O	JURONG SECONDARY SCHOOL 2023 GRADUATION EXAMINATION SECONDARY 4 EXPRESS/ SECONDARY 5 NORMAL ACADEMIC

CANDIDATE NAME	

CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS **4049/02**

PAPER 2 **24 August 2023**
Candidates answer on the Question Paper. **2 hours 15 minutes**
Additional Materials : Writing Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use
90

This document consists of **16** printed pages including this page.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer, and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 At the beginning of a virus outbreak, the number of cases of infected people increased with time. After t days, the number of recorded cases was N. It was observed that N can be modelled by the equation $N = 1200e^{kt}$.

(a) Write down the initial number of cases recorded. [1]

The number of cases recorded after 6 days rose to 4800.

(b) Estimate the number of cases recorded after 10 days. [4]

A pandemic is declared if the number reaches 20 000 cases.

(c) Assuming the trend continues, estimate after how many days will it take for a pandemic to be declared. [2]

2 The expression $x^3 + px^2 + qx + r$ is divisible by both x and $x - 2$ and it leaves a remainder of 8 when divided by $x + 2$.

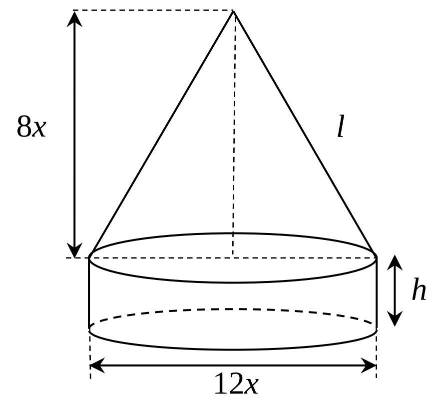
(a) Find the values of p, q and r. [4]

(b) Hence, find the remainder when it is divided by $x^2 + 2x - 3$. [2]

3 **(a)** Show that $2\cos\theta + \cot\theta - 1 = 2\cos\theta\cot\theta$ can be written as $(2\cos\theta - 1)(\sin\theta - \cos\theta) = 0$. [3]

(b) Hence, solve the equation $2\cos 2x + \cot 2x - 1 = 2\cos 2x\cot 2x$ for $0° < x < 180°$. [4]

4 The diagram below shows a mould made of a cylinder and a right circular cone. The diameter of the cylinder is $12x$ cm and its height is h cm. The vertical height of the cone is $8x$ cm.



(a) Find an expression, in terms of x, for the slant height l of the cone. [1]

(b) Given that the entire mould is covered with a plastic sheet whose area is 240π cm^2, express h in terms of x. [2]

(c) Show that the volume, V cm^3, of the mould is given by $V = 720\pi x - 192\pi x^3$. [3]

(d) Hence, find the value of x for which the volume has a stationary value and determine whether this value for the volume is a maximum or minimum. [4]

5 **(a)** Without using a calculator, show that $\cos 15° = \dfrac{\sqrt{2}+\sqrt{6}}{4}$. [3]

(b) Hence, state the value of $\cos(-15°)$. [1]

(c) Using your answer from **part (a)**, find the exact value of $\sec(15°)$ [3]

6 **(a)** Solve $6^x = \dfrac{10}{3} - 6^{-x}$. [4]

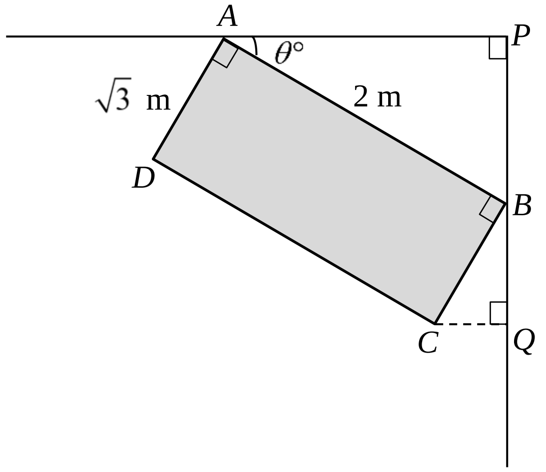
(b) Sketch the graph of $y = \ln x$, showing any points of intersection with the axes. [2]

(c) To solve $e^{1-2x} = x^3$, a straight line can be drawn on the same axes as the graph in **part (b)**.

(i) Determine the equation of the straight line to be drawn. [2]

(ii) Hence, state the number of solutions for $e^{1-2x} = x^3$. [1]

7 The diagram below shows a rectangular table, *ABCD* placed at the corner of a classroom. It is given that the table has length *AB* = 2 m and width *AD* = $\sqrt{3}$ m. It is also given that $\angle APB = 90°$ and $\angle PAB = \theta°$.



(a) Show that the length of *PQ*, *L*, can be expressed as $L = 2\sin\theta + \sqrt{3}\cos\theta$. [2]

(b) Express *L* in the form $R\sin(\theta + \alpha)$ where $0° < \alpha < 90°$ and $R > 0$. [3]

(c) Find the value of θ for *L* = 2.3 m. [2]

(d) Find the maximum value of *L* and the corresponding value of θ. [2]

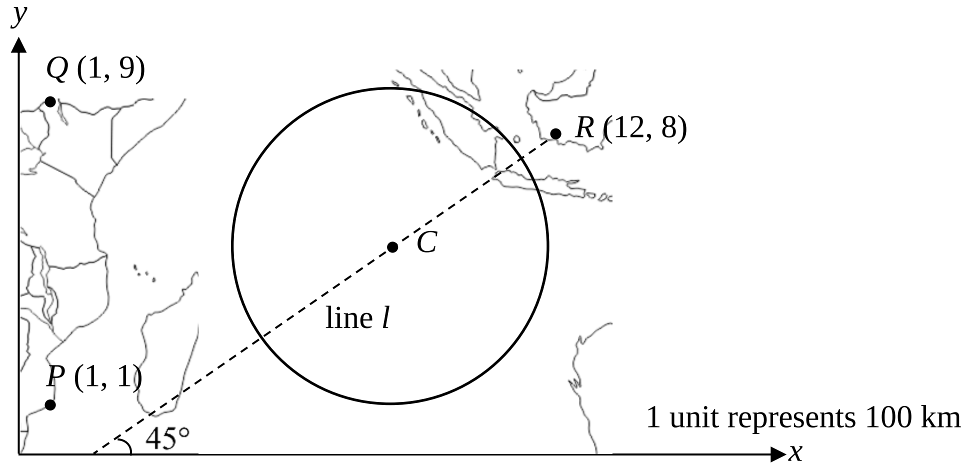
8 A curve has the equation $y = (3 - x)\sqrt{2x+5}$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{ax+b}{\sqrt{2x+5}}$, where a and b are constants to be determined. [3]

(b) A point (x, y) moves along the curve. When the y-coordinate is increasing at the same rate as the x-coordinate, find the x-coordinate.
Explain why you need to reject the positive value. [4]

(c) Using your answer in **(a)**, evaluate $\int_{-2}^{2} \dfrac{-3x}{\sqrt{2x+5}}\, dx$. [4]

9 The map below shows part of the Indian Ocean.
Geological stations $P(1, 1)$, $Q(1, 9)$ and $R(12, 8)$ detected an earthquake and a geologist is attempting to locate the epicentre, C of the earthquake.



Instruments at P and Q detected the earthquake at exactly the same time, indicating that the epicentre, C is equidistant from P and Q. Instrument at R detected it in the direction indicated by the line l, which makes an angle of $45°$ with the positive x-axis.

(a) Show that the line l can be represented by the equation $y = x - 4$. [2]

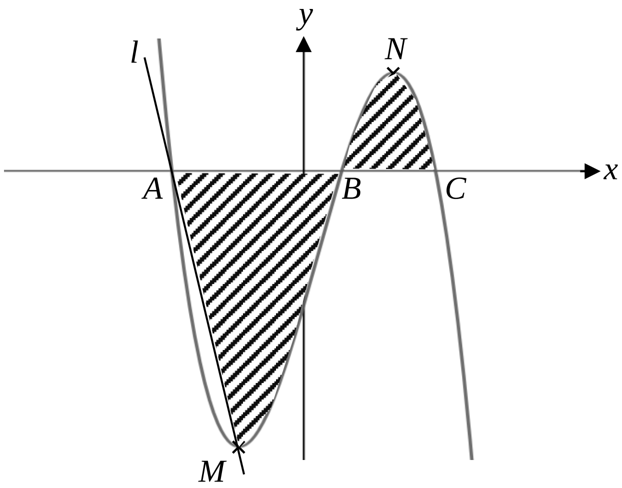
(b) Find the coordinates of C. [3]

(c) It is given that the earthquake detected can be felt at places as far as 450 km from the epicentre.
Find the equation of the circle that represents the places affected. [2]

(d) Hence, or otherwise, determine if geological station R is inside the circle. [2]

(e) Explain why it is not possible to draw such a circle that passes through all three geological stations P, Q and R, where PR is the diameter.
Support your answer with mathematical calculations. [3]

10 The diagram below shows part of the curve $y = (2x - 1)(3 - x^2)$. The curve has a minimum point at M and a maximum point at N. The curve intersects the x-axis at A, B and C respectively. The line l pass through A and M.



(a) Find the coordinates of A, B and C. [3]

(b) Find the coordinates of M. (You are not required to prove that it is the minimum point.) [3]

(c) Hence, find the area of the shaded region. [6]

--END OF PAPER--