PRESBYTERIAN HIGH SCHOOL

MATHEMATICS **4052/01**
PAPER 1

14 August 2023 Monday 2 hours 15 minutes

MARK SCHEME

Answer **all** the questions.

1 Solve $7x = 18 + 3x$.

$$7x = 18 + 3x$$
$$4x = 18$$
$$x = \frac{9}{2} \quad \text{--- B1}$$

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2 **(a)** Calculate $\dfrac{26.18^3}{\sqrt{4.52 - 0.4^2}}$.

Write your answer correct to 5 significant figures.

8593.4 --- B1

(b) Write your answer to **part (a)** in standard form.

8.5934×10^3 --- B1

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3 **(a)** Express 784 as a product of prime factors.

2	784
2	392
2	196
2	98
7	49
7	7
	1

$784 = 2^4 \times 7^2$ --- B1

(b) It is given that a and b are prime numbers.

Find the smallest values of a and b such that $784 \times \dfrac{a}{b}$ is a perfect cube.

$784 \times \dfrac{a}{b} = \left(2^4 \times 7^2\right) \times \dfrac{7}{2} \qquad \therefore \ a = 7$ and $b = 2$ --- B1, B1

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4 Expand and simplify $(w+5)(1-w)$.

$(w+5)(1-w)$

$= -w^2 + w - 5w + 5$ --- M1: at least 2 correctly expanded terms

$= -w^2 - 4w + 5$ --- A1

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5 The bar graph below shows the results of a survey conducted on the service quality of a hotel.

(a) Find the percentage of respondents who answered 'Strongly Satisfied' and 'Satisfied'.

$$\frac{18+27}{18+27+20+10+5}\times 100\%$$
$= 56.25\%$ --- B1

(b) Suggest the use of another statistical diagram to represent the results of the survey conducted, that can show the relative size of a part in relation to the whole.

Pie Chart --- B1

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6 Find the largest integer that satisfies $2y-3<4$.

$2y-3<4$

$2y<7$

$y<\frac{7}{2}$ B1: seen this answer

The largest integer is 3. ---- B1

7 P is directly proportional to Q^3.

When $Q = 2$, $P = 64$.
When the value of Q is halved, the value of P changes by a factor of m.
Find the value of m.

$P = kQ^3$

When $Q = 2$ and $P = 64$

$64 = k(2)^3$ ---- M1: attempt to find the proportionality constant by substitution

$k = 8$

$P = 8Q^3$

$P_{new} = 8\left(\frac{1}{2}Q\right)^3 = 8\left(\frac{Q^3}{8}\right) = Q^3$

Hence the factor m is $\frac{1}{8}$. --- A1

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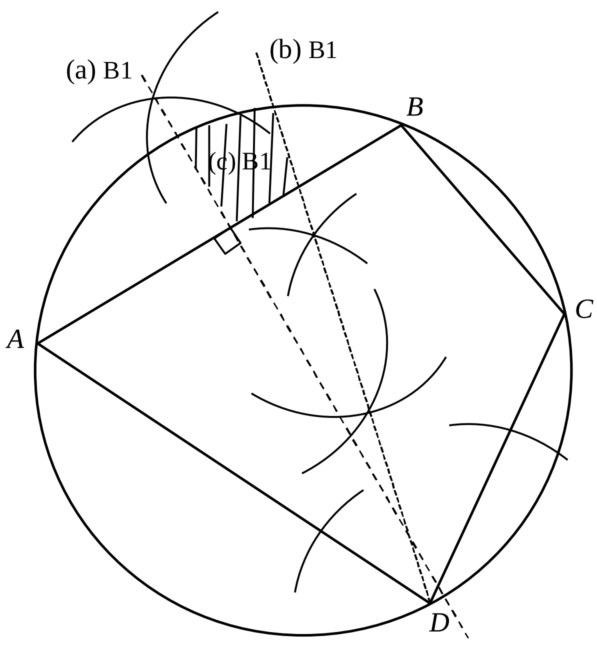
8 The diagram shows a quadrilateral playground $ABCD$.
A circular fence is constructed around the playground such that the vertices, A, B, C and D of the playground touch the circumference of the fence.

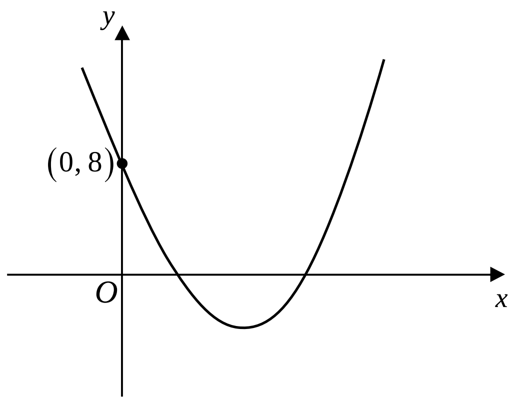


(a) Construct the perpendicular bisector of AB. [1]

(b) Construct the bisector of angle ADC. [1]

(c) A sand pit is to be constructed inside the circular fence but outside the quadrilateral playground. The sand pit is nearer to AD than CD and nearer to B than A.
Shade the region for the sand pit to be constructed. [1]

9 The diagram below shows the graph of $y = 3(x-h)^2 - 4$.



(a) Find the value of *h*.

Substitute $(0, 8)$:

$8 = 3(0-h)^2 - 4$ --- M1: shows subsitution

$h = 2$ or -2 (reject)

$\therefore h = 2$ --- A1

(b) Explain why the graph of $y = 3(x-h)^2 + 1$ does not cut the *x*-axis.

Either one

1. The minimum point of the graph $y = 3(x-h)^2 + 1$ is $(h, 1)$ or $(2, 1)$.
2. The equation $(x-h)^2 = -\frac{1}{3}$ has no solution for *x*. [1]

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10 A group of six students took a Mathematics quiz and the marks were recorded below.

8 10 9 13 10 9

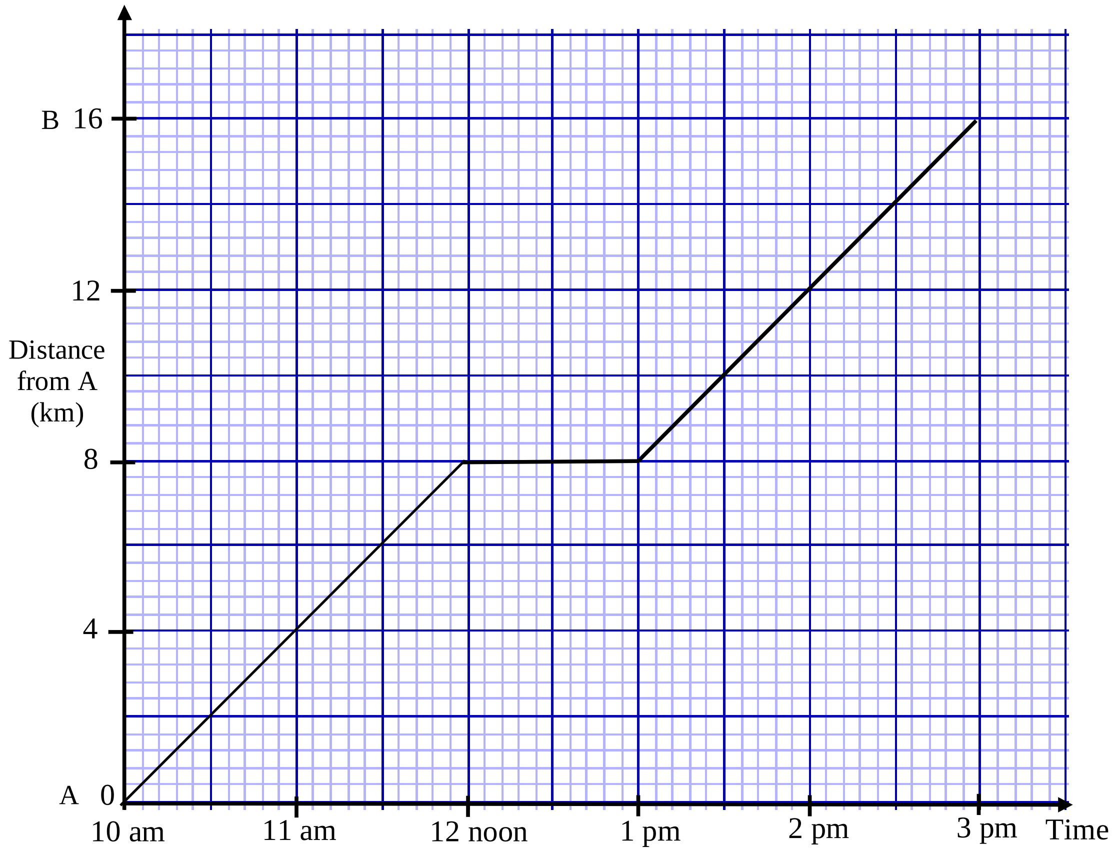
(a) Calculate the standard deviation.

1.57

(b) **Two** other students also took the quiz, and their marks were recorded.
Given that the mean mark obtained by the eight students was 10 and the mode was also 10, find the marks of these two students.

The two marks are 10 and 11. --- B1, B1

11



The distance-time graph shows the journey Tan took to run from town A to B.

(a) Find the distance Tan ran in the first two hours.

8 km --- B1

(b) Calculate the average speed, in m/s, for the whole journey Tan ran.

Average speed $= \dfrac{16000}{5\times60\times60}$ --- M1: attempt to convert km to m or h to sec

$= \dfrac{8}{9}$ m/sec or 0.889 m/sec --- A1

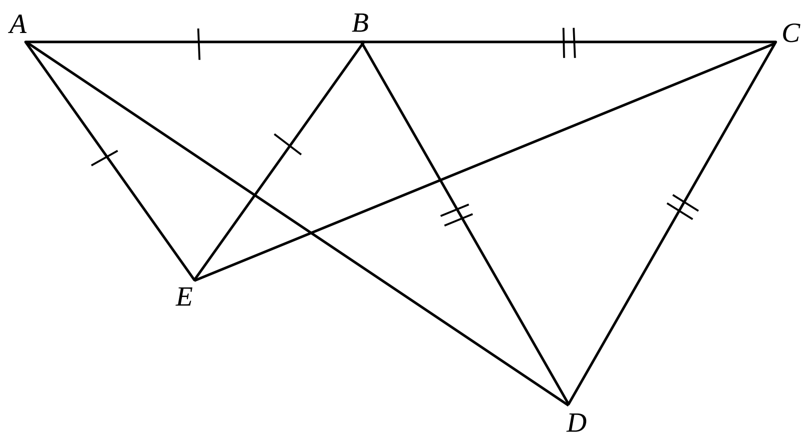
12 Simplify $\dfrac{2y^2+y-3}{4y^2-9}$.

$$\frac{2y^2+y-3}{4y^2-9}$$

$$=\frac{(2y+3)(y-1)}{(2y+3)(2y-3)}$$ --- M1, M1: factorise numerator and denominator

$$=\frac{y-1}{2y-3}$$ --- A1

13



In the diagram, *ABC* is a straight line and triangles *ABE* and *BCD* are equilateral triangles.

Show that triangle *ABD* and triangle *EBC* are congruent.
Give a reason for each statement you make.

Answer

1) $AB = EB$ (sides of an equilateral triangle / given)

2) $BD = BC$ (sides of an equilateral triangle / given) --- B1: at least one statement with reason pr

3) $\measuredangle ABD = 180^\circ - 60^\circ$ (adj. $\angle$ on a st. line)

$= \measuredangle EBC$ --- B1: show equivalent angles with explanation

$= 120^\circ$

$\therefore$ triangle *ABD* is congruent to triangle *EBC* (SAS) --- B1: with name of test [3]

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14 The first three terms in a sequence of numbers, $T_1,\ T_2,\ T_3,\ \ldots$ are given below.

$$T_1 = 1-\frac{1}{2}$$
$$T_2 = \frac{1}{2}-\frac{1}{3}$$
$$T_3 = \frac{1}{3}-\frac{1}{4}$$
$$\vdots$$

(a) Write down T_4.

$T_4 = \frac{1}{4}-\frac{1}{5}$ --- B1

(b) Show that the total sum of $T_1,\ T_2,\ T_3,\ \ldots,\ T_n$ in the above sequence is $1-\frac{1}{n+1}$.

$\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\ldots+\left(\frac{1}{n}-\frac{1}{n+1}\right)$ --- M1: seen either the formation or $\left(\frac{1}{n}-\frac{1}{n+1}\right)$

$= 1-\frac{1}{n+1}$ --- [AG1: shown]

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15 A, B and C are points $(-1, 0)$, $(3,\ 8)$ and $(2,\ 1)$ respectively.

(a) Find the length of AB.

Length of $AB = \sqrt{(-1-3)^2+(0-8)^2}$ --- M1: correct application of length formula

$= 8.94$ units (3s.f.) --- B1

(b) Find the equation of the line that passes through B and has the same gradient as AC.

$mAC = \frac{1-0}{2-(-1)} = \frac{1}{3}$ --- M1

Equation of line passing through B has the **same** gradient $= \frac{1}{3}$

The equation of the line:

$y = \frac{1}{3}x+c$ or $y-8 = \frac{1}{3}(x-3)$ (No marks without simplification)

$y = \frac{1}{3}x+7$ --- A1

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16 **(a)** Find the interior angle of a regular 18-sided polygon.

$$\frac{(18-2)\times 180^\circ}{18} \quad \text{--- M1}$$

$= 160^\circ$ --- A1

(b) An *n*-sided polygon has two of its exterior angles at 45° and 75°.
If the remaining exterior angles are each 20°, calculate the value of *n*.

$45+75+(n-2)(20)=360$ --- M1

$n=14$ --- A1

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17 **(a)** Simplify $\left(\frac{a^{-6}}{b^9}\right)^{\frac{1}{3}}$ and leave your answer in positive index notation.

$$\left(\frac{a^{-6}}{b^9}\right)^{\frac{1}{3}}$$

$=\frac{a^{-2}}{b^3}$ --- M1: applied indices law with at most one error

$=\frac{1}{a^2 b^3}$ --- A1

(b) Given that $2^{4x} \div 2^x = \sqrt[3]{2}$, find *x*.

$2^{4x} \div 2^x = \sqrt[3]{2}$

$2^{4x} \div 2^x = 2^{\frac{1}{3}}$ --- M1: able to convert to appropriate index form

$2^{3x} = 2^{\frac{1}{3}}$

$3x = \frac{1}{3}$

$x = \frac{1}{9}$ --- A1

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18 **(a)** Given that $m^2 - 8mn + 16n^2 = 0$, find the value of $\frac{m}{n}$.

Method 1

$m^2 - 8mn + 16n^2 = 0$

$(m-4n)^2 = 0$ --- M1: attempt to factorise into perfect square

$m - 4n = 0$

$m = 4n$

$\frac{m}{n} = 4$ ---- A1

Method 2

$m = \frac{-(-8) \pm \sqrt{(-8n)^2 - 4(1)(16n^2)}}{2(1)}$ --- B1

$= \frac{8n}{2} = 4n$

$\therefore \quad \frac{m}{n} = 4$ --- A1

(b) Factorise completely $3ac - 7c + 18ab - 42b$.

$3ac - 7c + 18ab - 42b$

$= c(3a-7) + 6b(3a-7)$ --- M1: identified one common linear factor correctly

$= (c+6b)(3a-7)$ --- A1

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19 A florist sells three types of bouquets, Bliss, Love and Commitment.
The number of stalks for each type of flower in each type of bouquet is shown in the table.

		Type of Flower			
		Rose	Lily	Gerbera	Sunflower
Type of Bouquet	Bliss	2	0	7	3
	Love	3	1	5	1
	Commitment	8	2	4	0

(a) Represent the above information in a 3×4 matrix, **F**.

$$\mathbf{F} = \begin{pmatrix} 2 & 0 & 7 & 3 \\ 3 & 1 & 5 & 1 \\ 8 & 2 & 4 & 0 \end{pmatrix}$$ --- B1

(b) The cost of each stalk of Rose, Lily, Gerbera and Sunflower are \$6, \$7.80, \$2.50 and \$3 respectively.

(i) Represent this information in a 4×1 column matrix, **H**.

$$\mathbf{H} = \begin{pmatrix} 6 \\ 7.80 \\ 2.50 \\ 3 \end{pmatrix} \quad \text{--- B1}$$

(ii) Evaluate $\mathbf{J} = \mathbf{FH}$.

$$\mathbf{J} = \begin{pmatrix} 2 & 0 & 7 & 3 \\ 3 & 1 & 5 & 1 \\ 8 & 2 & 4 & 0 \end{pmatrix} \begin{pmatrix} 6 \\ 7.80 \\ 2.50 \\ 3 \end{pmatrix} = \begin{pmatrix} 38.50 \\ 41.30 \\ 73.60 \end{pmatrix} \quad \text{--- B1}$$

(iii) State what the elements of **J** represent.

Answer

The elements of **J** represent the total cost of the four types of flowers - Rose, Lily, Gerbera and Sunflower in bouquet Bliss, Love and Commitment respectively. --- B1

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20 Box X contains 5 balls numbered 2, 3, 4, 7 and 9.
Box Y contains another 5 balls numbered 1, 5, 6, 8, and 10.
In a game, Ming drew a ball at random from each box, and the sum of both numbers is obtained.

(a) Complete the possibility diagram below.

Box Y

	+	1	5	6	8	10
Box X	2	3	7	8	10	12
	3	4	8	9	11	13
	4	5	9	10	12	14
	7	8	12	13	15	17
	9	10	14	15	17	19

B1: Every 8 correct values
B2: all correct

(b) Find the probability that

(i) the sum of both numbers is an odd number,

$$\frac{13}{25} \quad \text{--- B1}$$

(ii) the sum is a multiple of **one** of the two numbers drawn.

$\frac{10}{25}=\frac{2}{5}$ --- B1

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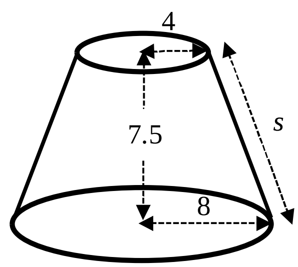
21 The upper part of a solid wooden right circular cone was cut off leaving the frustum as shown in the diagram. The frustum has top radius 4 cm, base radius 8 cm and height 7.5 cm.



(a) Show that the slant height, s, is 8.5 cm.

$s=\sqrt{4^2+7.5^2}$ = 8.5 cm (shown) --- AG1

(b) Find the curved surface area of the frustum.

Curved surface area $=\pi(8)(2\times 8.5)-\pi(4)(8.5)$

- M1: curved S.A. of the original right circular cone (BIG)
- M1: curved S.A. of the wooden right circular cone (SMALL)

$=320.44\approx 320\text{ cm}^2$ (3s.f.) --- A1

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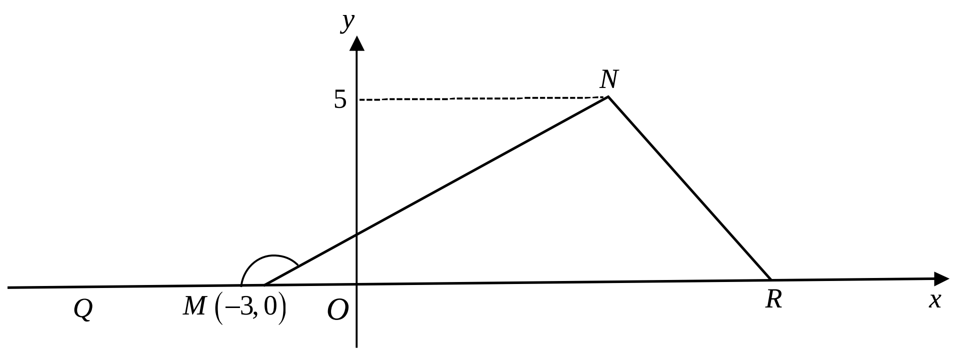
22 In triangle MNR, point M is $(-3, 0)$ and $\sin\angle NMR=\frac{5}{13}$.

Q is a point on the negative x-axis.



(a) Express the following as a fraction

(i) $\cos\angle NMQ$,

Length of "adjacent" = 12 units --- M1: using Pythagoras' Theorem

$-\frac{12}{13}$ --- A1

(ii) $\tan \angle NMR$.

$\frac{5}{12}$

(b) The area of triangle *MNR* is 50 square units.

Find the coordinates of *R*.

Area of triangle $MNR = \frac{1}{2} \times \text{ base } \times 5 = 50$

Base $= 20$ units --- M1

Coordinates of $R = (17,\ 0)$. --- A1

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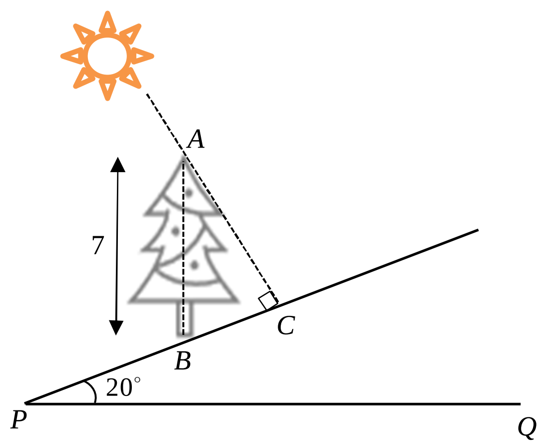
23 The diagram below shows a tree *AB* of height 7 m that stands vertically on a slope inclined at 20° with the horizontal *PQ*.

At a particular time in the morning, the tree casts a shadow, *BC*, on the slope.
AC is perpendicular to the slope.



(a) Calculate the length of the shadow, *BC*, on the slope.

$\sin 20^\circ = \frac{BC}{7}$ --- M1

$BC = 7 \times \sin 20^\circ$

$= 2.39$ m (3s.f.) --- A1

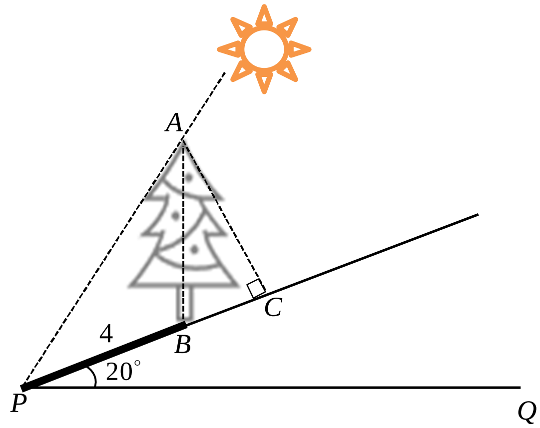
Or

$\cos 70^\circ = \frac{BC}{7}$ --- M1

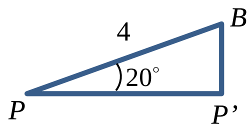
$BC = 7 \times \cos 70^\circ$

$= 2.39$ m (3s.f.) --- A1

After some time, the sun goes into a position as shown below.



(b) If the shadow, BP, of the tree on the slope is 4 m, find the angle that the sun ray makes with the horizontal PQ.



$\cos 20^\circ = \dfrac{PP'}{4}$

$PP' = 4 \times \cos 20^\circ$

$\quad = 3.7587 \text{ m}$

$\sin 20^\circ = \dfrac{BP'}{4}$

$BP' = 4 \times \sin 20^\circ$

$\quad = 1.3680 \text{ m}$ --- M1: either length seen

Height of A to the horizontal PQ

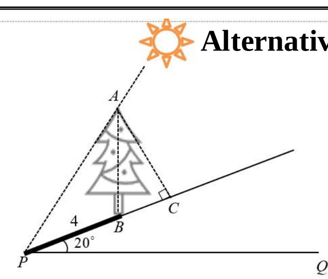
$= (7 + 1.3680) \text{ m}$

The required angle

$\tan \angle APP' = \dfrac{8.3680}{3.7587}$ --- M1: appropriate use of trigo ratio to find the angle

$\angle APP' = 65.8^\circ$ (1 d.p.) --- A1

Alternative method



$\sin 70^\circ = \dfrac{AC}{7} \quad (\angle ABC = 70^\circ)$

$AC = 6.57784 \text{ m}$

$PC = 4 + 2.39414 = 6.39414 \text{ m}$ --- M1: seen length of PC

$\tan \angle APC = \dfrac{AC}{PC} = \dfrac{6.57784}{6.39414}$

$\angle APC = \tan^{-1}\left(\dfrac{6.57784}{6.39414}\right)$ --- M1

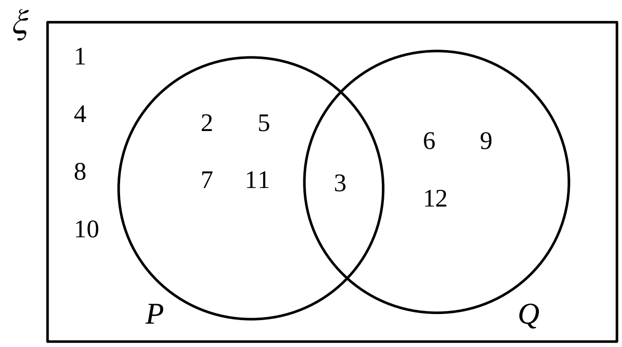
$\quad = 45.8113^\circ$

The required angle $= 45.8113^\circ + 20^\circ = 65.8^\circ$ --- A1

24 **(a)** $\xi = \{\text{integers } x : 1 \le x \le 12\}$
$P = \{\text{prime numbers}\}$
$Q = \{\text{multiples of 3}\}$

(i) Represent the above information on the Venn diagram shown in the answer space below.

Answer



B1: every 6 correct values

B2: correct representation

(ii) List the elements in $(P' \cap Q') \cup (P \cap Q)$.

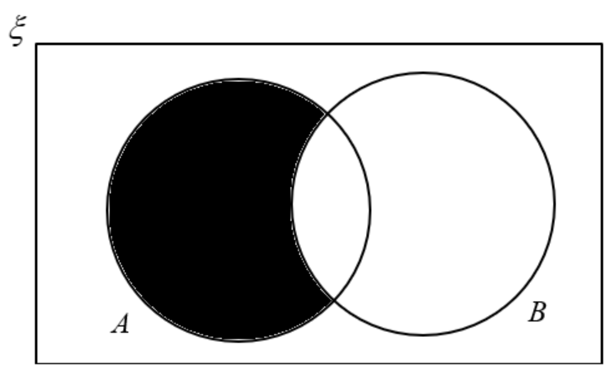
$(P' \cap Q') \cup (P \cap Q) = \{1, 3, 4, 8, 10\}$ --- B1

(iii) $R = \{x : x \text{ is a multiple of } 6\}$

Use set notation to describe the relationship between Q and R.

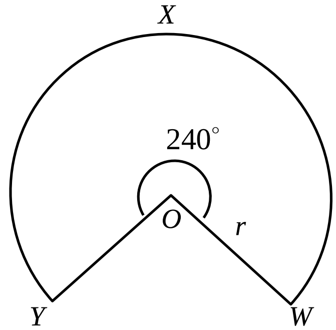
$R \subset Q$ --- B1

(b) On the Venn diagram, shade the region which represents the set $A \cap B'$.



B1: correct shading

25 *OWXY* is a sector of a circle, centre *O*, of radius *r* cm and reflex angle 240°.



The sector *OWXY* has an area of 150π cm^2.

(a) Express 240° in terms of π radians.

$$240^\circ = 240 \times \frac{\pi}{180} \text{ rad} \quad \text{--- M1}$$

$$= \frac{4}{3}\pi \text{ rad} \quad \text{--- A1}$$

(b) Show that $r = 15$.

Answer

$$\frac{1}{2}(r)^2 \frac{4\pi}{3} = 150\pi \qquad \text{--- M1: applied formula}$$

$$r = 15 \quad \text{--- AG1}$$

(c) The radii, *OW* and *OY*, are joined together to form a cone.

Find the base radius of the cone.

Answer

Method 1:

Arc length = Circumference of circular base

$$15\left(\frac{4}{3}\pi\right) = 2\pi x \quad \text{--- M1}$$

$$x = 10 \quad \text{--- A1}$$

Method 2:

$$\pi x l = 150\pi$$

$$x = \frac{150\pi}{15\pi} \quad \text{--- M1}$$

$$= 10 \quad \text{--- A1}$$

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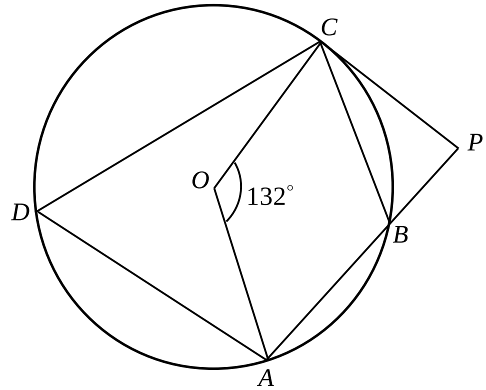

In the diagram above, O is the centre of the circle, such that angle $COA = 132^\circ$.
PC is a tangent to the circle at C and PBA is a straight line.

By giving a reason for each step of your working, find

(a) $\angle CDA$,

$\angle CDA = 132^\circ \div 2$

$= 66^\circ$($\angle$ at the centre = twice $\angle$ at circumference) --- B1 reason, B1 answer

(b) $\angle CBP$.

$\angle CBA = 180^\circ - 66^\circ$

$= 114^\circ$ ($\angle$s in opp. segment)

$\angle CBP = 180^\circ - 114^\circ$ (adj. $\angle$s on a st. line)

$= 66^\circ$ --- B1

B1: $\angle$s in opp. segment

(c) If the radius of the circle is 3.55 cm, calculate the area of triangle AOC.

Area of $\Delta\ AOC = \frac{1}{2} \times 3.55 \times 3.55 \times \sin 132^\circ$ --- M1: applied area of triangle formula

$= 4.68 \text{ cm}^2$ (3s.f.) --- A1