CHIJ ST. THERESA'S CONVENT

PRELIMINARY EXAMINATION 2024

SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

CANDIDATE NAME			
CLASS		INDEX NUMBER	

MATHEMATICS **4052/1**

Paper 1 **26 August 2024**

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answers in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta\text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta\text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 Calculate $\dfrac{6.4+\sqrt[5]{14.8^3-4.6^2}}{3\times 24}$.

0.159 (3 s.f.)

Answer ………………………………… [1]

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2 **(a)** Given that $6\sin x = 5$, find the two possible values for angle x, where $0° \le x \le 180°$

$6\sin x = 5$

$\sin x = \dfrac{5}{6}$

$\therefore x = 56.4^{\circ}$ or 123.6° (1 d.p.)

Answer x = ………………° or ………………° [2]

(b) Convert 138° into radians.

$138° = \dfrac{\pi}{180°}\times 138° \approx 2.41$ rad (3 s.f.)

Answer ………………………… radians [1]

---

3 Solve $4^x = \sqrt{32}$.

$4^x = \sqrt{32}$

$2^{2x} = 2^{\frac{5}{2}}$

$\Rightarrow 2x = \dfrac{5}{2}$

$\therefore x = \dfrac{5}{4}$

Answer x = ………………………………… [2]

---

4 Simplify

(a) $5-2(4x-3)$,

$$5-2(4x-3)=5-8x+6$$
$$=-8x+11$$

Answer ……………………..………… [1]

(b) $\left(\dfrac{27x^3}{y^6}\right)^{-\frac{2}{3}}$.

$$\left(\frac{27x^3}{y^6}\right)^{-\frac{2}{3}}=\left(\frac{y^6}{27x^3}\right)^{\frac{2}{3}}$$
$$=\frac{y^4}{9x^2}$$

Answer ………………………………….. [2]

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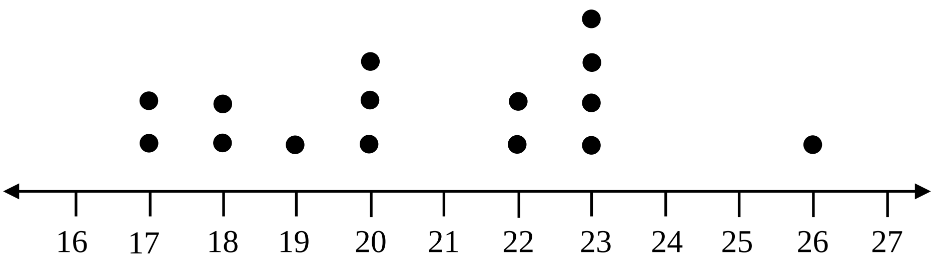
5 A group of 15 adults was surveyed on the number of hours they spent in a week watching Netflix. The results of the survey were represented in the dot diagram below.



(a) Write down the modal length of time.

Modal length = 23

Answer …………………………… hours [1]

(b) Find the median length of time.

Median = 20

Answer …………………………… hours [1]

---

6 **(a)** Factorise completely $24k-16$.

$24k-16=8(3k-2)$

Answer .... [1]

(b) Expand and simplify $(3a-7b)^2$.

$(3a-7b)^2=9a^2-42ab+49b^2$

Answer .... [1]

---

7 A pencil case contains 9 black pens, 5 red pens and 6 green pens.

(a) A pen is chosen at random and then replaced.
What is the probability that it is **not** a red pen?

$\frac{9+6}{9+5+6}=\frac{3}{4}$

Answer .... [1]

(b) x black pens are removed from the box.

The probability of choosing a green pen is now $\frac{3}{8}$.

Find the value of x.

$\frac{6}{9-x+5+6}=\frac{3}{8}$

$\frac{6}{20-x}=\frac{3}{8}$

$16=20-x$

$\therefore x=4$

Answer .... [2]

---

8 Rearrange the formula $y = \dfrac{2x+3}{5x-1}$ to make x the subject.

$$y = \frac{2x+3}{5x-1}$$
$$y(5x-1) = 2x+3$$
$$5xy - y = 2x+3$$
$$5xy - 2x = y+3$$
$$x(5y-2) = y+3$$
$$\therefore x = \frac{y+3}{5y-2}$$

Answer …………………..…..………… [3]

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9 Express $\dfrac{8}{3x-2} - \dfrac{3}{2x+1}$ as a single fraction in its simplest form.

$$\frac{8}{3x-2} - \frac{3}{2x+1}$$
$$= \frac{8(2x+1) - 3(3x-2)}{(3x-2)(2x+1)}$$
$$= \frac{16x+8-9x+6}{(3x-2)(2x+1)}$$
$$= \frac{7x+14}{(3x-2)(2x+1)}$$

Answer ……………………..………………… [3]

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10 The mean weight of a group of athletes is 72 kg.
There are 6 more male athletes than female athletes in the group.

The mean weight of the female athletes is 60.8 kg.
The mean weight of the male athletes is 80.4 kg.

Calculate the total number of athletes in the group.

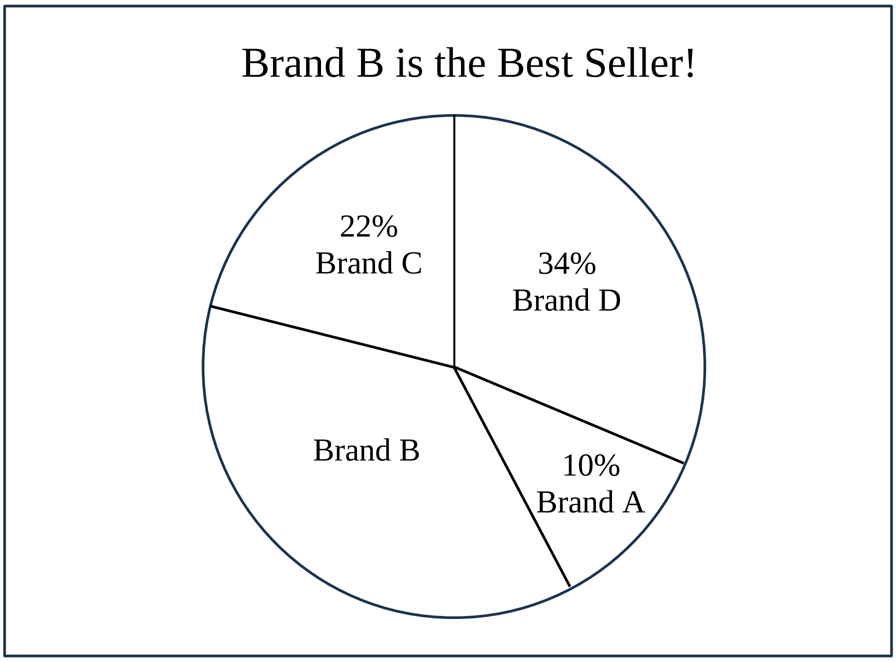
Let the number of female athletes be x.
$\therefore$ the number of male athletes is $x+6$.
$60.8x+80.4(x+6)=72(x+x+6)$
$60.8x+80.4x+482.4=144x+432$
$-2.8x=-50.4$
$\therefore x=\dfrac{-50.4}{-2.8}=18$
$\therefore$ total number of athletes $=18+(18+6)=42$

Alternative solution:

Let the number of male athletes be x.
$\therefore$ the number of female athletes is $x-6$.
$60.8(x-6)+80.4(x)=72(x+x-6)$
$60.8x-364.8+80.4x=144x-432$
$-2.8x=-67.2$
$\therefore x=\dfrac{-67.2}{-2.8}=24$
$\therefore$ total number of athletes $=24+(24-6)=42$

Answer .... [3]

11 The pie chart below shows the sales for four different brands of potato chips.



(a) State one misleading feature of the pie chart.

% of Brand $B = 100\% - 22\% - 34\% - 10\% = 34\%$
Brand B and Brand D have equal percentages but their proportions in the pie chart are not equal.

OR The title is biased

[1]

(b) Explain how this feature affects the reader's interpretation of the pie chart.

Answer ……………………………………………………………………………………..

It may mislead readers into believing that Brand B is selling better than Brand D.

OR It does not allow the reader to make his/her own judgement.

…………………………………………………………………………………….. [1]

12 Factorise $9x^2-6x-8$.

$9x^2-6x-8=(3x-4)(3x+2)$

Answer .... [2]

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13 Cerra is thinking of two numbers.
If she triples the first number and subtracts from it twice the second number, the answer is 6.
If she multiplies the first number by 5 and adds to it 6 times the second number, the answer is 38. Find these two numbers.

Let the first number be x and the second number be y.

$3x-2y=6$ ----- (1)

$5x+6y=38$ ----- (2)

Eqn $(1)\times 3$, we have,

$9x-6y=18$ ----- (3)

$(2)+(3)$, we have,

$14x=56$

$\therefore x=4$

$\therefore y=3$

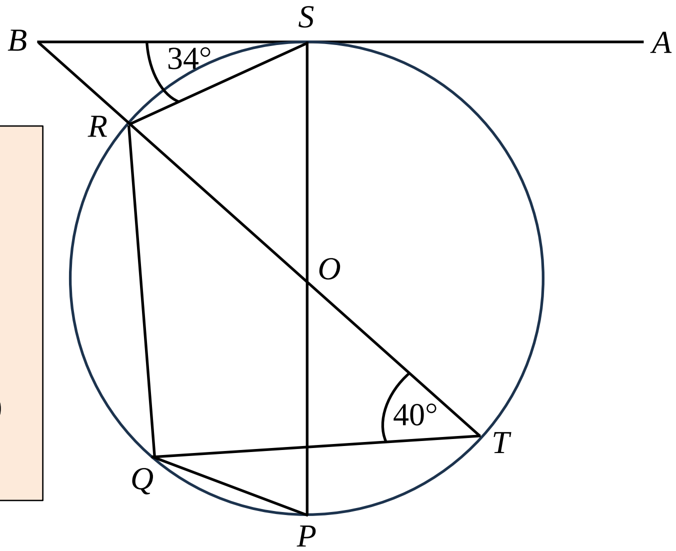
Answer .... and .... [3]

14 P, Q, R, S and T are points on the circumference of a circle, centre O.

PS intersects TR at O and AB is a tangent to the circle at S.
TR produced meets AS produced at point B.
$\angle RSB = 34°$ and $\angle QTR = 40°$.

Giving reasons for each step of your working, find

(i) $\angle RBS$,



$\angle OSB = 90^\circ$ (tangent perpendicular to radius)
$\angle OSR = 90^\circ - 34^\circ = 56^\circ$
$\therefore \angle ORS = 56^\circ$ (isoceles triangle)
$\therefore \angle BRS = 180^\circ - 56^\circ = 124^\circ$ (adj. $\angle$s on a straight line)
$\therefore \angle RBS = 180^\circ - 124^\circ - 34^\circ = 22^\circ$ (angle sum of triangle)

Answer $\angle RBS =$ .... ° [2]

(ii) $\angle PQR$,

$\angle PQR = 180^\circ - 56^\circ = 124^\circ$ ($\angle$s in opposite segment)

Answer $\angle PQR =$ .... ° [1]

(iii) reflex $\angle POR$,

Reflex $\angle POR = 124^\circ \times 2 = 248^\circ$ ($\angle$ at centre = twice $\angle$ at circumference)

Answer reflex $\angle POR =$ .... ° [1]

(iv) $\angle QRP$.

$\angle QPR = 40^\circ$ ($\angle$s in the same segment)
$\therefore \angle QRP = 180^\circ - 124^\circ - 40^\circ = 16^\circ$ (angle sum of triangle)

Answer $\angle QRP =$ .... ° [1]

15 **(a)** Given that $\dfrac{2a-b}{a-3b}=\dfrac{2}{9}$, find the value of $\dfrac{a}{b}$.

$$\frac{2a-b}{a-3b}=\frac{2}{9}$$
$$9(2a-b)=2(a-3b)$$
$$18a-9b=2a-6b$$
$$16a=3b$$
$$\therefore \frac{a}{b}=\frac{3}{16}$$

Answer .... [2]

(b) Consider the equation $(m+5)^{80}+(n-8)^{100}=0$.
Determine the value of $m+n$.

$$(m+5)^{80}+(n-8)^{100}=0$$

$(m+5)^{80}\geq 0$ and $(n-8)^{100}\geq 0$

$\Rightarrow m+5=0$ $\quad\quad$ $\Rightarrow n-8=0$

$\therefore m=-5$ $\quad\quad$ $\therefore n=8$

$$\therefore m+n=-5+8=3$$

Answer .... [2]

16 The table below shows the timings taken of 40 girls for a 2.4 km run.

Timing, t (minutes)	Frequency
$12:30 \le t < 13:30$	5
$13:30 \le t < 14:30$	8
$14:30 \le t < 15:30$	15
$15:30 \le t < 16:30$	12

(a) Calculate an estimate for

(i) the mean timing of the girls,

Mean $= 14.85$ min

Answer …………………………………. minutes [1]

(ii) the standard deviation of the timings of the girls.

Standard deviation ≈ 0.989 min (3 s.f.)

Answer …………………………………. minutes [1]

(b) It was later discovered that the stopwatch used for the timing of the 2.4 km run was faulty and the timing recorded of every girl should be 1 minute shorter.

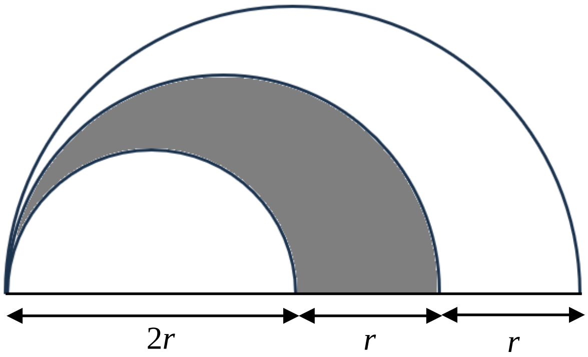
Explain how the mean and the standard deviation will be affected after the timings have been rectified.

Answer………………………………………………………………………………………..

The mean will decrease by 1 min and
there will be no change to the standard deviation. [B1]

……………………………………………………………………………………….. [2]

17 The diagram shows three semi-circles with diameters $2r$, $3r$ and $4r$ respectively.
Find the ratio of the unshaded region to that of the shaded region.



Area of shaded region:

$$=\frac{\pi(1.5r)^2}{2}-\frac{\pi(r)^2}{2}=\frac{5}{8}\pi r^2$$

Area of unshaded region:

$$=\frac{\pi(2r)^2}{2}-\frac{5}{8}\pi r^2=\frac{11}{8}\pi r^2$$

$\therefore$ Area of unshaded region : Area of shaded region

$$=\frac{11}{8}\pi r^2:\frac{5}{8}\pi r^2$$

$$=11:5$$

Answer .... : .... [3]

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18 **(i)** Solve the inequalities $2x+11<4x+5<137-2x$.

$2x+11<4x+5<137-2x$

$2x+11<4x+5$	and	$4x+5<137-2x$
$-2x<-6$		$6x<132$
$x>3$		$x<22$

$\therefore 3<x<22$

Answer .... [2]

(ii) Hence, write down the largest prime number value of x which satisfies $2x+11<4x+5<137-2x$.

Largest prime number $=19$

Answer .... [1]

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19 The number of students enrolled for Mathematics and Geography enrichment classes at 3 outlets are shown in the table below.

	Mathematics	Geography
Ang Mo Kio	125	130
Bishan	145	128
Clementi	80	115

The cost of Mathematics and Geography enrichment classes per month are $120 and $150 respectively.

The above information can be represented by the matrices **S** = $\begin{pmatrix} 125 & 130 \\ 145 & 128 \\ 80 & 115 \end{pmatrix}$ and **C** = $\begin{pmatrix} 120 \\ 150 \end{pmatrix}$.

(a) Evaluate the matrix **T** = **SC**.

$$T = \begin{pmatrix} 125 & 130 \\ 145 & 128 \\ 80 & 115 \end{pmatrix}\begin{pmatrix} 120 \\ 150 \end{pmatrix} = \begin{pmatrix} 34500 \\ 36600 \\ 26850 \end{pmatrix}$$

Answer **T** = ... [2]

(b) State what the elements of matrix **T** represent.

Answer..

The elements represent the total earnings (or revenue) for each outlet.

OR the elements represent the total cost of the Mathematics and Geography enrichment classes in each outlet respectively.

...... [1]

(c) The monthly operational costs of the Ang Mo Kio, Bishan and Clementi outlets are $6000, $8500 and $5500 respectively.

Represent the monthly operational costs using a 3×1 matrix **M**.

$$M = \begin{pmatrix} 6000 \\ 8500 \\ 5500 \end{pmatrix}$$

Answer **M** = ... [1]

(d) Hence, evaluate $\mathbf{P} = \frac{1}{3}\begin{pmatrix}1 & 1 & 1\end{pmatrix}(\mathbf{T} - \mathbf{M})$.

$$P = \frac{1}{3}\begin{pmatrix}1 & 1 & 1\end{pmatrix}\left[\begin{pmatrix}34500\\36600\\26850\end{pmatrix} - \begin{pmatrix}6000\\8500\\5500\end{pmatrix}\right]$$

$$= \frac{1}{3}\begin{pmatrix}77950\end{pmatrix}$$

$$= \begin{pmatrix}25983.33\end{pmatrix}$$

Answer **P** = ………………………………… [2]

(e) State what the element(s) of matrix **P** represent.

The element represents the average profit for the three outlets. ………..

………..

……………………………………………………………………………………….. [1]

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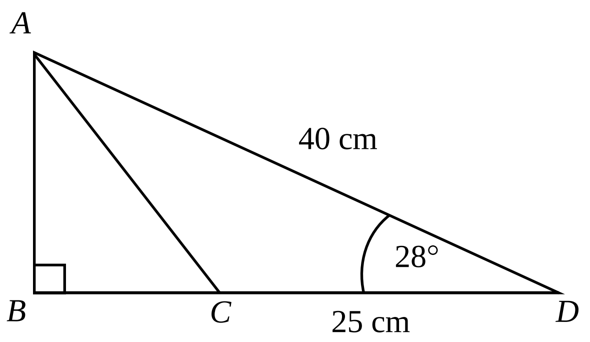
20 C is a point on BD such that $CD = 25$ cm.
$\angle ABD = 90°$, $\angle ADB = 28°$ and $AD = 40$ cm.
Calculate $\angle ACB$.



$$\sin 28^\circ = \frac{AB}{40}$$

$$\therefore AB = 40\sin 28^\circ = 18.77886251$$

$$\therefore BD = \sqrt{40^2 - \left(40\sin 28^\circ\right)^2} = 35.31790371$$

$$\therefore BC = \sqrt{40^2 - \left(40\sin 28^\circ\right)^2} - 25$$

$$= 10.31790371$$

$$\tan \angle ACB = \frac{18.77886251}{10.31790371}$$

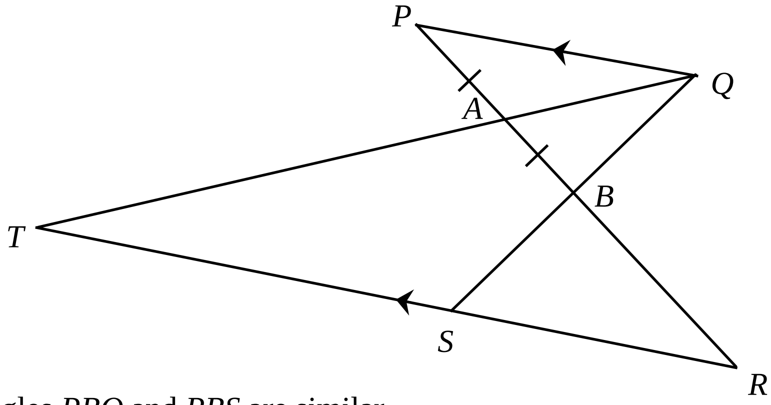
$$\therefore \angle ACB = \tan^{-1}\left(\frac{18.77886251}{10.31790371}\right)$$

$$= 61.21373934^\circ$$

$$\approx 61.2^\circ \quad \text{(2 d.p.)}$$

Answer $\angle ACB$ = ……………………………… ° [4]

21 In the diagram, PQ is parallel to RST. QBS and $PABR$ are straight lines.
It is given that $PA = AB$ and $5PA = 3BR$.



(a) Explain why triangles PBQ and RBS are similar.

Answer……………………………………………………………………………………..

$\angle QPB = \angle BRS$ (alternate $\angle$s, $PQ // RS$)

$\angle PBQ = \angle RBS$ (vertically opposite $\angle$s)

$\therefore$ triangles PBQ and RBS are similar (AA).

…………………………………………………………………………………….. [2]

(b) Calculate

(i) $\dfrac{\text{Area of triangle } QPA}{\text{Area of triangle } QBA}$,

$$\frac{\text{Area of triangle } QPA}{\text{Area of triangle } QBA} = \frac{\frac{1}{2} \times PA \times h}{\frac{1}{2} \times AB \times h} = \frac{PA}{AB} = 1$$

Answer ………………………………….. [1]

(ii) $\dfrac{\text{Area of triangle } PBQ}{\text{Area of triangle } RBS}$.

$5PA = 3BR$

$\therefore PA = \dfrac{3}{5} BR$

$\therefore PB = \dfrac{6}{5} BR$

$$\frac{\text{Area of triangle } PBQ}{\text{Area of triangle } RBS} = \left(\frac{PR}{BR}\right)^2 = \left(\frac{\frac{6}{5}}{1}\right)^2 = \frac{36}{25}$$

Answer ………………………………….. [1]

22 The terms T_1, T_2, T_3, T_4 of a sequence are given as follows:

$$T_1 = 4^2 - (-2)^2 = 12 \times 1$$
$$T_2 = 5^2 - (-1)^2 = 12 \times 2$$
$$T_3 = 6^2 - (0)^2 = 12 \times 3$$
$$T_4 = 7^2 - (1)^2 = 12 \times 4$$

(a) Write down an expression, in the same form and in terms of n, to represent T_n.

$$T_n = (n+3)^2 - (n-3)^2 = 12n$$

Answer T_n = ………………………………… = …………………… [2]

(b) Using your answer in part **(a)** or otherwise, find the positive value of a and of b such that $a^2 - b^2 = 1104$.

$$T_n = 12n = 1104$$
$$\therefore n = \frac{1104}{12} = 92$$
$$\therefore a = 92 + 3 = 95$$
$$\therefore b = 92 - 3 = 89$$

Answer a = ………………………………… [1]

b = ………………………………… [1]

23 Figure 1 shows a container formed by joining together a hemisphere of radius 5 cm and a cone with a base radius of 5 cm.

(a) Given that the volume of the cone is equal to $\frac{2}{3}$ of the volume of the hemisphere, find the vertical height of the cone.

Vol of hemisphere $= \frac{1}{2}\left(\frac{4}{3}\pi(5)^3\right)$

$= \frac{250\pi}{3}$ cm^3

$\frac{1}{3}\pi(5)^2 h = \frac{2}{3}\left(\frac{250\pi}{3}\right)$

$\therefore h = \frac{500\pi}{9} \div \frac{\pi(5)^2}{3} = 6\frac{2}{3}$ cm

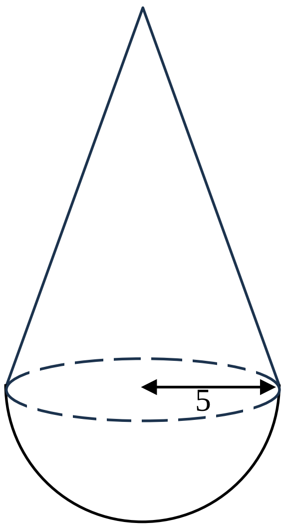


Figure 1

Answer …………………………………… cm [3]

(b) Figure 2 shows the vertical cross-section of the container. The container is partially filled with coloured liquid. The surface of the liquid is represented by *AB* which is 4 cm below the vertex, *V*, of the container.

Calculate the length of *AB*.

$\frac{k}{5} = \frac{4}{6\frac{2}{3}}$

$\therefore k = \frac{4 \times 5}{6\frac{2}{3}} = 3$

$\therefore AB = 3 \times 2 = 6$ cm

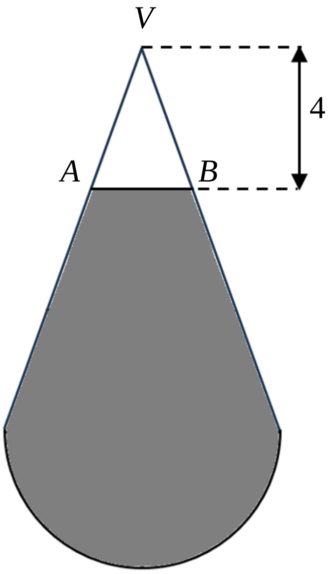


Figure 2

Answer …………………………………… cm [2]

(c) This container is geometrically similar to a larger container. Given that the volume of the larger container is $\frac{7}{2}$ of the volume of the container in Figure 1, find the total surface area of the container (in Figure 1) as a percentage of the total surface area of the larger container.

$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$

$$\frac{V_1}{\frac{7}{2}V_1} = \left(\frac{l_1}{l_2}\right)^3$$

$$\frac{2}{7} = \left(\frac{l_1}{l_2}\right)^3$$

$$\therefore \frac{l_1}{l_2} = \sqrt[3]{\frac{2}{7}} = 0.658633756$$

$$\therefore \frac{A_1}{A_2} = \left(\sqrt[3]{\frac{2}{7}}\right)^2 = 0.433798424$$

$$\therefore \text{Percentage} = \left(\sqrt[3]{\frac{2}{7}}\right)^2 \times 100\%$$

$$\approx 43.4\% \ \ (3 \text{ s.f.})$$

Answer ………………………………… % [3]

24 **(a)** Express 630 as a product of its prime factors.

$$630 = 2 \times 3^2 \times 5 \times 7$$

Answer ……………………………..…………… [1]

(b) 270 adults, 504 boys and 630 girls are to be grouped such that the adults, boys and girls are equally distributed among the groups. Find the greatest number of groups that can be formed.

$$270 = 2 \times 3^3 \times 5$$
$$504 = 2^3 \times 3^2 \times 7$$
$$630 = 2 \times 3^2 \times 5 \times 7$$
$$\therefore \text{ HCF} = 2 \times 3^2$$
$$= 18$$
$\therefore$ the greatest number of groups $= 18$

Answer ………………………………….. [2]

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25 It is given that y is inversely proportional to $\sqrt{x-1}$ and that the difference in the values of y is $\frac{1}{18}$ when the values of x are 10 and 17. Express y in terms of x.

$$y = \frac{k}{\sqrt{x-1}}$$

When $x = 10$, $y = \frac{k}{\sqrt{10-1}} = \frac{k}{3}$ When $x = 17$, $y = \frac{k}{\sqrt{17-1}} = \frac{k}{4}$

$$\therefore \frac{k}{3} - \frac{k}{4} = \frac{1}{18}$$
$$\therefore k = \frac{12}{18} = \frac{2}{3}$$
$$\therefore y = \frac{2}{3\sqrt{x-1}}$$

Answer ………………………………….. [3]

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26 In triangle PQR, $PQ = 10$ cm, $QR = 8$ cm and $\angle PQR = 70°$.

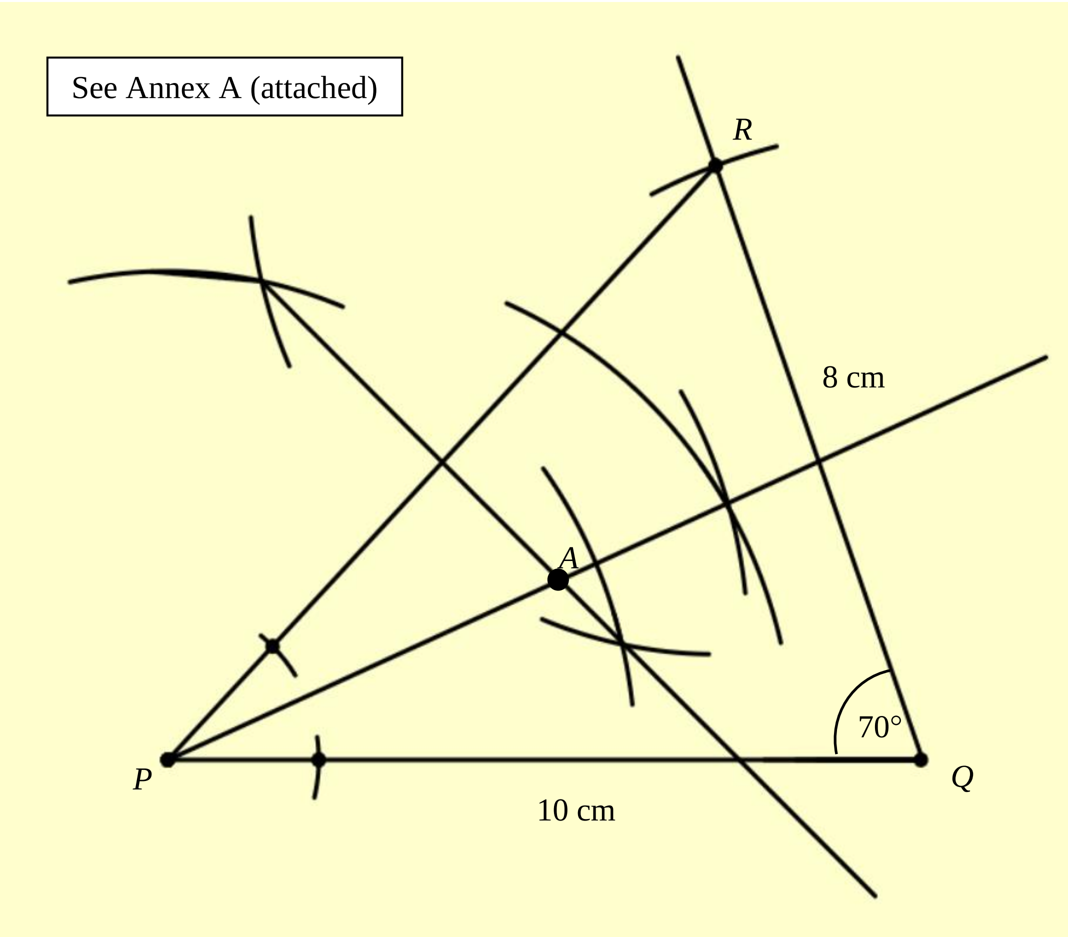
(a) Construct triangle PQR in the answer space below. [2]

(b) In triangle PQR, construct

(i) the perpendicular bisector of the line PR, [1]

(ii) the bisector of $\angle QPR$. [1]

Answer for **(a)** *and* **(b)**



(c) The line in **(b)(i)** and the line in **(b)(ii)** intersect at the point A.
Complete the sentence in the answer space.

Answer: The point A is equidistant from the points .... *P* .... and .... *R* [B1] .... and equidistant from the lines .... *PR* .... and .... *PQ* [B1] .... [2]

~~~ End of Paper ~~~
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