PEICAI SECONDARY SCHOOL
SECONDARY 4 EXPRESS
PRELIMINARY EXAMINATION 2021

CANDIDATE NAME: SOLUTIONS

CLASS:

REGISTER NUMBER:

ADDITIONAL MATHEMATICS — 4049/01

Paper 1

26 August 2021

2 hours 15 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your register number, class and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

This document consists of **19** printed pages and **1** blank page.

Setter: Mrs Ho Thuk Lan

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}ab\sin C$$

1 Given that $\sqrt[3]{289^x} = \dfrac{17^{1-x}}{289}$, find the value of $\sqrt[3]{289^x}$. [4]

$17^{\frac{2}{3}x} = 17^{-1-x}$

$\frac{2}{3}x = -1 - x$

$x = -\frac{3}{5}$

$\sqrt[3]{289^x}$ = 0.322 (3SF)

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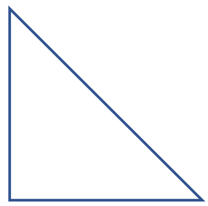
2 An equilateral triangle has a perimeter of $13 - 3\sqrt{3}$ cm. Find the area of the triangle, in cm^2, in the form $a + b\sqrt{3}$, where *a* and *b* are constants. [4]

<table>
<tr><th><u>Method 1:</u></th><th><u>Method 2</u>:</th></tr>
<tr>
<td>Length of one side = $\dfrac{13-3\sqrt{3}}{3}$
$\sin 60° = \dfrac{\sqrt{3}}{2}$

Area = $\frac{1}{2}\left(\frac{13-3\sqrt{3}}{2}\right)^2 \sin 60°$
Area = $\frac{1}{2}\left(\frac{13-3\sqrt{3}}{2}\right)^2 \times \frac{\sqrt{3}}{2}$

Area = $-\frac{13}{2}+\frac{49}{9}\sqrt{3}$</td>
<td>Height = $\sqrt{\left(\frac{13-3\sqrt{3}}{3}\right)^2 - \left(\frac{13-3\sqrt{3}}{6}\right)^2}$ (Pythagoras Theorem)

Height = $\sqrt{\frac{(13-3\sqrt{3})^2}{9} - \frac{(13-3\sqrt{3})^2}{36}}$
Height = $\sqrt{\frac{4(13-3\sqrt{3})^2 - (13-3\sqrt{3})^2}{36}}$
Height = $\sqrt{\frac{3(13-3\sqrt{3})^2}{36}}$
Height = $\frac{\sqrt{3}}{6}(13-3\sqrt{3})$

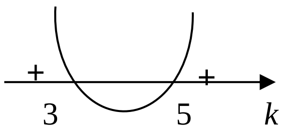
Area = $\frac{1}{2}\times\left(\frac{13-3\sqrt{3}}{3}\right)\times\frac{\sqrt{3}(13-3\sqrt{3})}{6}$
Area = $\frac{\sqrt{3}(13-3\sqrt{3})^2}{36}$
Area = $\frac{\sqrt{3}(196-78\sqrt{3})}{36}$
Area = $\frac{196\sqrt{3}-234}{36} = \frac{98\sqrt{3}-117}{18}$
Area = $-\frac{13}{2}+\frac{49}{9}\sqrt{3}$</td>
</tr>
</table>

3 The equation of a curve $y = 2x^2 + kx + 5$ intersects the line $y = x + 3$ at two points. Find the set of values of k. [4]

$2x^2 + (k-1)x + 2 = 0$

$(k-1)^2 - 4(2)(2) > 0$

$(k+3)(k-5) > 0$



$k < -3$ or $k > 5$

4 It is given that f(x) is defined for $x > 1$ and is such that $f'(x) = \dfrac{2x^2}{x^2-1}$.

By considering $f''(x)$, explain whether $f'(x) = \dfrac{2x^2}{x^2-1}$ is an increasing or decreasing function. [4]

$$f''(x) = \frac{(x^2-1)4x - 2x^2(2x)}{(x^2-1)^2}$$

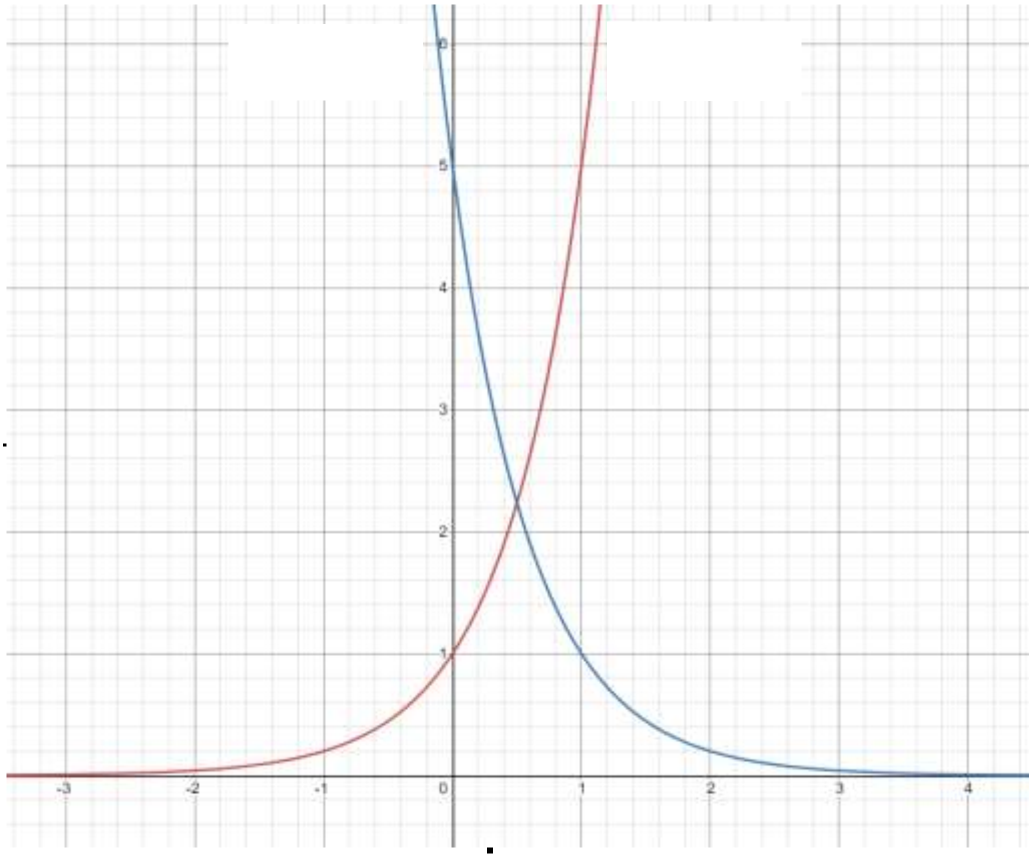
$$f''(x) = \frac{-4x}{(x^2-1)^2}$$

Given $x > 1$, $4x < 0$, $(x^2-1)^2 > 0$

▯ $\dfrac{-4x}{(x^2-1)^2} < 0$

Since $f''(x) < 0$, f'(x) is a decreasing function.

5 **(a)** On the same diagram, sketch the graphs of $y = 5^x$ and $y = 5(5^{-x})$. [3]



(b) Find the coordinates of the point of intersection of $y = 5^x$ and $y = 5(5^{-x})$. [2]

$5^x = 5(5^{-x})$

$x = 1 - x$

$x = \frac{1}{2}$

Coordinates = $\left(\frac{1}{2}, \sqrt{5}\right)$

6 A, B and C are the angles of a triangle.

(a) Show that $\cos C = \quad \cos (A + B)$ [2]

In a triangle, $A + B + C = 180°$

∴ $A + B = 180° \quad C$

$$\cos (A + B) = \cos (180° \quad C)$$
$$\cos (A + B) = \cos 180° \cos C \quad \sin 180° \sin C$$
$$\cos (A + B) = (\quad 1) \cos C \quad (0) \sin C$$
$$\cos (A + B) = \quad \cos C$$

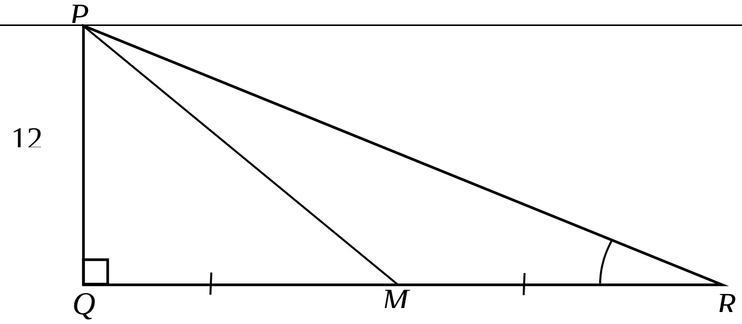
therefore $\cos C = \quad \cos (A + B)$

Given that $A = 45°$ and $B = 30°$.

(b) **Without using a calculator**, find cos C in the form $\frac{1}{4}\left(\sqrt{a}+\sqrt{b}\right)$, where a and b are integers. [3]

$$\cos C = -\cos(45° + 30°)$$
$$\cos C = -\cos 45° \cos 30° + \sin 45° \sin 30°$$
$$\cos C = -\frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2}$$
$$\cos C = \frac{1}{4}\left(\sqrt{2} - \sqrt{6}\right)$$

7



The diagram shows a triangle PQR in which angle $PRQ = \frac{\pi}{6}$ radians, angle PQR is a right angle, M is the midpoint of QR and the length of PQ = 12 m. **Without using a calculator**, find the value of the integer k such that angle $MPR = \sin^{-1}\left(\frac{\sqrt{k}}{14}\right)$. [5]

$\tan\frac{\pi}{6} = \frac{12}{QR}$ $QR = 12\sqrt{3}$ $QM = MR = 6\sqrt{3}$	Pythagoras Theorem: $PM = \sqrt{12^2 + QM^2} = \sqrt{252}$ $PM = 6\sqrt{7}$

$$\frac{\sin\angle MPR}{MR} = \frac{\sin\frac{\pi}{6}}{PM}$$

$$\sin\angle MPR = \frac{\sin\frac{\pi}{6}}{PM} \times MR$$

1

$$\sin\angle MPR = \frac{\frac{1}{2}}{6\sqrt{7}} \times 6\sqrt{3}$$

$$\sin\angle MPR = \frac{\sqrt{3}}{2\sqrt{7}}$$

$$\sin\angle MPR = \frac{\sqrt{3}}{2\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$$

$$\sin\angle MPR = \frac{\sqrt{21}}{14}$$

$k = 21$

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8 A curve is such that $\frac{d^2y}{dx^2} = 6x - 5$. The curve passes through the point (1, 5) and at this point, the gradient of the curve is 3.

(a) Find the x-coordinates of the stationary points of the curve. [4]

$$\frac{dy}{dx} = \int (6x - 5)\ dx$$

$$\frac{dy}{dx} = 3x^2 - 5x + c$$

At point (1, 5), gradient = 3

$$3 = 3(-1)^2 - 5(-1) + c$$

$$c = -5$$

$$\frac{dy}{dx} = 3x^2 - 5x - 5$$

$$3x^2 - 5x - 5 = 0$$

$$x = \frac{5 \pm \sqrt{85}}{6}$$

(b) Find the equation of the curve. [2]

$$y = x^3 - \frac{5}{2}x^2 - 5x + c$$

At point (1, 5),

$$5 = (-1)^3 - \frac{5}{2}(-1)^2 - 5(-1) + c$$

$$c = \frac{7}{2}$$

$$y = x^3 - \frac{5}{2}x^2 - 5x + \frac{7}{2}$$

9 Coffee is dripped at a constant rate of $2\pi\ cm^3/s$ into an empty cylindrical cup of radius 4 cm.

The volume, $V\ cm^3$, of coffee in the cup is given by $V = \pi x^2(10 - x)$ where x is the depth, in cm, of the coffee.

Find

(i) The time taken for the depth of the coffee to reach 3 cm, [3]

$x = 3$,

$V = \pi 3^2(10 - 3) = 63\pi$

Time = $\frac{volume}{rate}$

Time = $\frac{63\pi}{2\pi} = 31.5$ s

(ii) The rate of change of the depth of the coffee at this time. [4]

$V = \pi x^2(10 - x)$

$V = 10\pi x^2 - \pi x^3$

$\frac{dV}{dx} = 20\pi x - 3\pi x^2$

When $x = 3$, $\frac{dV}{dx} = 33\pi$

$\frac{dV}{dt} = 2\pi$

$\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt}$

$\frac{dx}{dt} = \frac{1}{33\pi} \times 2\pi$

$\frac{dx}{dt} = \frac{2}{33}$

Rate of change = $\frac{2}{33} cm/s$

10 **(i)** Write down and simplify the first three terms in the expansion, in ascending powers of x, of $(2-3x)^5$. [3]

First 3 terms of $$(2-3x)^5 = 2^5 + \binom{5}{1}2^4(-3x) + \binom{5}{2}2^3(-3x)^2$$

$$(2-3x)^5 = 32 - 240x + 720x^2$$

(b) Given that the expansion of $(p+qx)(2-3x)^5$ up to the term in x^2 is $8 + rx + 1680x^2$, find the value of p, or q and of r. [4]

$$(p+qx)(2-3x)^5 = (p+qx)\left(32 - 240x + 720x^2\right)$$
$$(p+qx)(2-3x)^5 = 32p - 240px + 720px^2 + 32qx - 240qx^2 + \ldots$$

Comparing

$$32p = 8$$
$$p = \frac{1}{4}$$
$$720p - 240q = 1680$$
$$720\left(\frac{1}{4}\right) - 240q = 1680$$
$$-240q = 1500$$
$$q = -\frac{25}{4}$$

$$32q - 240p = r$$
$$r = 32\left(-\frac{25}{4}\right) - 240\left(\frac{1}{4}\right) = -260$$

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11 **(a)** Solve the equation $2+\log_3(3x-1)=\log_3(2x^2-5)$. [4]

$2=\log_3(2x^2-5)-\log_3(3x-1)$

$\log_3\dfrac{2x^2-5}{3x-1}=2$

$\dfrac{2x^2-5}{3x-1}=3^2$

$\dfrac{2x^2-5}{3x-1}=9$

$2x^2-5=27x-9$

$2x^2-27x+4=0$

$x=\dfrac{27\pm\sqrt{27^2-4(2)(4)}}{2(2)}$

$x=\dfrac{27\pm\sqrt{697}}{4}$

$x=13.4$ (3SF) or $x=0.150$ (Rejected)

(b) **(i)** Show that $\log_2 x+\log_8 x=\dfrac{4\lg x}{3\lg 2}$. [3]

$\log_2 x+\log_8 x=\dfrac{\lg x}{\lg 2}+\dfrac{\lg x}{\lg 8}$

$\log_2 x+\log_8 x=\dfrac{\lg x}{\lg 2}+\dfrac{\lg x}{3\lg 2}$

$\log_2 x+\log_8 x=\dfrac{3\lg x+\lg x}{3\lg 2}=\dfrac{4\lg x}{3\lg 2}$

(ii) **Hence** solve the equation $\log_2 x+\log_8 x=5$. [2]

$\dfrac{4\lg x}{3\lg 2}=5$

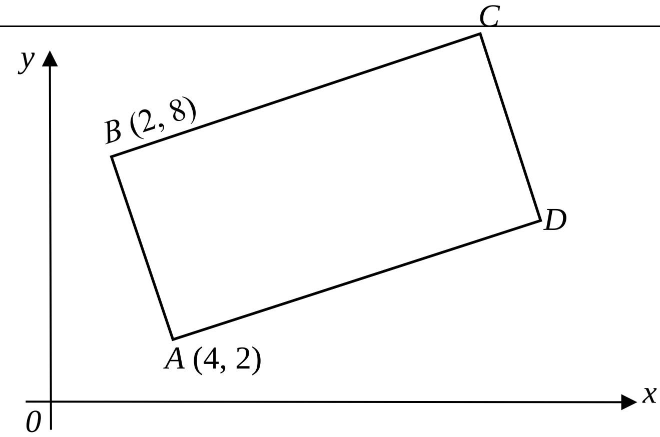
$4\lg x=15\lg 2$

$\lg x=\dfrac{15\lg 2}{4}$

$x=10^{\frac{15\lg 2}{4}}$

$x=13.5$ (3SF)

12



The diagram shows a rectangle *ABCD* with vertices *A* (4, 2) and *B* (2, 8).

(i) Find the equation of *BC*, [3]

Gradient of *AB* = $\frac{8-2}{2-4} = -3$

Gradient of *BC* = $\frac{1}{3}$

$8 = \frac{1}{3}(2) + c$

$c = \frac{22}{3}$

$y = \frac{1}{3}x + \frac{22}{3}$

(ii) Given that the line $y = x - 2$ passes through point *C*, find the coordinates of [2]

$x - 2 = \frac{1}{3}x + \frac{22}{3}$

$x = 14$

$y = 12$

C = (14, 12)

(iii) Find the coordinates of the midpoint of *AC*. [1]

Midpoint of *AC* = $\left(\frac{14+4}{2}, \frac{12+2}{2}\right) = (9, 7)$

(iv) Hence, find the coordinates of *D*. [1]

$\left(\frac{x+2}{2}, \frac{y+8}{2}\right) = (9, 7)$

D = (16, 6)

(v) Find the area of the rectangle *ABCD*. [2]

Area = $\frac{1}{2}\begin{vmatrix} 2 & 4 & 16 & 14 & 2 \\ 8 & 2 & 6 & 12 & 8 \end{vmatrix}$

Area = 80 square units

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13 **(i)** Express $\frac{8x^2-9x+1}{x(2x-1)^2}$ in partial fractions.

$$\frac{8x^2-9x+1}{x(2x-1)^2}=\frac{A}{x}+\frac{B}{2x-1}+\frac{C}{(2x-1)^2}$$

$$8x^2-9x+1=A(2x-1)^2+Bx(2x-1)+cx$$

Let $x=\frac{1}{2}$ $8\left(\frac{1}{2}\right)^2-9\left(\frac{1}{2}\right)+1=\frac{1}{2}c$ $c=-3$	Let x = 0 A = 1	Let x = 1 $8-9+1=1+B-3$ $B=2$

$$\frac{8x^2-9x+1}{x(2x-1)^2}=\frac{1}{x}+\frac{2}{2x-1}-\frac{3}{(2x-1)^2}$$

(ii) **Hence** find $\int_1^2 \frac{8x^2-9x+1}{x(2x-1)^2}\,\mathrm{d}x$ in the form $\ln a-b$ where a and b are integers. [6]

$$\int_1^2 \frac{8x^2-9x+1}{x(2x-1)^2}\,\mathrm{d}x=\int_1^2\left[\frac{1}{x}+\frac{2}{2x-1}-\frac{3}{(2x-1)^2}\right]\mathrm{d}x$$

$$\int_1^2 \frac{8x^2-9x+1}{x(2x-1)^2}\,\mathrm{d}x=\int_1^2\left[\frac{1}{x}+\frac{2}{2x-1}-\frac{3}{(2x-1)^2}\right]\mathrm{d}x$$

$$\int_1^2 \frac{8x^2-9x+1}{x(2x-1)^2}\,\mathrm{d}x=\left[\ln x+\frac{2\ln(2x-1)}{2}-\frac{3(2x-1)^{-1}}{-1(2)}\right]_1^2$$

$$\int_1^2 \frac{8x^2-9x+1}{x(2x-1)^2}\,\mathrm{d}x=\left[\ln x+\ln(2x-1)+\frac{3}{2(2x-1)}\right]_1^2$$

$$\int_1^2 \frac{8x^2-9x+1}{x(2x-1)^2}\,dx = \left[\ln x + \ln(2x-1) + \frac{3}{2(2x-1)}\right]_1^2$$

$$\int_1^2 \frac{8x^2-9x+1}{x(2x-1)^2}\,dx = \left[\ln 2 + \ln 3 + \frac{1}{2}\right] - \left[\ln 1 + \ln 1 + \frac{3}{2}\right]$$

$$\int_1^2 \frac{8x^2-9x+1}{x(2x-1)^2}\,dx = \left[\ln 6 + \frac{1}{2}\right] - \left[\frac{3}{2}\right]$$

$$= \ln 6 - 1$$

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14 **(a)** **(i)** Express $4x^2-4x-19$ in the form $a(x+b)^2+c$, where a, b and c are constants. [2]

$$4x^2-4x-19$$
$$=4\left(x^2-x-\frac{19}{4}\right)$$
$$=4\left(x^2-x+\left(\frac{-1}{2}\right)^2-\left(\frac{-1}{2}\right)^2-\frac{19}{4}\right)$$
$$=4\left[\left(x-\frac{1}{2}\right)^2-\frac{1}{4}-\frac{19}{4}\right]$$
$$=4\left[\left(x-\frac{1}{2}\right)^2-\frac{20}{4}\right]$$
$$=4\left(x-\frac{1}{2}\right)^2-20$$

(ii) Find the coordinates of the turning point on the graph $y=4x^2-4x-19$. [1]

$$\left(\frac{1}{2},-20\right)$$

(iii) Find the exact values of the x intercepts. [3]

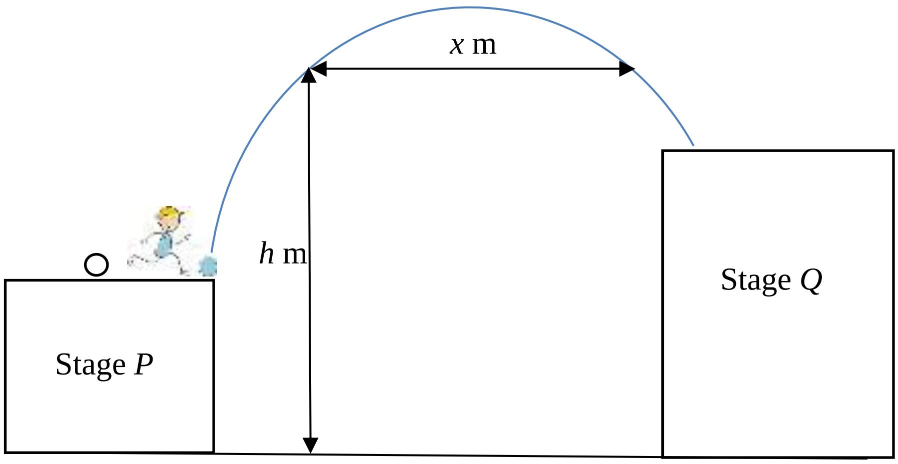
$$4\left(x-\frac{1}{2}\right)^2-20=0$$

$$\left(x-\frac{1}{2}\right)^2=5$$

$$\left(x-\frac{1}{2}\right)=\pm\sqrt{5}$$

$$x=\frac{1}{2}\pm\sqrt{5}$$

(b) A boy kicked a ball from the floor of the top of a stage *P* to the top of stage *Q*.



The height, *h* m, of the ball above the ground is represented by the equation

$$h=-x^2+8x+9$$

where *x* m is the horizontal distance travelled by the ball.

(i) Find the height of stage *P*. [1]
$x = 0$
Height = 9 m

(ii) Express $h=-x^2+8x+9$ in the form $h=p-(x+q)^2$ where *p* and *q* are integers. [2]

$$h = -x^2 + 8x + 9$$
$$h = -\left(x^2 - 8x - 9\right)$$
$$h = -\left[x^2 - 8x + \left(\frac{-8}{2}\right)^2 - \left(\frac{-8}{2}\right)^2 - 9\right]$$
$$h = -\left[\left(x-4\right)^2 - 16 - 9\right]$$
$$h = -\left[(x-4)^2 - 25\right]$$
$$h = 25 - (x-4)^2$$

(iii) State the coordinates of the turning point of $h = -x^2 + 8x + 9$. [1]

Turning point = (4, 25)

End of Paper

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