NAME:		( )	CLASS:	

YISHUN TOWN SECONDARY SCHOOL

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PRELIMINARY EXAMINATION 2023
SECONDARY 4 NORMAL ACADEMIC
ADDITIONAL MATHEMATICS PAPER 1 (4051/01)

DATE : 14 August 2023 **DAY : Monday**

DURATION : 1 hour 45 minutes **MARKS : 70**

READ THESE INSTRUCTIONS FIRST

Do not turn over the cover page until you are told to do so.
Write your name, class and class index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 70.

	MARKS	
	OBTAINED	FULL
1		4
2		5
3		4
4		5
5		4
6		4
7		4
8		4
9		6
10		3
11		6
12		4
13		8
14		9
TOTAL		70

This question paper consists of **17** printed pages including this cover page and 1 blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

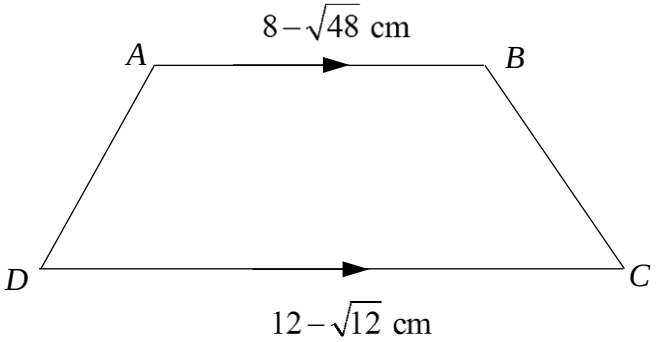
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Express $\frac{2+x^2}{(x-2)^2(x+4)}$ in partial fractions. [4]

2 Do not use a calculator in answering this question.



ABCD is a trapezium where *AB* is parallel to *DC*.
The area of the trapezium is $12+11\sqrt{3}$ cm^2.
$AB = 8-\sqrt{48}$ cm and $DC = 12-\sqrt{12}$ cm.

Find the perpendicular distance between *AB* and *DC*, leaving your answer in the form $a+b\sqrt{3}$, where *a* and *b* are integers. [5]

3 Solve the simultaneous equations

$$x^2 - xy + y^2 - 7 = 0,$$
$$y - 3x + 7 = 0.$$ [4]

4 The function $f(x)$ is defined by $f(x) = 3x^3 + hx^2 + kx - 4$ for all real x. Given that $3x-1$ is a factor of $f(x)$ and that when $f(x)$ is divided by $x+1$, the remainder is -4, find the value of each of the constants h and k. [5]

5 Find the set of values of the constant k for which the curve $y = x^2 + 12x - 4k + 41$ lies completely above the line $y = kx + \frac{9}{4}k$. [4]

6 **(a)** Factorise $27x^3 + 125$. [2]

(b) Explain why $x = -\frac{5}{3}$ is the only real root of the equation $27x^3 + 125 = 0$. [2]

7 **(a)** Show that $\dfrac{\sin(A+B)+\sin(A-B)}{\cos(A+B)+\cos(A-B)} = \tan A$. [2]

(b) Hence, find the exact value of $\dfrac{\sin 105^o + \sin 15^o}{\cos 105^o + \cos 15^o}$. [2]

8 Given that $\cos ecA = \frac{p}{2}$, where A is obtuse, express each of the following in terms of p.

(a) $\cot A$, [2]

(b) $\cos 2A$. [2]

9 **(a)** Express, in terms of π, the principal value of

(i) $\sin^{-1}\left(-\frac{1}{2}\right)$, [1]

(ii) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$. [1]

(b) State the set of values of x for which $\cos^{-1}\left(-\frac{1}{2}x\right)$ is defined. [1]

(c) Solve $\sin(45^o - 2x) = 0.75$ for $0^o \le x \le 180^o$. [3]

10 Without using a calculator and showing all your working, find the value of

$$(\sin 45^o + \cos 30^o)(\sin 60^o - \cos 45^o).$$ [3]

11 **(a)** Given that $\mathrm{P}(x) = (x-2)\sqrt{2x+1}$, find $\frac{\mathrm{dP}}{\mathrm{d}x}$, giving your answer in the form $\frac{ax-1}{\sqrt{2x+1}}$ where a is an integer. [3]

(b) Hence find the value of $\int_0^4 \frac{6x-2}{\sqrt{2x+1}}\,\mathrm{d}x$. [3]

12 The volume V cm^3, of liquid contained in a tank is given by $V = \frac{2}{3}\pi h^3$, where h cm is the depth of the liquid. If liquid is pumped into the tank at a constant rate of 30 cm^3/s, determine the rate of increase of h when $h =$ 2.5 cm, leaving your answer in terms of π. [4]

13 **(a)** Write $\sqrt{3}\cos\theta+\sin\theta$ in the form $R\cos(\theta-\alpha)$ where $R>0$ and $0<\alpha<\frac{\pi}{2}$ radians. [3]

(b) Find the minimum value of $\sqrt{3}\cos\theta+\sin\theta$ and the corresponding value of θ. [2]

(c) Solve the equation $\sqrt{3}\cos\theta + \sin\theta = -1$ for $0 < \theta < 2\pi$. [3]

14 The points $A(2,1)$ and $B(11,-2)$ lie on a circle.

(a) Find the equation of the perpendicular bisector of the chord AB. [4]

(b) The line with equation $y = \frac{4}{3}x - 10$ is a normal to the circle.
Find the equation of the circle. [4]

(c) Find the coordinates of the point on the circle which is at the greatest distance from the x-axis. [1]

Answer Key

1	$\frac{1}{2(x-2)}+\frac{1}{(x-2)^2}+\frac{1}{2(x+4)}$	9c	$x=136.8^o, 178.2^o$
2	$3+2\sqrt{3}$	10	$\frac{1}{4}$
3	$x=3$ or $x=2$ $y=2$ or $y=-1$	11a	$\frac{3x-1}{\sqrt{2x+1}}$
4	$k=8$ $h=11$	11b	16
5	$-5<k<4$	12	$\frac{12}{5\pi}$
6a	$(3x+5)(9x^2-15x+25)$	13a	$2\sin\theta+2\cos\left(\theta+\frac{\pi}{6}\right)=2\cos\left(\theta-\frac{\pi}{6}\right)$
6b	Consider $9x^2-15x+25$ b^2-4ac $=(-15)^2-4(9)(25)$ $=-675$ Since $b^2-4ac<0$, therefore $9x^2-15x+25$ cannot be factorised. Thus, $x=-\frac{5}{3}$	13b	Minimum value $=-2$ Occurs when $\theta=\pi+\frac{\pi}{6}=\frac{7\pi}{6}$
7a	$\frac{\sin(A+B)+\sin(A-B)}{\cos(A+B)+\cos(A-B)}$ $=\frac{\sin A\cos B+\cos A\sin B+\sin A\cos B-\cos A\sin B}{\cos A\cos B-\sin A\sin B+\cos A\cos B+\sin A\sin B}$ $=\frac{2\sin A\cos B}{2\cos A\cos B}$ $=\tan A$	13c	$\theta=\frac{5\pi}{6},\frac{3\pi}{2}$
7b	$\sqrt{3}$	14a	$y=3x-20$
8a	$-\frac{\sqrt{p^2-4}}{2}$	14b	$(x-6)^2+(y+2)^2=25$
8b	$\frac{p^2-8}{p^2}$	14c	point $=(6,-7)$
9ai	$-\frac{\pi}{6}$		
9aii	$\frac{3\pi}{4}$		
9b	$-2\le x\le 2$		