Name:_____ **Class:**_____

JURONG PIONEER JUNIOR COLLEGE

JC1 Year End Examination 2022

MATHEMATICS **9758/01**
Higher 2 **29 Sept 2022**

Paper 1 **3 hours**

Additional materials: Answer Paper
Cover Page
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and civics class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given by [ ] at the end of each question or part question.

This document consists of **6** printed pages.

[Turn over

1 **(i)** On the same axes, sketch the graphs of $y = \dfrac{1}{x-2}$ and $y = 4x - 2$, indicating clearly the equations of the asymptotes and the coordinates of the axial intercepts. [2]

(ii) Hence, or otherwise, solve the inequality $\dfrac{1}{x-2} \le 4x - 2$. [3]

2 **(a)** Differentiate the following with respect to x.

(i) $\ln\left(9 + 3e^{3x}\right)$, [1]

(ii) $\sin^4 x^2$, [2]

(iii) $\sec 3x \sin^{-1} 2x$. [3]

(b) A curve has equation $y^3 - 4y + x^2 - 9x + 10 = 0$. Find $\dfrac{dy}{dx}$ in terms of x and y. [2]

3 By using the substitution $u = 1 + t^3$, find the exact value of $\displaystyle\int_0^2 \frac{t^5}{\left(1+t^3\right)^3}\,dt$ without using a calculator. [5]

4 The curve C has equation $y = \dfrac{x^2 + kx + 4}{x + 1}$, where k is a constant.

(i) Find the range of values of k if C has two distinct stationary points. [3]

For the rest of this question, use $k = 4$.

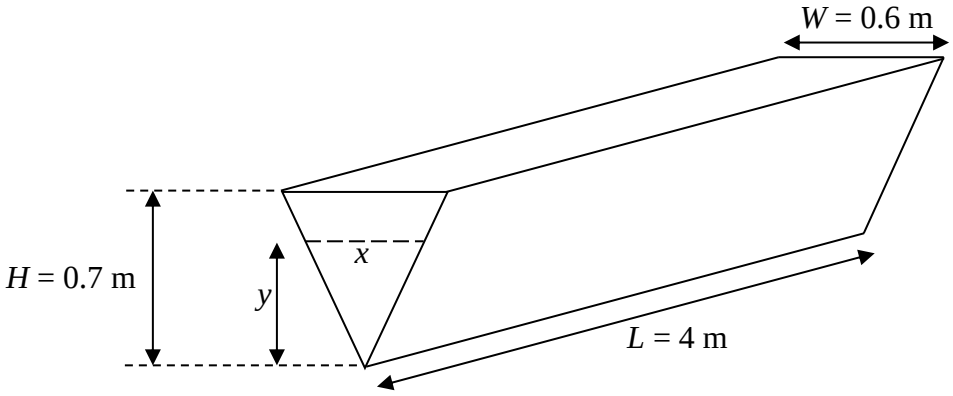
(ii) Sketch the graph of C, indicating clearly the equations of any asymptotes and the coordinates of any turning points and axial-intercepts. [3]

(iii) Using your sketch in **(ii)**, find the range of values of m, where m is a positive real number for which the equation

$$x^2 + kx + 4 - (mx + 3)(x + 1) = 0$$

has no real roots. [2]

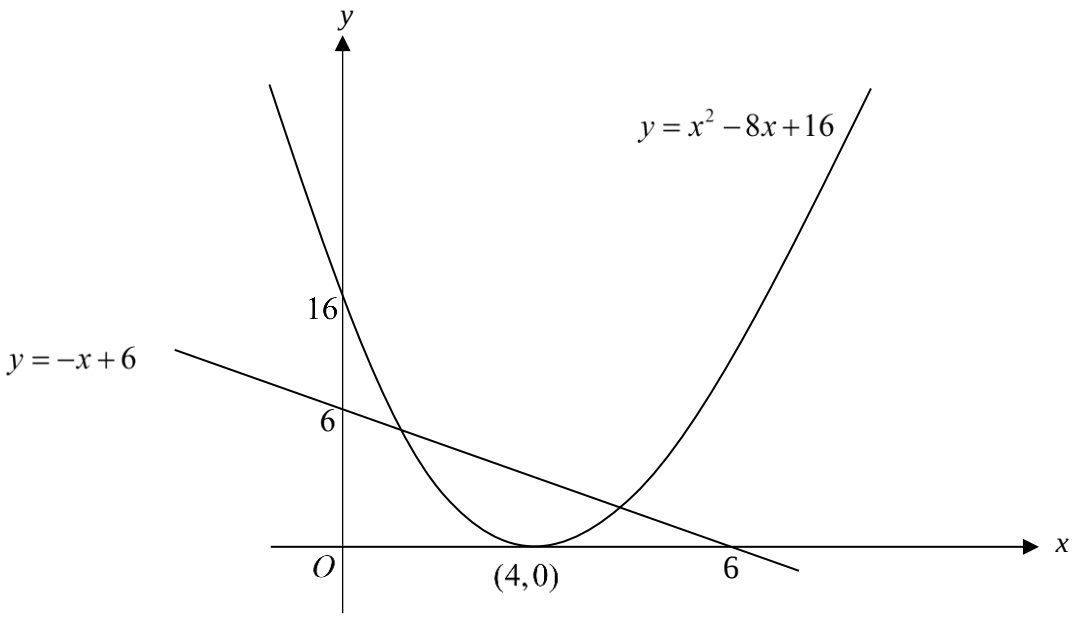
5 The diagram shows a V-shaped tank with dimensions $L = 4$ m, $W = 0.6$ m and $H = 0.7$ m. The tank is initially empty. Water is pumped into the tank at a rate of 0.0025 m^3/s. At any instant from the start of water flowing into the tank, the water in the tank has a depth of y m and a surface width of x m.



(i) Find the rate of change of the water depth when $y = 0.4$ m, leaving your answer to 4 decimal places. [4]

(ii) Find, to the nearest second, the time taken to completely fill up the tank from the instant when $y = 0.4$ m. [2]

6 The diagram shows the curve C and the line l with equations $y = x^2 - 8x + 16$ and $y = -x + 6$ respectively.



(i) Find the intersection points of C and l. [1]

(ii) The region R is bounded by C, l, the x-axis and the line $x = a$, where $5 < a < 6$. Find the area of region R, leaving your answer in terms of a. [4]

7 Find

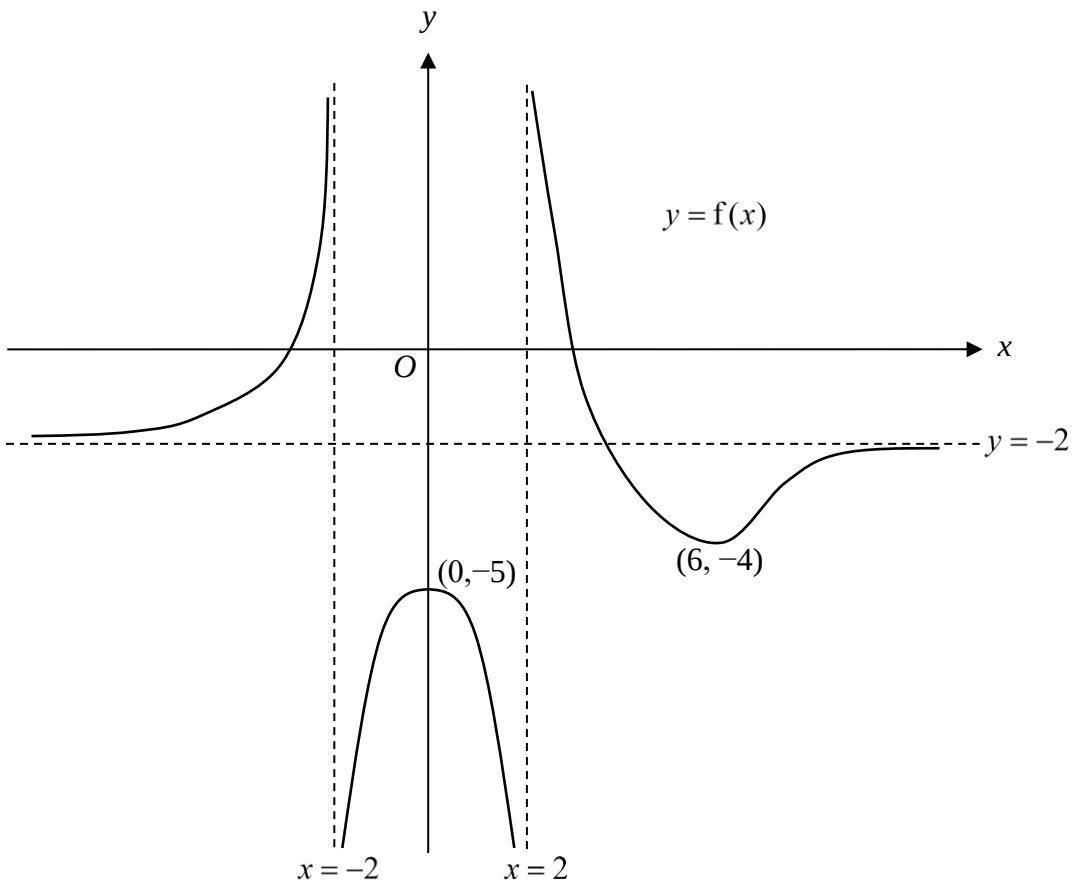
(a) $\int \frac{10e^x}{5 - 2e^x} dx$, [2]

(b) $\int \frac{x}{\sqrt{1 + 8x^2}} dx$, [2]

(c) $\int x(\ln x)^2 dx$. [5]

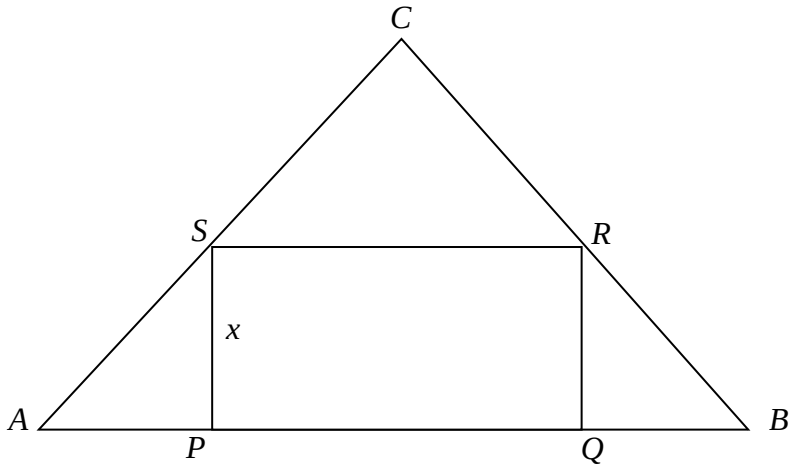
8 **(a)** By expressing the equation of the curve $y = \frac{12x + 11}{2x + 1}$ in the form $y = A + \frac{B}{2x + 1}$, where A and B are constants, describe a sequence of three transformations which maps the graph of $y = \frac{1}{2x - 3}$ onto the graph of $y = \frac{12x + 11}{2x + 1}$. [4]

(b) The diagram shows the graph of $y = f(x)$. The curve has a maximum point at $(0, -5)$ and a minimum point at $(6, -4)$. The equations of the asymptotes of the curve are $x = -2$, $x = 2$ and $y = -2$.



Sketch the graph of $y = \mathrm{f}\left(-x+2\right)+4$, indicating clearly the equations of the asymptotes and the coordinates of the axial intercepts and turning points. [3]

9



In the isosceles triangle *ABC*, $AC = BC$, $AB = 20$ cm and $\angle BAC = 30^\circ$. A rectangle *PQRS* is inscribed in *ABC* with points *P* and *Q* on *AB*, point *R* on *BC* and point *S* on *AC* as shown in the diagram.

Taking *PS* to be *x* cm, show that the area of *PQRS* may be expressed as $20x - 2\sqrt{3}x^2$. [2]

Hence, as *x* varies, find the exact values of *PS* and *PQ* such that the area of *PQRS* is a maximum, and find the corresponding area of *PQRS*. [4]

10 Referred to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, where $\mathbf{a}$ and $\mathbf{b}$ are non-zero and non-parallel vectors. The point C lies on OA such that $OC : CA = 2 : 1$. The point D lies on OB produced such that $OD : BD = 4 : 1$.

(i) Find the position vectors $\overrightarrow{OC}$ and $\overrightarrow{OD}$, giving your answers in terms of $\mathbf{a}$ and $\mathbf{b}$. [2]

(ii) The lines BC and AD meet at the point E. Show that E has position vector $4\mathbf{b} - 2\mathbf{a}$. [4]

(iii) Show that the area of triangle CDE can be written as $k\left|\mathbf{a}\times\mathbf{b}\right|$, where k is a constant to be found. [4]

11 A curve C has parametric equations

$$x = 3t^2, \qquad y = 6t^3, \qquad \text{where } t \text{ is a real parameter.}$$

(i) Sketch C. [1]

(ii) Find the exact coordinates of the point P on C where the tangent is parallel to the line $y = 4 - 2x$. [3]

(iii) Show that the equation of the tangent to C at the point $Q\left(\frac{4}{3}, \frac{16}{9}\right)$ is $9y = 18x - 8$. [2]

(iv) The tangent at Q cuts the x-axis at the point R. Find the area of triangle PQR. [2]

(v) The tangent at Q cuts C again at the point S. Find the coordinates of S. [3]

12 The function f is defined by $\mathrm{f} : x \mapsto \frac{2x}{x^2 - 1}, x \in \mathbb{R}, x > 1$.

(i) Sketch the graph of f and show that f has an inverse. [2]

(ii) Find $\mathrm{f}^{-1}(x)$ and state its domain. [4]

(iii) Write down the equation of the line in which the graph of f must be reflected in order to obtain the graph of f^{-1} and hence find the exact solution of the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$. [3]

13 The planes p_1 and p_2 have equations $\mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + s\begin{pmatrix} 3 \\ -1 \\ 7 \end{pmatrix} + t\begin{pmatrix} 1 \\ -3 \\ 12 \end{pmatrix}$ and $x + 3y = 1$ respectively, where s and t are parameters.

(i) Find the line of intersection of p_1 and p_2. [3]

(ii) Find the acute angle between p_1 and p_2. [2]

The point A has position vector $5\mathbf{i} - 4\mathbf{j} + 15\mathbf{k}$ and the point B has position vector $\mathbf{i} - 2\mathbf{k}$.

(iii) Find the foot of perpendicular from A to p_2. [3]

(iv) Find the length of projection of AB onto p_2. [3]