Suggested Solutions

Tutorial 10C: Single Slit and Rayleigh's Criterion

Self-Practice Questions

S1

This question involves the use of the **single slit diffraction formula** $\sin\theta = \dfrac{\lambda}{b}$.

Using $\sin\theta = \dfrac{\lambda}{b}$,

$$\begin{aligned}\theta &= \sin^{-1}\left(\frac{\lambda}{b}\right)\\ &= \sin^{-1}\left(\frac{589.0\times10^{-9}}{0.250\times10^{-3}}\right)\\ &= 0.135^\circ\end{aligned}$$

Common Error:

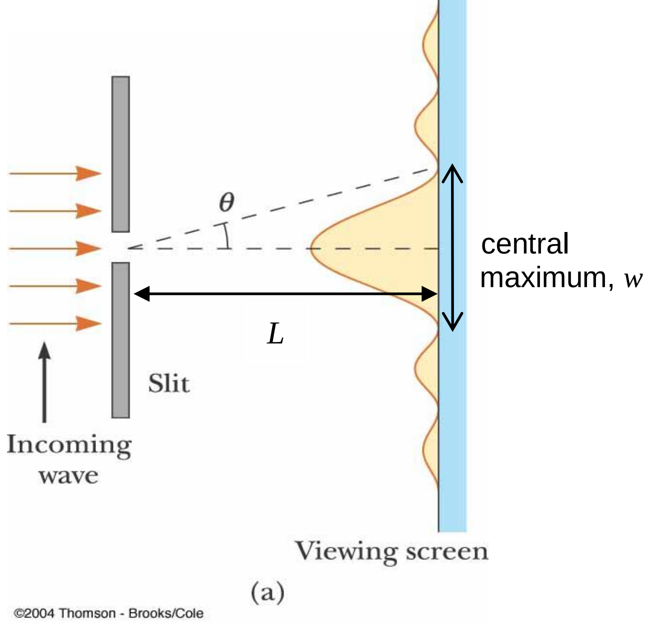
Remember that similar to the equation for the diffraction grating equation, θ is taken **with respect to the principle axis.**

Let w be the width of the central maximum and L be the slit-screen distance.

Then, $\tan\theta = \left(\dfrac{0.5w}{L}\right) \Rightarrow$

$$\begin{aligned}w &= 2L\tan\theta\\ &= 2(2.50)\tan(0.135^\circ)\\ &= 0.0118m\\ &= 1.18cm\end{aligned}$$

Single-slit Diffraction of light



Note:

In this answer, we show how the width of the central maximum can be calculated exactly. This is in contrast to Lecture Example 10.7.1 in your notes, where we showed the use of small angle approximation to simplify our working.

This question can also be solved in similar fashion by taking small angle approximation, which will still give you an answer of 1.18cm. Persuade yourself why this approximation is reasonable.

S2

This question involves the use of the **single slit diffraction formula** $\sin\theta = \frac{\lambda}{b}$.

Using trigonometry,

$$\tan\theta = (\frac{0.5w}{L}) = \frac{0.5\times 3.8\times 10^{-2}}{1.5} \Rightarrow \theta = 0.726°$$

The first minimum intensity occurs at angle θ from the central maximum.

$$\sin\theta = \frac{\lambda}{b}$$

$$\lambda = b\sin\theta = (0.050\times 10^{-3})\times \sin(0.726°) = 633nm$$

> Note:
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> This question is essentially the 'reverse' of Question S1. Now, you are given the width of the central maximum (which effectively tells you the position of the first minimum intensity), from which you can use the formula to calculate a value for the unknown wavelength.

S3

This question involves **qualitative** understanding of the **Rayleigh criterion**.

The Rayleigh criterion states that two images are just resolved or distinguishable when the central maximum of one image coincides with the first minimum of the other image.

Ans: (D)

S4

This question involves **quantitative** calculation using the **Rayleigh criterion** $\theta_{\min} \approx \frac{\lambda}{b}$**.**

The Rayleigh criterion for the resolving power of a single aperture is given by $\theta_{\min} \approx \frac{\lambda}{b}$ for a small $\theta_{\min}$, where *b* is the diameter of the aperture and $\theta_{\min}$ is measured in radians.

$$\begin{aligned}\theta_{\min} &= \frac{\lambda}{b} \\ &= \frac{6.0\times 10^{-2}}{120} \\ &= 5.0\times 10^{-4}\, rad\end{aligned}$$

> Important:
>
> Remember to switch your calculator to **RADIAN** mode when solving calculation problems on resolution.

i.e. $\theta_{\min} \sim 10^{-4}$

Ans: (D)