* and the force on the small charge in its final position was *F*.

If the small charge is now placed at 1.0 m from the centre of the sphere, what are the values of work done and force?

	work done	force
A	*W*/4	*F*/4
B	*W*/4	*F*/2
C	*W*/2	*F*/4
D	*W*/2	*F*/2

J00/I/6

P8 Two charged plates are 0.020 m apart, producing a uniform electric field. The potential energy E_P of an electron in the field varies with displacement x from one of the plates as shown.

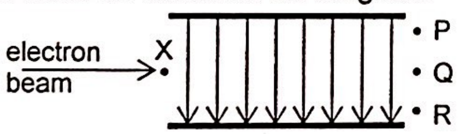


What is the magnitude of the force on the electron?

A 3.0×10^{-18} N
B 7.5×10^{-17} N
C 3.8×10^{-15} N
D 7.5×10^{-15} N

N88/I/27

P9 An electron enters a region in an evacuated tube in which there is a uniform electric field directed as shown in the diagram.

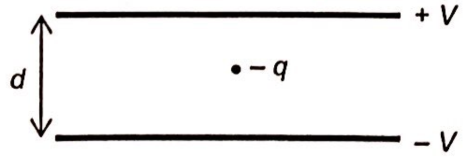


Which one of the following is a possible path for the beam?

A A curved line from X to P
B A straight line from X to P
C A curved line from X to R
D A straight line from X to R
E The straight line XQ

J94/I/15

P10 An oil droplet has a charge $-q$ and is situated between two parallel horizontal metal plates as shown in the diagram.



The separation of plates is d. The droplet is observed to be stationary when the upper plate is at potential $+V$ and the lower potential $-V$.

For this to occur, the weight of the droplet is equal in magnitude to

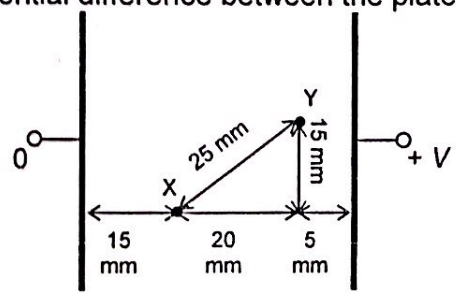
A $\frac{Vq}{d}$ B $\frac{2Vq}{d}$ C $\frac{Vd}{q}$ D $\frac{2Vd}{q}$

P11 With reference to the diagram in question **10**, if the oil droplet acquires additional negative charge, which of the following changes should be made for the droplet to remain stationary?

A Move the plates closer together.
B Reverse the charges on the plates.
C Give the positive plate more positive charge.
D Move both plates the same distance upwards.
E Decrease the potential difference between the plates.

J97/I/16

P12 Two large plane parallel conducting plates are situated 40 mm apart as shown. The potential difference between the plates is V.

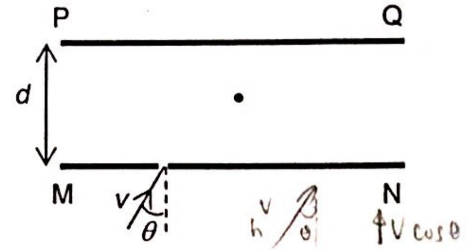


What is the potential difference between point X and point Y?

A $\frac{15}{40}V$ B $\frac{20}{40}V$ C $\frac{25}{40}V$ D $\frac{40}{40}V$

N81/II/7

P13 In an evacuated enclosure, a metal plate PQ is maintained at a negative potential V relative to a second plate MN. Electrons of velocity v enter the space between the plates as shown.



Given that the electron charge is e and that the electron mass is m_e, electrons just reach the plate PQ if

A $\frac{1}{2}m_e v^2 = \frac{eV}{d}$

B $\frac{1}{2}m_e (v\cos\theta)^2 = eV$

C $\frac{1}{2}m_e (v\cos\theta)^2 = \frac{eV}{d}$

D $\frac{1}{2}m_e (v\sin\theta)^2 = eV$

E $\frac{1}{2}m_e (v\sin\theta)^2 = \frac{eV}{d}$

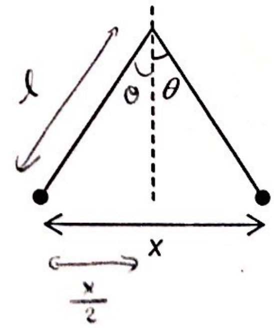
Answers

1	B	8	D
2	A	9	A
3	E	10	B
4	A	11	E
5	B	12	B
6	A	13	B
7	C		

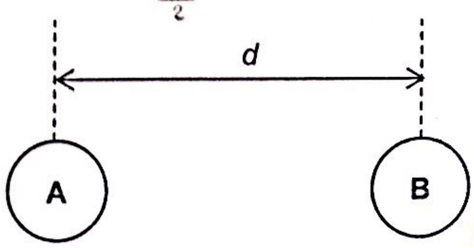
Discussion Questions

D1. **(a)** Two similar conducting balls of mass m are hung from non-conducting threads of length l and carry similar charges q as shown. Using the small angle approximation $\tan\theta \approx \sin\theta \approx \theta$, show that the separation x, between the balls is given by

$$x = \left[\frac{q^2 l}{2\pi\varepsilon_0 mg}\right]^{\frac{1}{3}}.$$



(b) A and B are two identical conducting spheres, each carrying a positive charge Q and are placed close to each other in a vacuum with their centres distance d apart as shown.



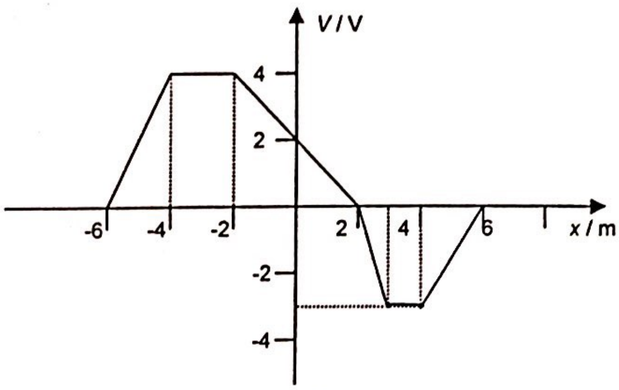
Explain why the force F between them is ***not*** given by the expression $F = \frac{Q^2}{4\pi\varepsilon_0 d^2}$.

D2. Two electrons are fixed 2.0 cm apart. A third electron is shot from infinity and comes to rest midway between the two. Determine the initial velocity of the third electron.

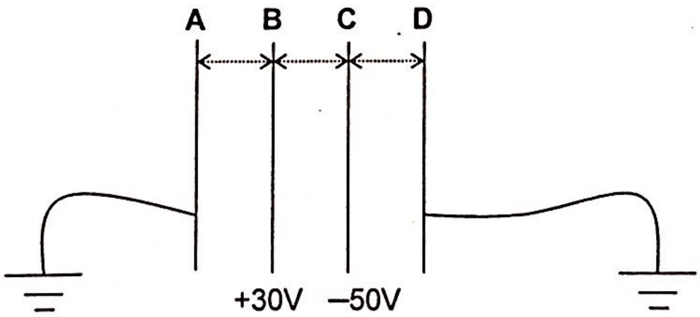
D3. One particle has a mass of 3.00×10^{-3} kg and a charge of +8.00 μC. A second particle has a mass of 6.00×10^{-3} kg and the same charge. The two particles are initially held in place and then released. The particles fly apart, and when the separation between them is 0.100m, the speed of the 3.00×10^{-3} kg particle is 125 m s^{-1}. Determine the initial separation between the particles.

D4. The figure shows the variation of electrical potential V with the distance x from a fixed point along the x-axis.

Sketch a graph to show the variation of the electric intensity E with the distance x.



D5. Four square conducting plates **A**, **B**, **C** and **D**, each of side 0.70 m and negligible thickness, are arranged as shown in the figure below. The distance between adjacent plates is 15 mm. The outer plates **A** and **D** are earthed, **B** is maintained at a potential of +30 V, and C at –50 V.



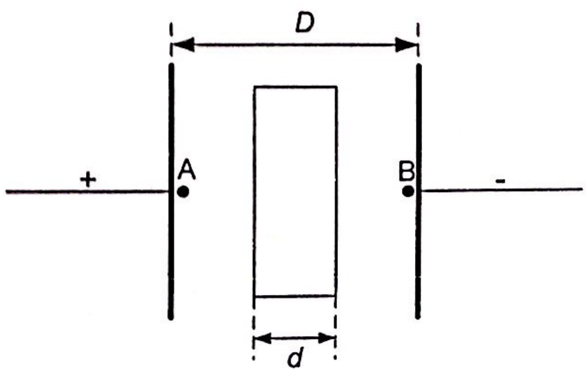
Sketch labelled graphs to show how the electric potential and electric field strength varies along a line through the centre of the plates perpendicular to their planes.

D6. J87/III/6
Two large metal plates are oppositely charged and placed a distance D apart. A conductor of thickness d is situated centrally between the plates.

Sketch graphs, one in each case, to show the variation from A to B of

(a) the electric potential V,

(b) the electric field strength E.



D7. N07/II/2

Electrons are emitted from cathode C and are accelerated towards anode A, as shown in Fig.1.

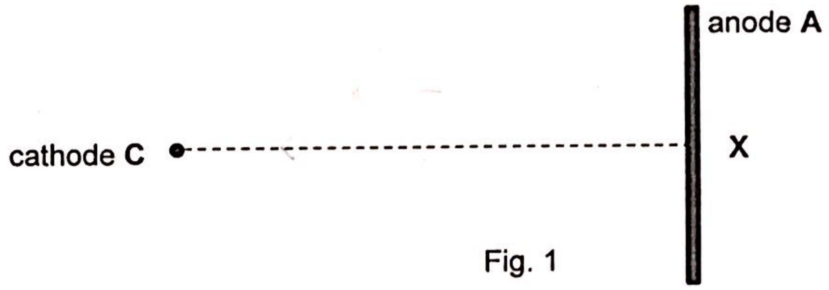


Fig. 1

The anode is earthed. CX is a line drawn from C normal to the anode A. The distance CX is 4.0 cm.

The variation with distance *d* from C along CX of the magnitude of the electric field strength E is shown in Fig 2.

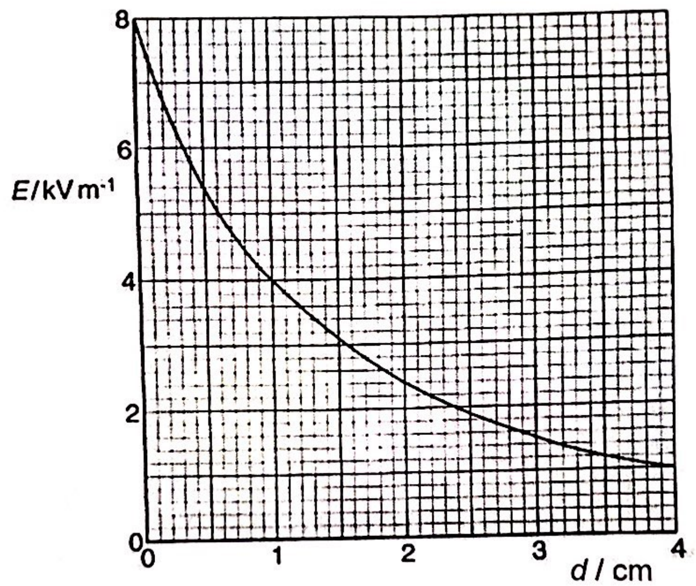


Fig. 2

(a) Define *electric field strength* at a point. [1]

(b) (i) On Fig. 1, mark with an arrow the direction of the electric field along CX. [1]

(ii) Use Fig 2 to determine the force *F* on an electron at a point mid-way between C and X. [2]

(c) (i) A student assumes that the force *F* on the electron remains constant as the electron moves from C to X. Use the value of F calculated in **(b)(ii)** to estimate, on the basis of this assumption, the potential difference between C and X. [2]

(ii) Suggest, with a reason, whether the magnitude of the potential difference calculated in **(i)** will be an over-estimate or an under-estimate of the actual potential difference. [1]

D8. N07/III/2

Fig. 8 is drawn to full scale and shows the pattern of the electric field (solid lines) in and around a pair of parallel, charged metal plates. It also shows lines joining points at the same potential (dotted lines).

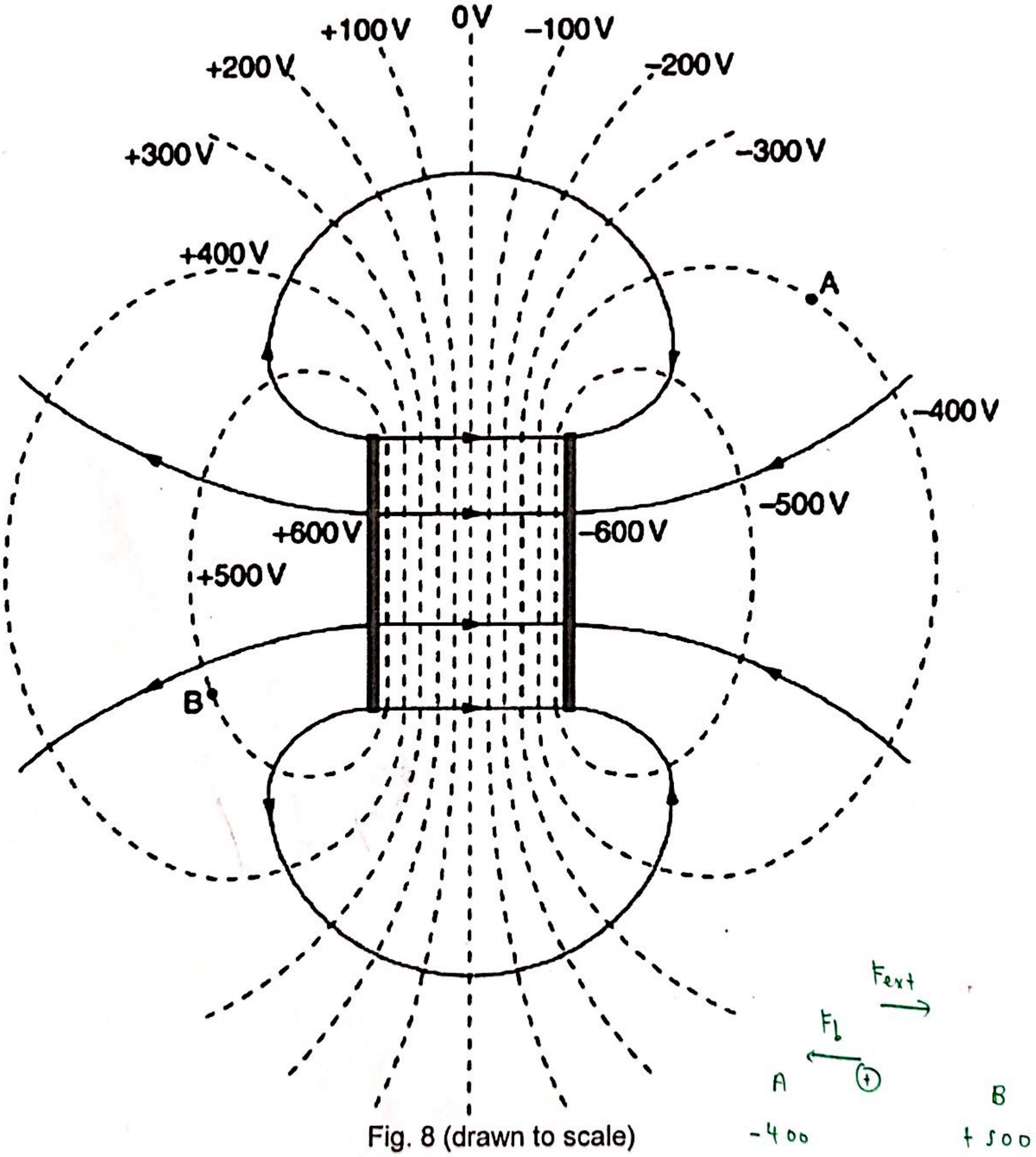


Fig. 8 (drawn to scale)

(a) From the diagram, deduce

(i) the value of the electric field strength between the plates, [3]

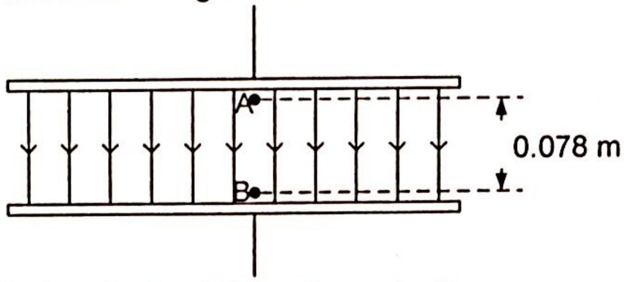
(ii) the work that needs to be done (by an external agent) to move a charge of 8.0×10^{-19} C from point A to point B. [2]

(b) Explain why, in the absence of any other charged bodies, the potential will be zero along the centre line joining the plates. [2]

D9. J93/III/14

(a) Calculate the force exerted on a proton, which has a charge of $+1.6\times10^{-19}$ C, when it is in a uniform field strength of 2.7×10^{5} N C^{-1}. [2]

(b) A proton is moved in a vacuum by a uniform electric field of 2.7×10^{5} N C^{-1} from A to B, a distance of 0.078 m as shown in the figure below.

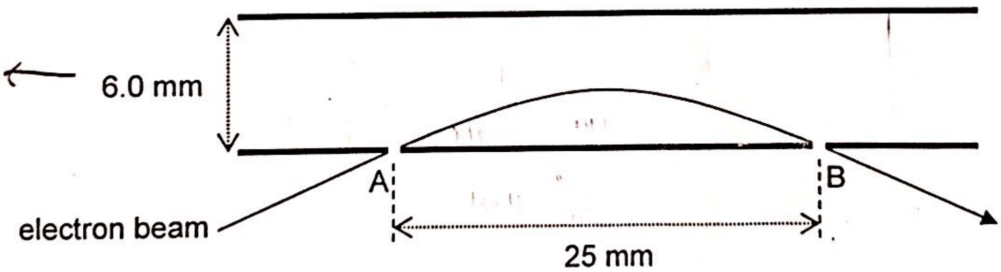


(i) How much work is done by the field on the proton? [2]

(ii) What is the gain in the kinetic energy of the proton? [1]

(iii) Calculate the difference in potential between A and B. State whether A or B is at the higher potential. [3]

D10. The figure below shows the principle of a type of velocity selector for electrons. Two parallel metal plates are arranged 6.0 mm apart in a vacuum. The lower plate is at a potential of + 45V relative to the upper, and has two parallel slits A and B in it which are 25 mm apart. A collimated beam containing electrons of different speeds is directed towards slit A at an angle of 15 degrees with the plate. Electrons emerging from B have the same velocity. Neglect the effect of gravity.



(a) **(i)** What is the magnitude and direction of the acceleration of an electron while it is between the plates? [2]

(ii) Draw the electric field lines between the plates. [1]

(iii) Describe the path of the beam in between the plates. [2]

(b) Determine the speed of the electrons which come out of slit B. [3]

Answers to Discussion Questions

2. 318 m s^{-1}
3. 0.0141 m
7. (b)(ii) 3.84×10^{-16} N, (c)(i) 96 V
9. (a) 4.32×10^{-14} N, (b)(i) 3.37×10^{-15} J, (b)(ii) 3.37×10^{-15} J, (b)(iii) 21.1 kV
10. (a)(i) 1.32×10^{15} m s^{-2} downwards, (b) 8.12×10^{6} m s^{-1}

Challenging Question

C1. A certain molecule consists of two singly-charged ions, A^+ and B^-. An equation for the potential energy U_P of the molecule is

$$U_p = -\frac{e^2}{4\pi\varepsilon_0 r} + \frac{C}{r^9}$$

where r is the separation of the ions and C is a positive constant.

(a) Give a physical interpretation of each of the two terms on the right-hand side of this equation. Explain why the exponent of r in the first term is -1.

(b) Given that force F and potential energy U_P are related by the expression $F = -\frac{dU_P}{dr}$, show that $C = \frac{e^2 r_0^{\,8}}{36\pi\varepsilon_0}$, where r_0 is the equilibrium separation of the ions.

(c) Determine U_{min}, the minimum potential energy of the molecule, in terms of r_0, e and ε_0.

(d) It is known that, in this molecule, r_0 is about 0.3 nm. Deduce the energy required to break the bond between the ions.

Ans: (c) $-\frac{2}{9}\frac{e^2}{\pi\varepsilon_0 r_0}$; (d) 6.8×10^{-19} J

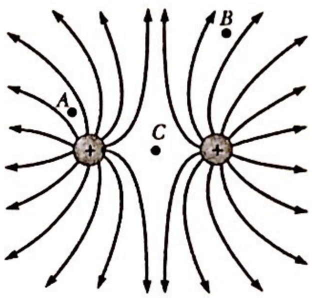
Self-Check: Suggested Solutions

S1. The **electric field strength** at a point is defined as the electric force exerted per unit positive charge placed at that point.

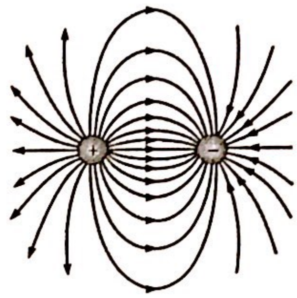
The **electric potential** at a point is defined as the work done per unit positive charge by an external force in bringing a small test charge from infinity to that point.

S2.

(a)



(b)



S3. The positively charged particle will experience a constant acceleration towards the plate of lower potential, while the component of the velocity perpendicular to the electric field remaining constant. Thus, the path is parabolic between the plates. On exiting the region between the plates, the particle moves in a straight line.

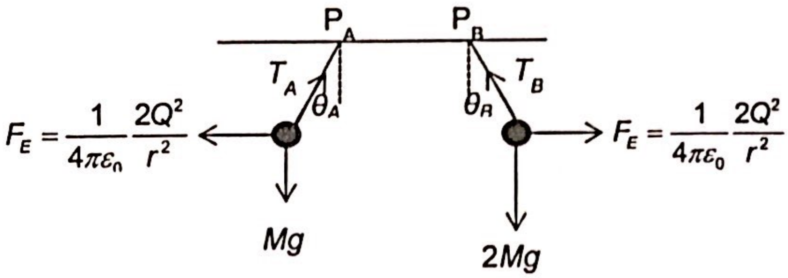
S4. The electric field strength at a point is numerically equal to the potential gradient at a point; or $E = -\frac{dV}{dx}$.

The uniform electric field strength between charged parallel plates is determined by the potential difference between the two plates divided by the separation between them.

S5. Similarities: Both forces are inverse-square laws.
Differences: Electric force can be attractive and repulsive in nature, while gravitational force is only attractive. Electric force exists between charges, while gravitational force exists between masses.

Self-Practice: Suggested Solutions

1 B



From the figure above, $\tan\theta_A = \dfrac{\frac{1}{4\pi\varepsilon_0}\frac{2Q^2}{r^2}}{Mg}$ & $\tan\theta_B = \dfrac{\frac{1}{4\pi\varepsilon_0}\frac{2Q^2}{r^2}}{2Mg}$.

Since the angles are very small, $\tan\theta \approx \theta$, $\theta_A : \theta_B \approx 2.0$.

2 A If a positive test charge is placed at the fourth corner, it will be repelled by +Q and attracted by –Q. The resultant force exerted by the two +Q (direction of A) will be larger than the force exerted by –Q (direction of D). Hence, resultant force due to the three charges is in the direction of A.

3 E By definition.

4 A Electric Field points from high to low Potential, towards Q.

$$V_X = V_P + V_Q$$
$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{+2}{r} + \frac{-2}{r}\right)\times 10^{-6} = 0$$

5 B $\Delta V = V_Y - V_X = -ve \Rightarrow \Delta U = q\,\Delta V = +ve$. (A✗)
Since E points from X to Y, a –ve charge will be experience a force form Y to X.

6 A

$$W_{\text{by electric force}} = -U = -q\Delta V = Q(V_i - V_f)$$
$$= Q\left(6\times\frac{Q}{4\pi\varepsilon_0 r} - 0\right) = \frac{6Q^2}{4\pi\varepsilon_0 r}$$

7 C

$$W \propto \frac{1}{r} \quad \Rightarrow \frac{W_{1.0}}{W_{0.5}} = \frac{0.5}{1.0} = \frac{1}{2}$$
$$F_E \propto \frac{1}{r^2} \quad \Rightarrow \frac{F_{1.0}}{F_{0.5}} = \left(\frac{0.5}{1.0}\right)^2 = \frac{1}{4}$$

8 D

$$F = -\frac{dE_P}{dx} = \frac{1.5\times10^{-16}}{0.020} = 7.5\times10^{-15}\ \text{N}$$

9 A Electrons experience a vertical, upward force. Hence, a parabolic path upwards.

10 B

$$mg = q\frac{\Delta V}{d} = q\frac{(V-(-V))}{d} = \frac{2Vq}{d}$$

11 E Based on above equation, the electric force on the larger charge can be reduced (to achieve equilibrium) by decreasing *V* or increasing *d*.

12 B Since the field is uniform, equipotential lines are parallel to the plates. (Draw 7 vertical dotted lines every 5cm to represent a ΔV of $V/8$).

Since 40mm≡*V*, 20mm≡20/40 *V* (the slanted 25mm serves no purpose).

13 B For the electron to just reach PQ, it's velocity should be solely horizontal by then (i.e. vertically, velocity is zero).

By the principle of conservation of energy

loss in KE = gain in EPE

$$\frac{1}{2}mv^2 - \frac{1}{2}m(v\sin\theta)^2 = q\Delta V$$

or $\frac{1}{2}m(v\cos\theta)^2 = eV$