RAFFLES INSTITUTION
2023 Year 5 H2 Mathematics Promotion Exam

1 The first 3 terms of a sequence are given by $u_1 = 1823$, $u_2 = 200$ and $u_3 = 2023$. Given that u_n is a quadratic polynomial in n, find u_n in terms of n. [4]

2 **Do not use a calculator in answering this question.**
Solve the inequality $x^2 + 6x + 5 \leq \frac{x+5}{3x+1}$. [4]

Hence solve

(a) $x^4 + 6x^2 + 5 \leq \frac{x^2+5}{3x^2+1}$, [2]

(b) $(\ln x)^2 + 6\ln x + 5 \leq \frac{5+\ln x}{1+3\ln x}$. [2]

3 Vectors **a** and **b** are such that the magnitude of **a** is 2 and **b** is a unit vector perpendicular to **a**.

(a) Find the area of the parallelogram with adjacent sides formed by the vectors $\mathbf{a}+3\mathbf{b}$ and $5\mathbf{a}-4\mathbf{b}$. [3]

A vector **c** is such that $\mathbf{b}\times\mathbf{c} = 21(\mathbf{a}\times\mathbf{b})$.

(b) Show that $\mathbf{c} = \lambda\mathbf{b} - 21\mathbf{a}$, where λ is a constant. [2]

(c) Give the geometrical meaning of $|\mathbf{b}.\mathbf{c}|$ and find the possible values of λ if $|\mathbf{b}.\mathbf{c}| = 5$. [3]

4 (a) Write $\frac{1}{r(r+1)}$ in partial fractions. [1]

(b) Using your answer to part **(a)**, find $\sum_{r=1}^{n} \frac{1}{r^2+r}$. [2]

Hence find

(i) $\sum_{r=n+1}^{2n} \frac{1}{r^2+r}$, [2]

(ii) $\frac{1}{2\times 3} + \frac{1}{3\times 4} + \frac{1}{4\times 5} + \cdots$ [2]

5 In this question you may use expansions from the List of Formulae (MF26).

(a) Find the first four non-zero terms of the Maclaurin series of $e^{-x}(1+\cos 3x)$, in ascending powers of x. [4]

(b) It is given that the first three terms of this series are equal to the first three terms in the series expansion, in ascending powers of x, of $\frac{1}{a+bx} + cx^2$. Find the values of a, b and c. [4]

6 (a) Let $y = a^x$, where a is a positive constant. Show that $\dfrac{dy}{dx} = a^x \ln a$. [2]

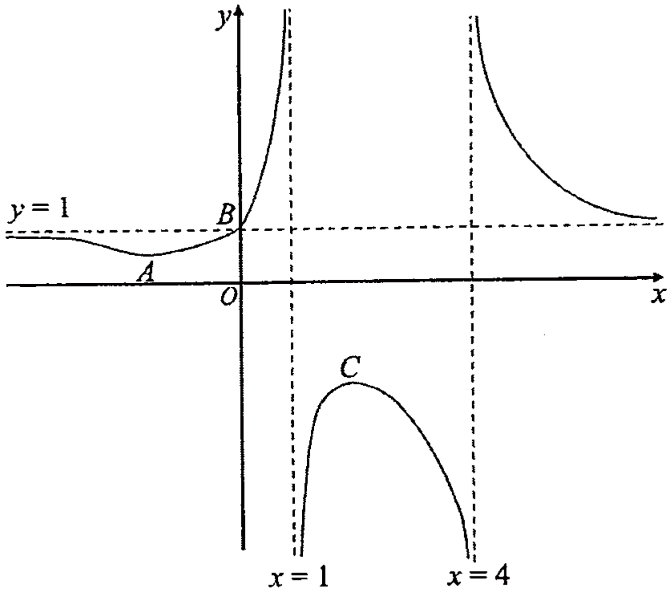
(b) A curve has equation

$$y\left(3^x\right) + 2^{y-1} = 2.$$

Find the equation of the normal to the curve at the point $(0, 1)$. Give your answer in the form $y = Ax + B$, where A and B are constants in the exact form. [6]

7 (a) A curve C has equation $y = \dfrac{ax-3}{(x-3)(x-1)}$ where a is a real constant such that $a \neq 1,\ 3$. Determine the range of values of a for which the curve C has no turning points. [4]

(b)



The diagram above shows the graph of $y = \mathrm{f}(x)$. It has asymptotes $y = 1$, $x = 1$ and $x = 4$. The curve cuts the y-axis at the point $B(0, 1)$, has a minimum at the point $A\left(-2, \dfrac{2}{3}\right)$ and a maximum at the point $C(2, -2)$.

By showing clearly the equations of asymptotes and the coordinates of the points corresponding to A, B and C where possible, sketch, on **separate diagrams**, the graphs of

(i) $y = 3\mathrm{f}(x+1)$, [4]

(ii) $y = \dfrac{1}{\mathrm{f}(x)}$. [4]

8 **Do not use a calculator in answering this question.**

(a) One root of the equation $2z^3 - 5z^2 + \alpha z - 5 = 0$, where $\alpha \in$, is $z = 1 - 2\mathrm{i}$. Find the value of α and the other roots. [5]

(b) The complex number z is given by

$$z = \frac{\left(-2+2\sqrt{3}\,\mathrm{i}\right)^2}{\cos\frac{1}{12}\pi + \mathrm{i}\sin\frac{1}{12}\pi}.$$

Find $|z|$ and $\arg(z)$. [4]

9 It is given that

$$\mathrm{f} : x \mapsto \left|\frac{1}{x-4}\right|, \text{ where } x \in \ \ , x \neq 4,$$

$$\mathrm{g} : x \mapsto \ln(x+2), \text{ where } x \in \ \ , x > -2.$$

(a) Explain why the composite function gf exists and find gf in a similar form. [3]

(b) Find the range of gf. [1]

(c) Explain why f does not have an inverse. [1]

(d) If the domain of f is further restricted to $x < k$, state the maximum value of k such that the function f^{-1} exist. [1]

In the rest of this question, the domain of f is $x \in \ \ , x < 3$.

(e) Find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [3]

(f) Sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ on the same diagram and state the geometrical relation between the two graphs. [3]

10 In long-track speed skating, races are run counter-clockwise on a 400-metre two-lane oval rink. A full 400-metre round the rink is known as a lap. The current world records for the 10000 metres men's race and 500 metres men's race are 12 minutes 30.74 seconds (11 Feb 2022) and 33.61 seconds (9 Mar 2019) respectively.

Abel and Caine are 2 skaters training for the 10000 metres men's race. During a particular training session for Abel, he completes the first lap in b seconds and he takes 10% longer to complete each succeeding lap than he does in the previous lap.

(a) Write down an expression for the time taken by Abel to complete n laps, in terms of b and n.
Hence find the value of b that will enable Abel to complete 10000 metres in 13 minutes. [3]

(b) Comment on the feasibility of this value of b in the context of the question. [1]

After training for 6 months, Abel and Caine both entered a 10000 metres men's race. During the race, Abel completes the first lap in k seconds and then he takes 1% longer to complete each succeeding lap than he does in the previous lap. Caine also completes the first lap in k seconds and on each subsequent lap he spends d seconds more than he spent on the previous lap. They arrive at the finish line at the same time.

(c) Find the value of $\frac{d}{k}$, correct to 5 decimal places. [4]

(d) Given that Abel completes his 25th lap in p seconds and Caine completes his 25th lap in q seconds, evaluate $\frac{q}{p}$, giving your answer correct to 3 decimal places.
Hence determine the skater who has the faster time for the first 24 laps. [4]

11 The lines l_1 and l_2 have equations

$$\mathbf{r}=\begin{pmatrix}10\\-1\\2\end{pmatrix}+\lambda\begin{pmatrix}3\\-1\\1\end{pmatrix} \quad \text{and} \quad \mathbf{r}=\begin{pmatrix}4\\4\\5\end{pmatrix}+\mu\begin{pmatrix}-1\\2\\3\end{pmatrix}$$

respectively, where λ and μ are parameters.

(a) Without the use of a calculator, show that l_1 and l_2 are skew lines. [3]

(b) Find a vector, $\mathbf{n}$, that is perpendicular to both l_1 and l_2. [1]

Referred to the origin O, points P and Q have position vectors $\begin{pmatrix}4\\4\\5\end{pmatrix}$ and $\begin{pmatrix}10\\-1\\2\end{pmatrix}$ respectively.

(c) Find the exact length of projection of $\overrightarrow{PQ}$ onto $\mathbf{n}$. [2]

A plane π has equation $3x+z=11$.

(d) Find, in degrees, the acute angles between l_1 and π, and between l_2 and π. Hence, or otherwise, determine which one of l_1 or l_2 is not intersecting π. [4]

(e) Find the exact distance between the line determined in part **(d)** and π. [2]

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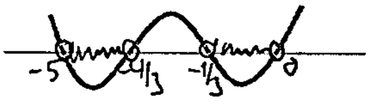
Solutions

1	Let $u_n = an^2 + bn + c$. $u_1 = 1823: \ a + b + c = 1823$ $u_2 = 200: \ 4a + 2b + c = 200$ $u_3 = 2023: \ 9a + 3b + c = 2023$ Using GC, we have $a = 1723, b = -6792, c = 6892.$ Therefore, $u_n = 1723n^2 - 6792n + 6892.$

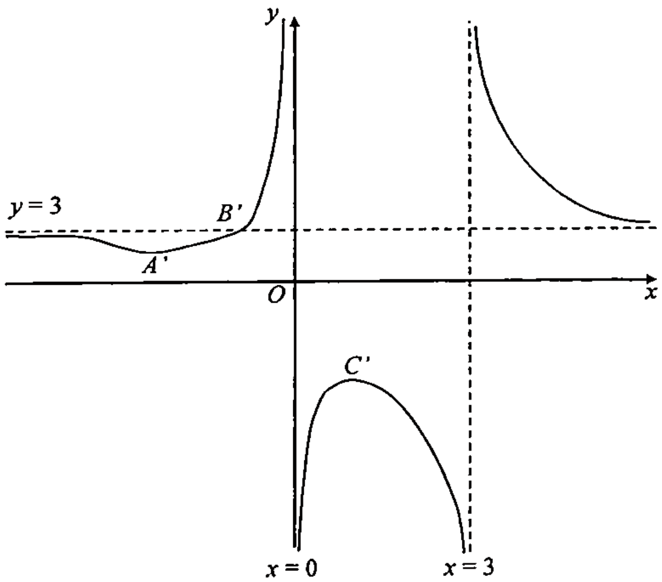
2	$x^2+6x+5-\frac{x+5}{3x+1}\le 0$ $(x+5)(x+1)-\frac{x+5}{3x+1}\le 0$ $(x+5)\frac{(x+1)(3x+1)-1}{3x+1}\le 0$ $\frac{(x+5)\left[3x^2+4x+1-1\right]}{3x+1}\le 0$ $\frac{(x+5)(x)(3x+4)}{3x+1}\le 0$  $\therefore -5\le x\le -\frac{4}{3}$ or $-\frac{1}{3}<x\le 0$
(a)	Replace x with x^2, $x^4+6x^2+5\le \frac{x^2+5}{3x^2+1}$ $\therefore -5\le x^2\le -\frac{4}{3}$ or $-\frac{1}{3}<x^2\le 0$ No solution, since $x^2\ge 0$ or $x=0$ $\Rightarrow x=0$
(b)	Replace x with $\ln x$, $(\ln x)^2+6\ln x+5\le \frac{\ln x+5}{3\ln x+1}$ $\therefore -5\le \ln x\le -\frac{4}{3}$ or $-\frac{1}{3}<\ln x\le 0$ $\Rightarrow e^{-5}\le x\le e^{-4/3}$ or $e^{-1/3}<x\le 1$

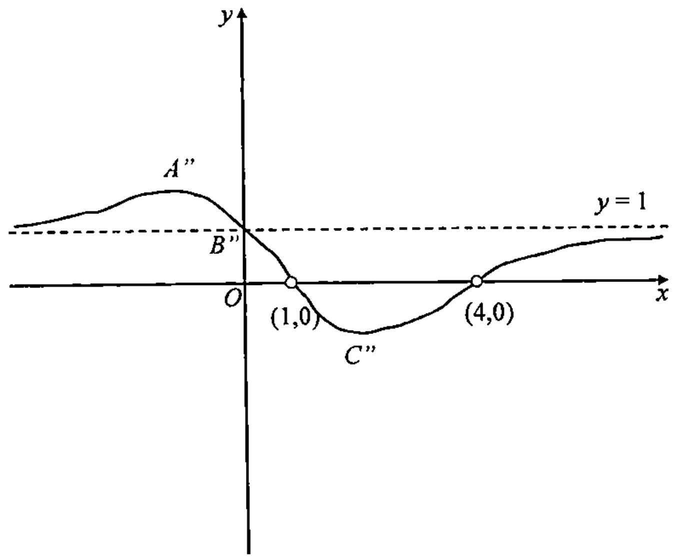
3(a)	Area of the parallelogram $=\|(\mathbf{a}+3\mathbf{b})\times(5\mathbf{a}-4\mathbf{b})\|$ $=\|5(\mathbf{a}\times\mathbf{a})+15(\mathbf{b}\times\mathbf{a})-4(\mathbf{a}\times\mathbf{b})-12(\mathbf{b}\times\mathbf{b})\|$ $=\|19(\mathbf{b}\times\mathbf{a})\|$ $=19\|\mathbf{b}\|\|\mathbf{a}\|\sin 90°$ $=38$
(b)	$\mathbf{b}\times\mathbf{c}=21(\mathbf{a}\times\mathbf{b})$ $\Rightarrow(\mathbf{b}\times\mathbf{c})-21(\mathbf{a}\times\mathbf{b})=\mathbf{0}$ $\Rightarrow(\mathbf{b}\times\mathbf{c})+21(\mathbf{b}\times\mathbf{a})=\mathbf{0}$ $\Rightarrow\mathbf{b}\times(\mathbf{c}+21\mathbf{a})=\mathbf{0}$ <u>Case 1: If $\mathbf{c}=-21\mathbf{a}$</u> Note that $\mathbf{c}=-21\mathbf{a}$ satisfy the equation $\mathbf{b}\times(\mathbf{c}+21\mathbf{a})=\mathbf{0}$ <u>Case 2: If $\mathbf{c}\neq-21\mathbf{a}$</u> Since **b** and $\mathbf{c}+21\mathbf{a}$ are nonzero vectors, then $\mathbf{c}+21\mathbf{a}$ is parallel to **b** and so $\mathbf{c}+21\mathbf{a}=\lambda\mathbf{b}$, where λ is a nonzero constant. $\mathbf{c}=\lambda\mathbf{b}-21\mathbf{a}$ Note that $\lambda=0$ corresponds to Case 1. Thus we can say that $\mathbf{c}=\lambda\mathbf{b}-21\mathbf{a}$, where λ is a constant. (Shown)
(c)	$\|\mathbf{b}\cdot\mathbf{c}\|=\|\mathbf{c}\cdot\mathbf{b}\|$ is the length of projection of **c** onto **b**. $\|\mathbf{b}\cdot\mathbf{c}\|=\|\mathbf{b}\cdot(\lambda\mathbf{b}-21\mathbf{a})\|$ $=\|\lambda(\mathbf{b}\cdot\mathbf{b})-21(\mathbf{b}\cdot\mathbf{a})\|$ $=\left\|\lambda\|\mathbf{b}\|^2-21(0)\right\|$ $=\|\lambda\|$ Since $\|\mathbf{b}\cdot\mathbf{c}\|=5$, $\lambda=\pm5$.

4(a)	$\dfrac{1}{r(r+1)}=\dfrac{1}{r}-\dfrac{1}{r+1}$
(b)	$\displaystyle\sum_{r=1}^{n}\frac{1}{r^2+r}=\sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{r+1}\right)$ $=\dfrac{1}{1}-\dfrac{1}{2}$ $+\dfrac{1}{2}-\dfrac{1}{3}$ $+\dfrac{1}{3}-\dfrac{1}{4}$ . . . $+\dfrac{1}{n-1}-\dfrac{1}{n}$ $+\dfrac{1}{n}-\dfrac{1}{n+1}$ $=1-\dfrac{1}{n+1}$
(i)	$\displaystyle\sum_{r=n+1}^{2n}\frac{1}{r^2+r}=\sum_{r=1}^{2n}\frac{1}{r^2+r}-\sum_{r=1}^{n}\frac{1}{r^2+r}=\left(1-\frac{1}{2n+1}\right)-\left(1-\frac{1}{n+1}\right)=\frac{1}{n+1}-\frac{1}{2n+1}$
(ii)	$\displaystyle\frac{1}{2\times3}+\frac{1}{3\times4}+\frac{1}{4\times5}+\cdots=\sum_{r=2}^{\infty}\frac{1}{r(r+1)}=\sum_{r=1}^{\infty}\frac{1}{r(r+1)}-\frac{1}{1\times2}=\lim_{n\to\infty}\left(1-\frac{1}{n+1}\right)-\frac{1}{2}=\frac{1}{2}$

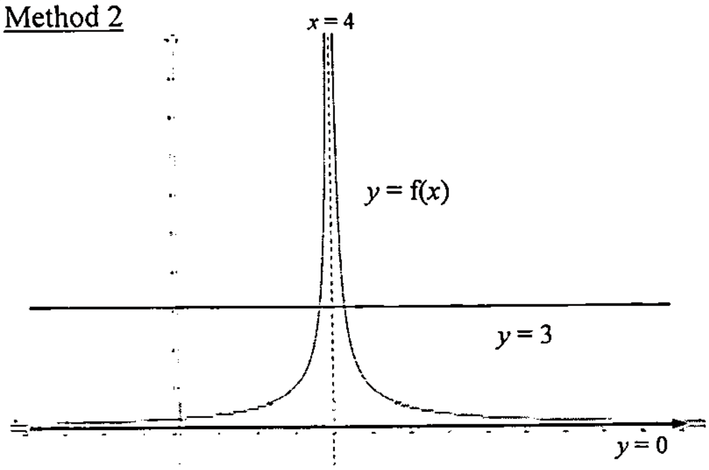
5(a)	$e^{-x}(1+\cos 3x)$ $=\left(1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\cdots\right)\left(1+1-\frac{(3x)^2}{2!}+\cdots\right)$ $=\left(1-x+\frac{x^2}{2}-\frac{x^3}{6}+\cdots\right)\left(2-\frac{9x^2}{2}+\cdots\right)$ $=2-2x+\left(1-\frac{9}{2}\right)x^2+\left(-\frac{1}{3}+\frac{9}{2}\right)x^3+\cdots$ $=2-2x-\frac{7}{2}x^2+\frac{25}{6}x^3+\cdots$
(b)	$\frac{1}{a+bx}+cx^2$ $=a^{-1}\left(1+\frac{b}{a}x\right)^{-1}+cx^2$ $=\frac{1}{a}\left(1-\frac{b}{a}x+\left(\frac{b}{a}\right)^2x^2+\cdots\right)+cx^2$ $=\frac{1}{a}-\frac{b}{a^2}x+\left(\frac{b^2}{a^3}+c\right)x^2+\cdots$ Comparing with the series expansion in **(a)**, $\frac{1}{a}=2 \quad \Rightarrow \quad a=\frac{1}{2}$ $-\frac{b}{a^2}=-2 \quad \Rightarrow \quad b=2a^2=\frac{1}{2}$ $\frac{b^2}{a^3}+c=-\frac{7}{2} \quad \Rightarrow \quad c=-\frac{7}{2}-\frac{b^2}{a^3}=-\frac{7}{2}-2=-\frac{11}{2}$

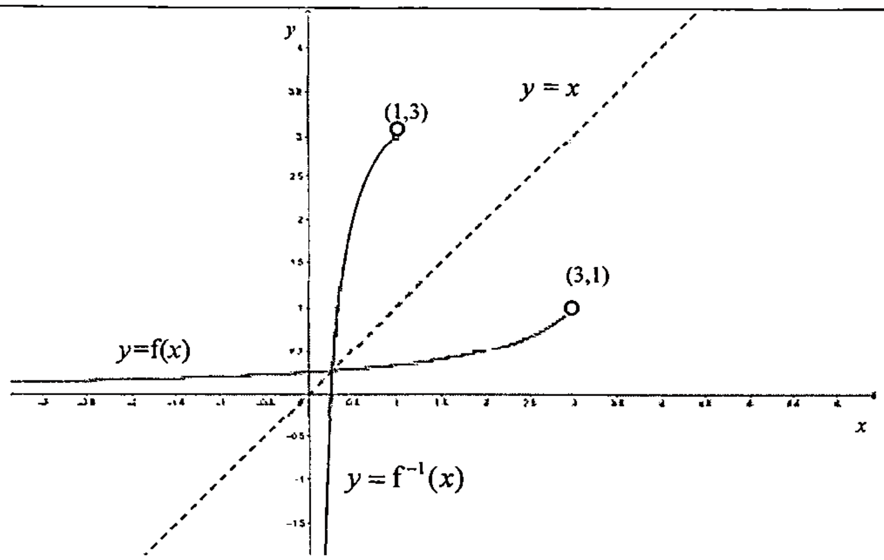
6(a)	$y = a^x \Rightarrow \ln y = x \ln a$. Differentiate with respect to x: $\frac{1}{y}\frac{dy}{dx} = \ln a \Rightarrow \frac{dy}{dx} = y \ln a = a^x \ln a$ (shown). Alternatively, $y = a^x = e^{\ln a^x} = e^{x \ln a}$. $\frac{dy}{dx} = e^{x \ln a}(\ln a) = a^x \ln a$ (shown).
(b)	$y(3^x) + 2^{y-1} = 2$ Differentiate with respect to x: $\frac{dy}{dx}3^x + y3^x \ln 3 + 2^{y-1}\frac{dy}{dx}\ln 2 = 0$ $\Rightarrow \frac{dy}{dx} = \frac{-y3^x \ln 3}{3^x + 2^{y-1}\ln 2}$. At $(0, 1)$, $\frac{dy}{dx} = \frac{-(1)(1)\ln 3}{(1)+(1)\ln 2} = \frac{-\ln 3}{1+\ln 2}$. Equation of normal at $(0, 1)$ is $y - 1 = \frac{1+\ln 2}{\ln 3}(x - 0)$ $\Rightarrow y = \left(\frac{1+\ln 2}{\ln 3}\right)x + 1$, where $A = \frac{1+\ln 2}{\ln 3}$, $B = 1$ (shown).

7(a)	$y=\dfrac{ax-3}{(x-1)(x-3)}$ $\dfrac{dy}{dx}=\dfrac{a(x^2-4x+3)-(2x-4)(ax-3)}{[(x-1)(x-3)]^2}$ $=\dfrac{-ax^2+6x+3a-12}{[(x-1)(x-3)]^2}$ To find turning points, $\dfrac{dy}{dx}=0$ So, $-ax^2+6x+3a-12=0$ For the curve to have no turning points, discriminant of $-ax^2+6x+3a-12=0$ is <0 $36-4(-a)(3a-12)<0$ $12a^2-48a+36<0$ $12(a-1)(a-3)<0$ $\Rightarrow 1<a<3$
(b) (i)	 $A'(-3,2)$, $B'(-1,3)$ and $C'(1,-6)$.

(ii)	 $A''(-2,\frac{3}{2})$, $B''(0,1)$ and $C''(2,-\frac{1}{2})$.

8(a)	Since coefficients of equation are real, $z = 1+2\mathrm{i}$ is also a root. So, the quadratic factor is $(z-(1-2\mathrm{i}))(z-(1+2\mathrm{i})) = z^2 - 2z + 5$. Thus, by observation, $2z^3 - 5z^2 + \alpha z - 5 = (z^2 - 2z + 5)(2z - 1)$ Comparing coefficients of z, $\alpha = 10 + 2 = 12$. The other roots of the equation are $1+2\mathrm{i}$ and $\frac{1}{2}$.
	Alternatively, $(1-2\mathrm{i})^2 = 1 - 4\mathrm{i} - 4 = -3 - 4\mathrm{i}$ $(1-2\mathrm{i})^3 = (-3-4\mathrm{i})(1-2\mathrm{i}) = -3 + 2\mathrm{i} - 8 = -11 + 2\mathrm{i}$ Substitute $z = 1 - 2\mathrm{i}$ into $2z^3 - 5z^2 + \alpha z - 5 = 0$, $2(-11+2\mathrm{i}) - 5(-3-4\mathrm{i}) + \alpha(1-2\mathrm{i}) - 5 = 0$ $\alpha(1-2\mathrm{i}) = 12 - 24\mathrm{i} = 12(1-2\mathrm{i})$ $\alpha = 12$ Since coefficients of equation are real, $z = 1+2\mathrm{i}$ is also a root. So, the quadratic factor is $(z-(1-2\mathrm{i}))(z-(1+2\mathrm{i})) = z^2 - 2z + 5$. Thus, $2z^3 - 5z^2 + 12z - 5 = (z^2 - 2z + 5)(2z - 1)$ $z = \frac{1}{2}$ is the third root of the equation.
(b)	$\left\|-2+2\sqrt{3}\mathrm{i}\right\| = \sqrt{2^2 + (2\sqrt{3})^2} = \sqrt{4+12} = 4$ $\arg(-2+2\sqrt{3}\mathrm{i}) = \frac{2\pi}{3}$ So, $\|z\| = \frac{4^2}{1} = 16$ and $\arg(z) = 2\left(\frac{2\pi}{3}\right) - \frac{\pi}{12} - 2\pi = \frac{5\pi}{4} - 2\pi = -\frac{3\pi}{4}$ Alternatively, $z = \frac{(-2+2\sqrt{3}\mathrm{i})^2}{\cos\frac{1}{12}\pi + \mathrm{i}\sin\frac{1}{12}\pi} = \frac{\left(4\mathrm{e}^{\frac{2\pi\mathrm{i}}{3}}\right)^2}{\mathrm{e}^{\frac{\pi\mathrm{i}}{12}}} = 16\mathrm{e}^{\left(\frac{4\pi}{3} - \frac{\pi}{12}\right)\mathrm{i}} = 16\mathrm{e}^{\left(\frac{5\pi}{4}\right)\mathrm{i}} = 16\mathrm{e}^{\left(-\frac{3\pi}{4}\right)\mathrm{i}}$ $\|z\| = 16$ and $\arg(z) = -\frac{3\pi}{4}$

9(a)	Since $R_f = (0,\infty) \subseteq D_g = (-2,\infty)$, gf exists. $gf: x \mapsto \ln\left(\left\lvert\frac{1}{x-4}\right\rvert + 2\right)$, where $x \in \mathbb{R}, x \neq 4$
(b)	$R_{gf} = (\ln 2, \infty)$
(c)	<u>Method 1</u> Since $3, 5 \in D_f$ and $3 \neq 5$ such that $f(3) = 1 = f(5)$, f is not one-one. Hence f^{-1} does not exist.
	<u>Method 2</u>  Since the horizontal line $y = 3$ cuts the graph of f twice, f is not one-one. Hence f^{-1} does not exist.
(d)	Maximum $k = 4$
(e)	Let $y = \left\lvert\frac{1}{x-4}\right\rvert = -\frac{1}{x-4}$, since $x < 3$ $x - 4 = -\frac{1}{y}$ $x = 4 - \frac{1}{y}$ $\therefore f^{-1}(x) = 4 - \frac{1}{x}$ $D_{f^{-1}} = R_f = (0,1)$

(f)	 The graph of $y = f(x)$ is a reflection of the graph of $y = f^{-1}(x)$ in the line $y = x$.

10 (a)	The lap times for Abel forms a GP with first term b and common ratio 1.1. Time taken to complete n laps, $S_n = \dfrac{b(1.1^n - 1)}{1.1-1} = 10b(1.1^n - 1)$ 10000 metres equals 25 laps, so we let $S_{25} = 13 \times 60$ $10b(1.1^{25} - 1) = 780$ $b = \dfrac{78}{(1.1^{25}-1)} = 7.9311 = 7.93 \quad \text{(3 s.f.)}$
(b)	The current world record for 500 metres is 33.61 seconds and this means that the average time per lap is around 26.89 seconds. Hence, Abel's training strategy is not feasible as he is unlikely to complete the first 400 metres in 7.93 seconds.
(c)	Abel's times form a GP with first term k and common ratio 1.01. Caines's times form an AP with first term k and common difference d. We have $S_{25}(\text{Abel}) = S_{25}(\text{Caine})$ $\dfrac{k(1.01^{25}-1)}{1.01-1} = \dfrac{25}{2}(2k+24d)$ $100k(1.01^{25}-1) = 25k + 300d$ $k\left(100(1.01^{25}) - 125\right) = 300d$ $\dfrac{d}{k} = \dfrac{100(1.01^{25})-125}{300} \approx 0.01081 \quad \text{(5 dp)}$
(d)	We have $\dfrac{q}{p} = \dfrac{k+24d}{k(1.01^{24})}$ $= \dfrac{1}{1.01^{24}} + \dfrac{24}{1.01^{24}}\left(\dfrac{d}{k}\right)$ $\approx \dfrac{1}{1.01^{24}} + \dfrac{24}{1.01^{24}}(0.01081)$ $\approx 0.99190 = 0.992 \ (3 \text{ dp}).$ Hence, $q \approx 0.992p < p$.

	This means Abel spent more time on the last lap than Caine. Since both of them arrived at the finish line at the same time, Abel is the skater who has the faster time for the first 24 laps.

11(a)

Since $\begin{pmatrix}3\\-1\\1\end{pmatrix} \neq k\begin{pmatrix}-1\\2\\3\end{pmatrix}$ for all $k \in \mathbb{R}$, so l_1 and l_2 are not parallel.

Consider $\begin{pmatrix}10\\-1\\2\end{pmatrix}+\lambda\begin{pmatrix}3\\-1\\1\end{pmatrix}=\begin{pmatrix}4\\4\\5\end{pmatrix}+\mu\begin{pmatrix}-1\\2\\3\end{pmatrix}$

$$\Rightarrow \begin{cases}10+3\lambda=4-\mu\\-1-\lambda=4+2\mu\\2+\lambda=5+3\mu\end{cases} \Rightarrow \begin{cases}3\lambda+\mu=-6 \quad ----(1)\\ \lambda+2\mu=-5 \quad ----(2)\\ \lambda-3\mu=3 \quad ----(3)\end{cases}$$

Using (1) and (2) to solve, we get $\lambda=-\frac{7}{5}$ and $\mu=-\frac{9}{5}$.

LHS of (3) $=\lambda-3\mu=-\frac{7}{5}-3\left(-\frac{9}{5}\right)=4\neq$ RHS of (3)

So l_1 and l_2 do not intersect.

Hence, l_1 and l_2 are skew lines (shown).

(b)

$\begin{pmatrix}3\\-1\\1\end{pmatrix}\times\begin{pmatrix}-1\\2\\3\end{pmatrix}=\begin{pmatrix}-5\\-10\\5\end{pmatrix}=-5\begin{pmatrix}1\\2\\-1\end{pmatrix}$. $\therefore \mathbf{n}=\begin{pmatrix}1\\2\\-1\end{pmatrix}$.

(c)

Length of projection of $\overrightarrow{PQ}$ onto $\mathbf{n}$

$$=\frac{\left|\left[\begin{pmatrix}10\\-1\\2\end{pmatrix}-\begin{pmatrix}4\\4\\5\end{pmatrix}\right]\cdot\begin{pmatrix}1\\2\\-1\end{pmatrix}\right|}{\sqrt{1+4+1}}=\frac{\left|\begin{pmatrix}6\\-5\\-3\end{pmatrix}\cdot\begin{pmatrix}1\\2\\-1\end{pmatrix}\right|}{\sqrt{6}}=\frac{|6-10+3|}{\sqrt{6}}=\frac{1}{\sqrt{6}}.$$

(d)

π: $\mathbf{r}\cdot\begin{pmatrix}3\\0\\1\end{pmatrix}=11$

Acute angle between l_1 and π

$$=\sin^{-1}\frac{\left|\begin{pmatrix}3\\0\\1\end{pmatrix}\cdot\begin{pmatrix}3\\-1\\1\end{pmatrix}\right|}{\sqrt{10}\sqrt{11}}=\sin^{-1}\frac{10}{\sqrt{10\times11}}=72.5^\circ \text{ (nearest } 0.1^\circ).$$

Acute angle between l_2 and π

	$= \sin^{-1} \dfrac{\left\lvert \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \right\rvert}{\sqrt{10}\sqrt{14}} = \sin^{-1} \dfrac{0}{\sqrt{10 \times 14}} = 0^{\circ}.$ Hence, either l_2 is on π or l_2 is parallel to π, but not on π. Since $\begin{pmatrix} 4 \\ 4 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} = 12 + 0 + 5 = 17 \neq 11$, so l_2 is not on π but is parallel to π. Thus, l_2 is not intersecting π.
(e)	$(0, 0, 11)$ is on π since $\begin{pmatrix} 0 \\ 0 \\ 11 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} = 11.$ Distance between l_2 and π $= \dfrac{\left\lvert \left[\begin{pmatrix} 4 \\ 4 \\ 5 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 11 \end{pmatrix} \right] \cdot \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \right\rvert}{\sqrt{9+0+1}} = \dfrac{\left\lvert \begin{pmatrix} 4 \\ 4 \\ -6 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \right\rvert}{\sqrt{10}} = \dfrac{\lvert 12 + 0 - 6 \rvert}{\sqrt{10}} = \dfrac{6}{\sqrt{10}}$ or $\dfrac{3\sqrt{10}}{5}.$