# Chapter 18 ALTERNATING CURRENTS

*prepared by Mr fRancis Chong*

## CONTENT

- Characteristics of alternating currents
- Root Mean Square value
- The transformer
- Transmission of electrical energy
- Rectification with a diode

## LEARNING OUTCOMES

Candidates should be able to:

(a) show an understanding of and use the terms period, frequency, peak value and root-mean-square (r.m.s.) value as applied to an alternating current or voltage.

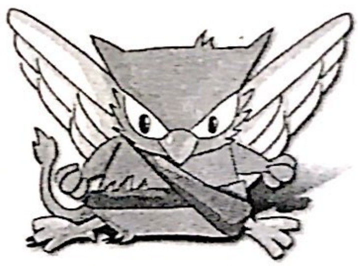
(b) deduce that the mean power in a resistive load is half the maximum power for a sinusoidal alternating current.

(c) represent an alternating current or an alternating voltage by an equation of the form $x = x_0 \sin \omega t$.

(d) distinguish between r.m.s. and peak values and recall and solve problems using the relationship $I_{rms} = \frac{I_0}{\sqrt{2}}$ for the sinusoidal case.

(e) show an understanding of the principle of operation of a simple iron-core transformer and recall and solve problems using $\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{I_p}{I_s}$ for an ideal transformer.

(f) explain the use of a single diode for the half-wave rectification of an alternating current.



Ace gryphon

## Introduction

**Why alternating current**

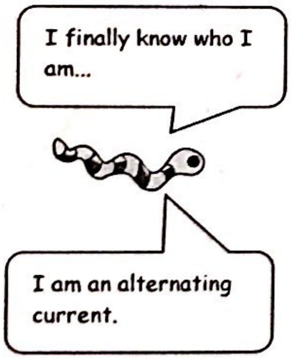
When Thomas Edison formed the Edison Electric Light Company in 1878, he made a bold claim that he "will make electricity so cheap that only the rich will burn candles". He went on to set up the Edison Electric Illuminating Company in 1880 which generated and supplied direct current (d.c.) and it earned him great success. Yet in less than a decade, d.c. was quickly replaced by alternating current (a.c.) which was largely brought into the scene by George Westinghouse.

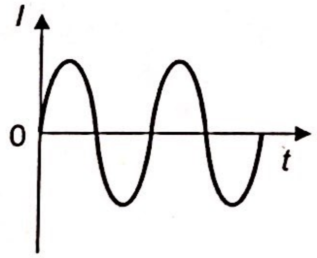
In applications where electricity is used to dissipate energy in the form of heat, a.c. does not hold any practical advantage over d.c. In fact, most electric appliances require d.c. to operate! Now you are probably starting to wonder – why the need of alternating current then? Why is electricity supplied in the form of a.c. in the first place? Instead of answering these questions, I will leave you to discover the answers as you go through this topic. Have fun!

## 18.1 Characteristics of alternating currents

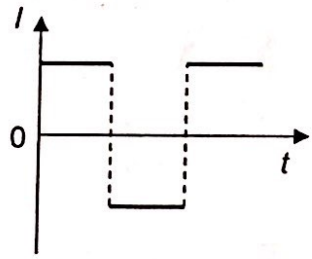
**Characteristics of alternating currents**

An alternating current varies periodically with time in ~~magnitude and~~ direction. Examples of a.c. waveforms:

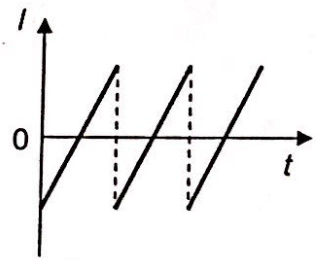


Sinusoidal a.c.



Rectangular a.c.



Saw-tooth a.c.

| Term | Definition |
|---|---|
| **Period, $T$** | of an alternating current is the time taken for one complete cycle. |
| **Frequency, $f$** | of an alternating current is the number of complete cycles per unit time. |
| **Peak current, $I_0$** | of an alternating current is the amplitude of the current. |

varying direct current

**Sinusoidal a.c.**

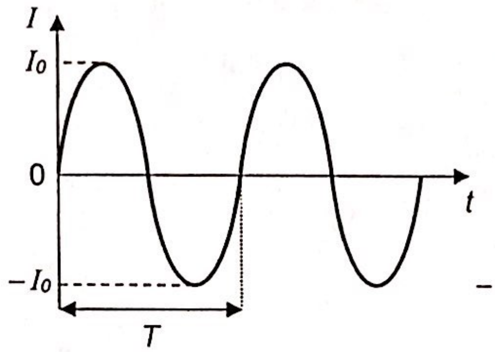
The sinusoidal alternating current and alternating voltage can be represented by the equations

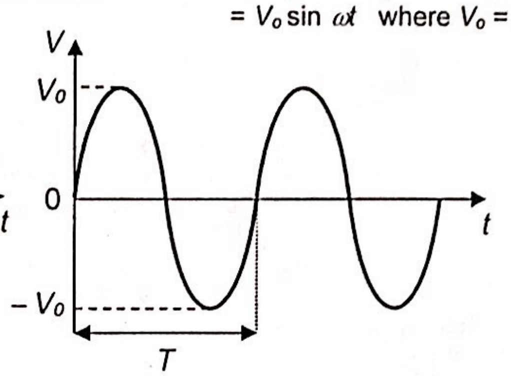
*Formula*

$$\boxed{\begin{aligned} I &= I_o \sin \omega t \\ V &= V_o \sin \omega t \end{aligned}}$$

or cos, depending on starting condition.

since $V = RI$
$= R\,(I_o \sin \omega t)$
$= V_o \sin \omega t$ where $V_o = I_oR$




## 18.2 Root Mean Square value

**Root mean square (r.m.s.) value**

In a.c. circuits, the direction of current changes periodically with time and its magnitude may also vary. A problem arises as to which value of current (or voltage) to use to compute electrical quantities such as the average power dissipated in a resistor. For instance, the average value of a sinusoidal alternating current is zero as the current is maintained in one direction for the same amount of time and at the same magnitudes as it is maintained in the opposite direction.

A resistor heats up when an alternating current is passed through it, much the same way as when a direct current is used. As free electrons move through the resistor when a potential difference is applied across it, they collide against fixed ions in the resistor and transfer some of their energies to these ions, resulting in an increase in the internal energy and correspondingly, the temperature of the resistor. Although the increase in temperature depends on the magnitude of the current, it is independent of the direction of current.

This indicates that some power is dissipated in the resistor. Thus the effective value of the sinusoidal alternating current cannot be zero. More often, root-mean-square or r.m.s values, rather than the average values.

To find r.m.s. current, start the procedure from the right of the letters

**r m s**

i.e. Step 1: We <u>s</u>quare the instantaneous current $I$

Step 2: Find the <u>m</u>ean (or average) value of $I^2$

(area under $I^2\!-\!t$ graph over one period divided by the period)

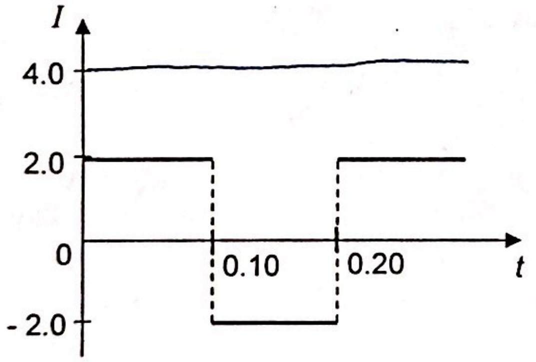
Step 3: Take the square <u>r</u>oot of that mean value.

not in syllabus

Mathematically, $I_{rms} = \sqrt{\dfrac{\int_0^T I^2 dt}{T}} = \sqrt{\langle I^2 \rangle}$

**Example 1** **Calculate the r.m.s. value of the alternating current shown.**

rectangular a.c.




1. Square the current

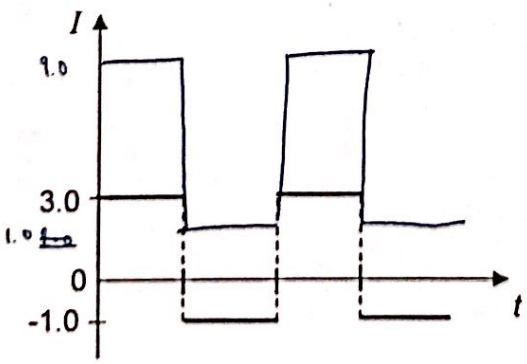
2. Mean = $\dfrac{\text{area under } I^2 - t \text{ graph over a period}}{\text{period}}$

$\dfrac{4.0 \times 0.20}{0.20}$

= 4.0

3. $I_{rms}$ = square root of mean = $\sqrt{4.0} = 2.0$

**Example 2** **Calculate $I_{rms}$ in the following case.**




1. Square the current

2. Mean = $\dfrac{(9.0+1.0)(\frac{T}{2})}{T} = 5.0$

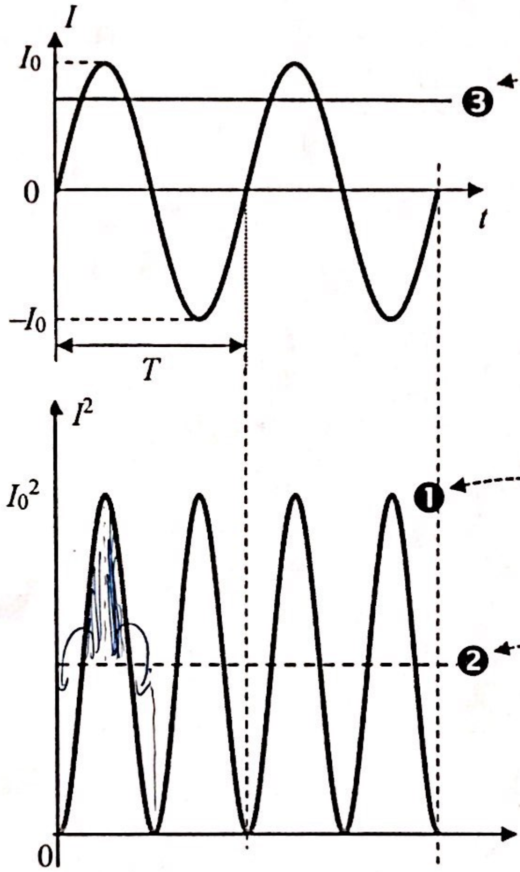
3. $I_{rms}$ = $\sqrt{5} = 2.24$ A



prepared by Mr fRancis Chong

**r.m.s. value of sinusoidal alternating current**

Find $I_{rms}$ in terms of $I_o$ in the following case.




❸ $I_{rms} = \sqrt{\langle I^2 \rangle} = \sqrt{\dfrac{I_0^2}{2}} = \dfrac{I_0}{\sqrt{2}}$

❶ $I^2 = I_0^2 \sin^2 \omega t$

area fits nicely

❷ $\langle I^2 \rangle = \dfrac{I_0^2}{2}$

$\langle I \rangle : \dfrac{\frac{I_0^2}{2} \times T}{T} = \dfrac{I_0^2}{2}$

**Formula**

no need to show derivation

For sinusoidal a.c.

$$I_{rms} = \frac{I_o}{\sqrt{2}}$$

$$V_{rms} = \frac{V_o}{\sqrt{2}}$$

$$\langle P \rangle = I_{rms} V_{rms} = \left(\frac{I_o}{\sqrt{2}}\right)\left(\frac{V_o}{\sqrt{2}}\right) = \frac{P_o}{2}$$

Mathematically, (not in syllabus)

$$I_{rms} = \sqrt{\frac{\int_0^T I^2 dt}{T}} = \sqrt{\frac{\int_0^T I_o^2 (\sin^2 \omega t)\, dt}{T}}$$

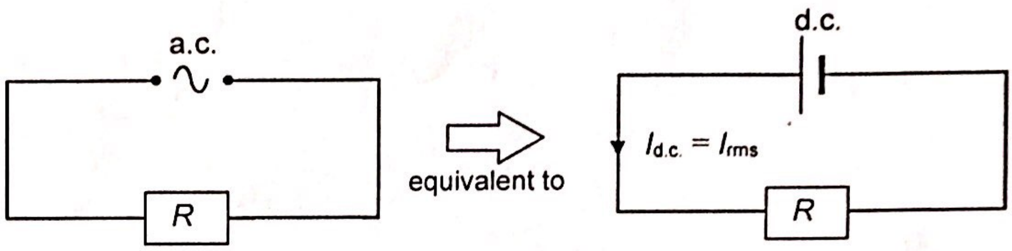
$$= \sqrt{\frac{I_o^2 \int_0^T \frac{1}{2}(1 - \cos 2\omega t)\, dt}{T}} = \sqrt{\frac{I_o^2 \left[\frac{1}{2}t - \frac{1}{2}\sin 2\omega t\right]_0^T}{T}}$$

$$= \sqrt{\frac{I_o^2 \left[\frac{1}{2}T\right]}{T}} = \sqrt{\frac{I_o^2}{2}} = \frac{I_o}{\sqrt{2}}$$

**r.m.s. and mean power**

Most of the time, it is more meaningful for us to know the average or mean power rather than the variation of power in an a.c. circuit.

Consider the two circuits below. The diagram on the left shows a resistor $R$ connected to an a.c. source, while the diagram on the right shows the same resistor connected to a d.c. source.




$$P_R = \langle P_{ac} \rangle = \langle I_{ac}^2 \rangle R \qquad\qquad P_R = \langle P_{dc} \rangle = I_{dc}^2 R$$

Suppose the resistor in both circuits dissipates heat at the same rate, we can then equate the average power dissipated as:

mean power more important than peak power

$$\langle P_{ac} \rangle = \langle P_{dc} \rangle$$
$$\langle I_{ac}^2 \rangle R = I_{dc}^2 R$$
$$\sqrt{\langle I_{ac}^2 \rangle} = I_{dc}$$
$$\Rightarrow \quad I_{rms} = I_{dc}$$

This shows that we can find the average power of an a.c. source by treating it as a d.c. source with $I_{dc} = I_{rms}$. Hence, the average power dissipated in the resistor,

*Formula*

$$\boxed{\langle P_{ac} \rangle = I_{rms}^2 R \text{ or } \frac{V_{rms}^2}{R} \text{ or } I_{rms} V_{rms}}$$

*Definition*

The **root-mean-square** value of the alternating current or voltage is that value of the direct current or voltage that would produce thermal energy at the same rate in a resistor.

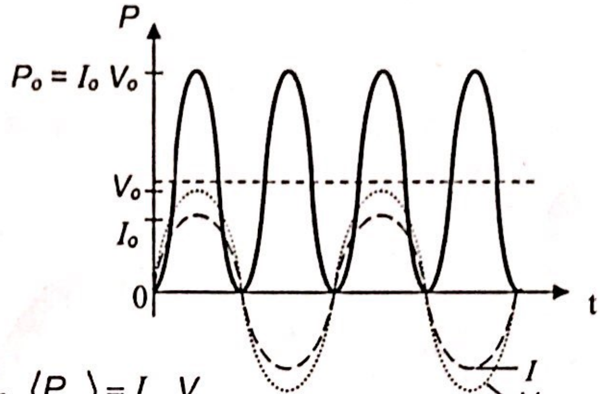
mean power

**Mean power of sinusoidal a.c.**

For sinusoidal a.c.,

$$I = I_o \sin \omega t$$
$$V = V_o \sin \omega t$$

Therefore, instantaneous power $P = IV = I_o V_o \sin^2 \omega t = P_o \sin^2 \omega t$




Mean power, $\langle P_{ac} \rangle = I_{rms} V_{rms}$

$$= \left(\frac{I_o}{\sqrt{2}}\right)\left(\frac{V_o}{\sqrt{2}}\right)$$

$$= \frac{I_o V_o}{2}$$

***Formula***

For sinusoidal a.c. $\boxed{\langle P_{ac} \rangle = \frac{P_o}{2}}$

**NOTE**

It can be seen from the graph that the frequency of the power is twice the frequency of the sinusoidal a.c.

**Example 3**

**When a domestic electric heater is operated from a 240 V a.c. supply, a r.m.s. current of 8.0 A flows. Assume that the heater is purely resistive, calculate**

**(a) its resistance**
**(b) the mean power output**
**(c) the maximum instantaneous power**

**J89/III/6**

(a) *If question does not specify whether the given value of the source is rms or peak value, assume that it is rms value.*

$R = \frac{V_{rms}}{I_{rms}} = \frac{240}{8.0} = 30\,\Omega$

(b) $\langle P \rangle = I_{rms} V_{rms} = (8.0)(240) = 1920\text{ W}$

(c) $\langle P \rangle = \frac{P_o}{2}$

$\Rightarrow P_o = 2\langle P \rangle = (2)(1920) = 3840\text{ W}$

## 18.3 The transformer

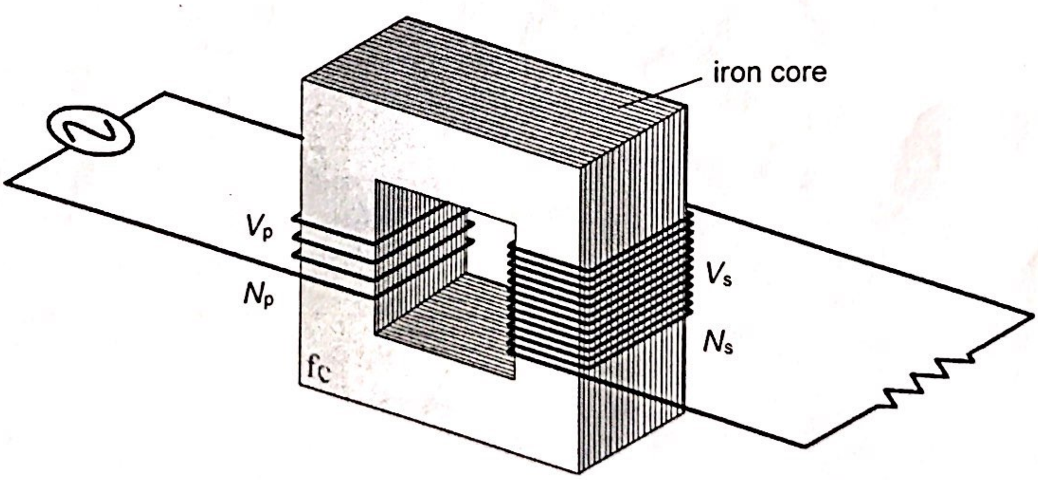
**Transformer**

Function: A device that uses mutual electromagnetic induction to step up or step down an alternating voltage.

Structure: It consists of two coils, a primary and a secondary coil, wound on the same soft iron core which is typically made from 'E' and 'I' shaped laminations.

Principle: When an alternating current flows through the primary coil, it sets up a varying magnetic field in the iron core which links the primary coil with the secondary coil.

In accordance to Faraday's law of electromagnetic induction, the changing magnetic field induces an alternating e.m.f. across each turn of wire in both the primary and secondary coils.




Consider a transformer with $N_p$ number of turns in the primary coil and $N_s$ number of turns in the secondary coil.

In the primary coil

Magnetic flux linkage through the primary coil $= N_p \Phi_p$

E.m.f. induced in the primary coil $V_p = N_p \dfrac{d\Phi_p}{dt}$ -------- (1)

In the secondary coil

Magnetic flux linkage through the secondary coil $= N_s \Phi_s$

E.m.f. induced in the secondary coil $V_s = N_s \dfrac{d\Phi_s}{dt}$ -------- (2)

In an ideal transformer, there is no flux leakage, $\Phi_p = \Phi_s \Rightarrow \frac{d\Phi_p}{dt} = \frac{d\Phi_s}{dt}$

Dividing (2) by (1), $\frac{V_s}{V_p} = \frac{N_s}{N_p}$

Step-up transformer : $N_S > N_P$, $V_S > V_P$
Step-down transformer : $N_S < N_P$, $V_S < V_P$

For an **ideal transformer** (i.e. 100% efficient), by the principle of the conservation of energy,

$$\text{Power output} = \text{Power input}$$

$$V_s I_s = V_p I_p$$

$$\boxed{\frac{V_s}{V_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p}}$$

**NOTE**

An ideal transformer has the following features:
1. Resistances of primary and secondary coils are zero.
2. No flux leakage. Both coils have the same magnetic flux through them.
3. It is 100% efficient i.e. power output = power input.

**Power loss in a transformer**

current flowing = powerloss
cannot eliminate current but make core out of laminate

| Cause of power loss in a transformer | Design features to reduce power loss |
|---|---|
| Heating effect of the current in the copper wires of the coils. $P_{lost} = I^2R = I^2\frac{\rho L}{A}$ | Thick copper wire of low resistance is used, particularly for the coil carrying high current at low voltage. |
| Heating effect due to eddy currents induced in the iron core. ↳ bulk matter | The iron core is made of laminated sheets, cutting across the path of eddy currents. |
| Energy is used in the process of magnetising the iron and reversing this magnetisation every time the current reverses. This heats up the iron core. | The iron core is made of *soft iron, which can be easily magnetised and demagnetised by the magnetic field of the primary coil. |
| Some of the magnetic field lines produced by the primary coil do not link with the secondary coil, reducing the e.m.f. induced in the secondary coil. | The presence of iron core maximises the flux linkage between the primary and secondary coils.<br><br>Flux leakage can also be minimized by having a good design. In the 'E-I' shaped iron core, the secondary coil is wound on top of the primary coil and the iron core forms a closed loop. |

*soft iron means "annealed iron" which has high values of permeability and lower reluctance.


prepared by Mr fRancis Chong

**Example 4**

**An ideal transformer has 550 turns and 30 turns on the primary and secondary coils respectively.**

**(a) What is the maximum output potential difference if $V_p$ is 3.3 kV?**

**(b) What is the maximum primary current required if a maximum current of 11 A is drawn by a resistive load?**

rms

(a) $$\frac{V_s}{V_p} = \frac{N_s}{N_p} \quad \Rightarrow V_s = \frac{30}{550}(3.3) = 0.18 \text{ kV}$$

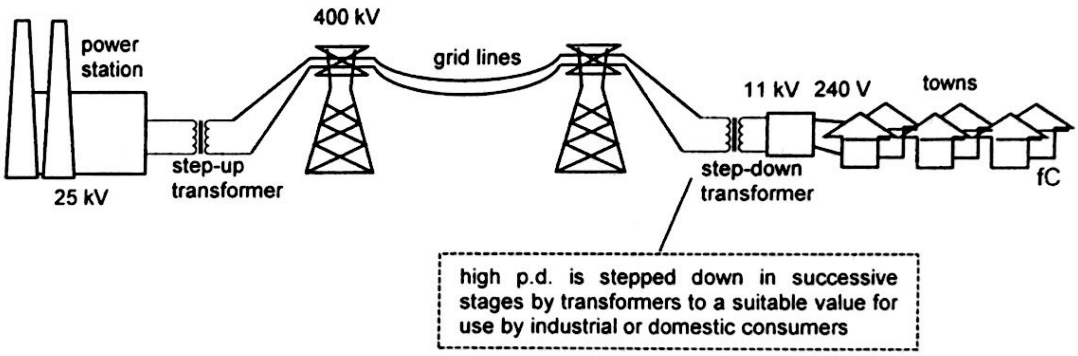
$V_{rms} = \frac{V_0}{\sqrt{2}}$

Maximum $V_s = 0.18\sqrt{2} = 0.25$ kV

(b) $$\frac{N_s}{N_p} = \frac{I_p}{I_s} \quad \Rightarrow I_p = \frac{30}{550}(11) = 0.60 \text{ A}$$

## 18.4 Transmission of electrical energy

**Power Transmission**




When it comes to heating and lighting applications, a.c. has no practical advantage over d.c. because the heating effect of a current is independent of its direction. In fact, many practical systems like large motors and railway systems can only run on d.c.

**Advantage of using a.c.**

The main advantage of using a.c. is that **it can be stepped up or down easily compared with d.c. for high voltage transmission.**

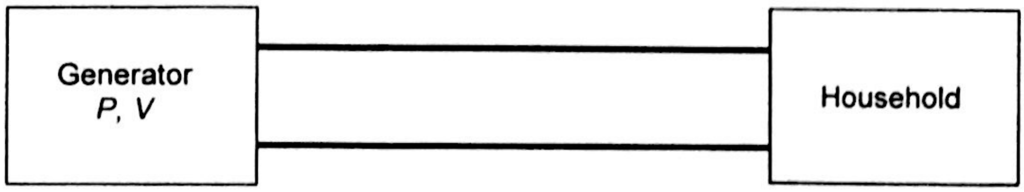
High voltage transmission of electrical power is **more efficient due to low power losses in the cables.**



prepared by Mr fRancis Chong

**NOTE**

**Why does high voltage transmission leads to low power losses in the cables?**




constant

For constant power generated $P$ and transmission voltage $V$,

current supplied $I = \frac{P}{V}$

If total resistance of cables = $R_{cables}$

Power lost in cables = $P_{lost} = I^2 R_{cables} = \left(\frac{P}{V}\right)^2 R_{cables}$

$\times \frac{V^2}{R_{cables}}$

Hence electrical power is transmitted at high voltage to minimise power lost in the cables.

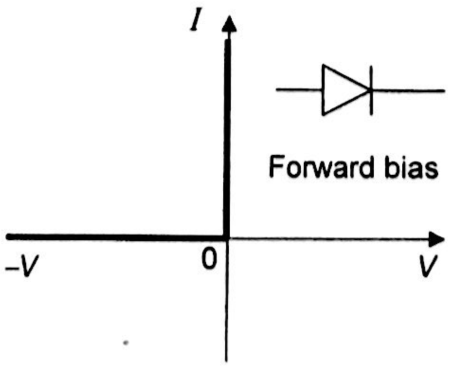
## 18.5 Rectification

**Rectification**

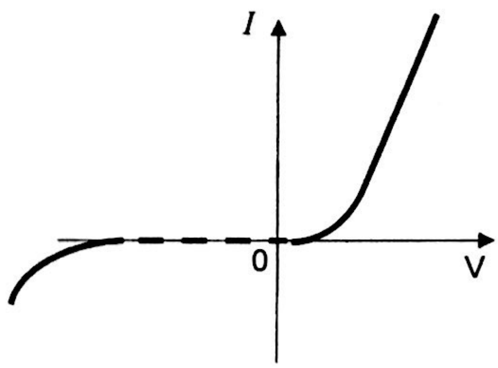
Rectification is the process of converting an alternating current to a direct current. Diodes are used for rectification.

**Diode**

A diode conducts current only in one direction i.e. when the p.d. applied is forward bias. For the reverse bias, the resistance of the diode is very high.



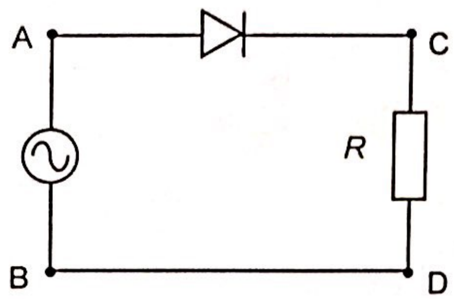

$I$-$V$ characteristic of an ideal diode



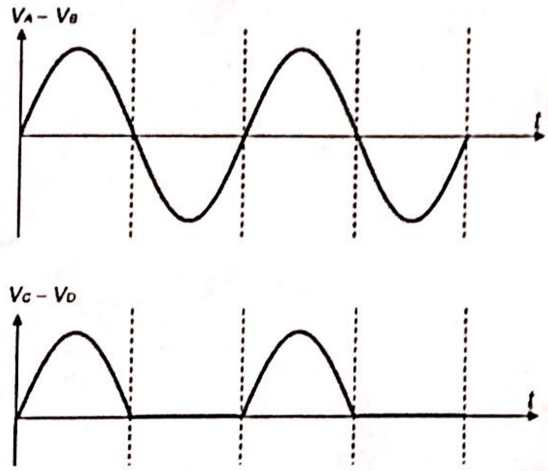

$I$-$V$ characteristic of a real diode

**Half-wave rectification**

Only one diode is required in this circuit.




Half-wave rectifier

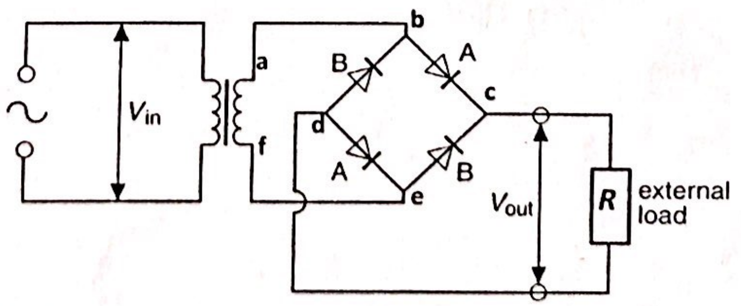


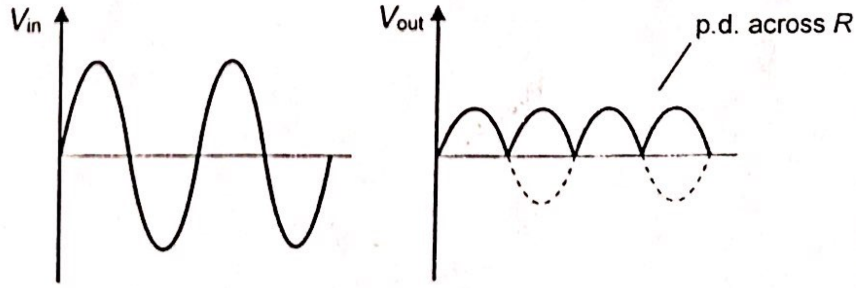

Current through the external load (resistor) flows in one direction only. For the first half of the cycle, the diode is forward bias, hence current flows through the diode and external load. For the second half of the cycle, the diode is reverse bias and negligible current flows.

**Full-wave rectification**
**(not in syllabus)**

Four diodes are arranged in a bridge network below.

If point **a** is positive during the first half-cycle, the two diodes A conduct and current takes the path **abc**, ***R***, **def**. On the next half-cycle when **f** is positive, the two diodes B are forward biased and current follows the path **fec**, ***R***, **dba**. Current flows through ***R*** in the same direction during both half-cycles of input $V_{in}$ and a d.c. output $V_{out}$ is obtained.



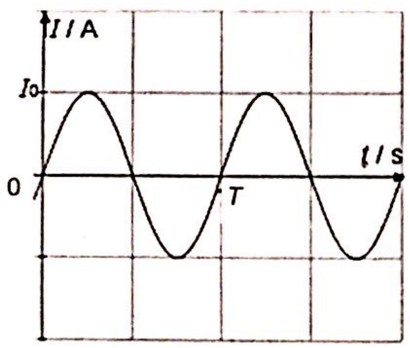
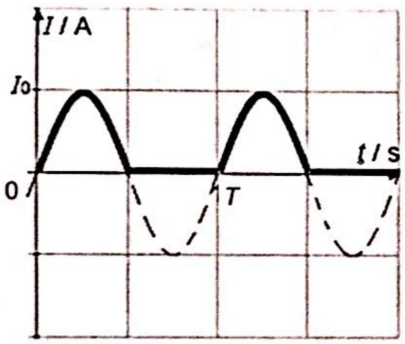
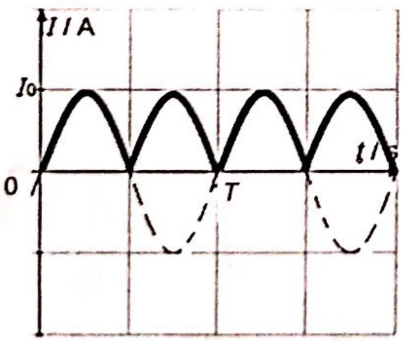
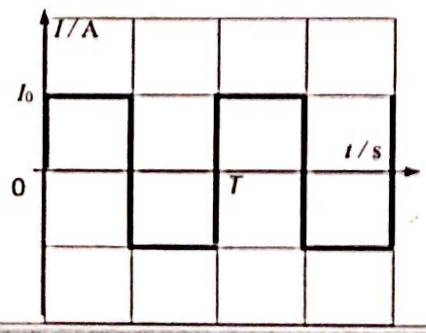



prepared by Mr fRancis Chong

derive + memorise

can ignore

## *Comparison of $I_{rms}$ and <I>*

| Variation of current $I$ with time $t$ | Waveform | r.m.s. current | average current |
|---|---|---|---|
|  | *Sinusoidal*<br>$\langle p \rangle : \frac{P_0}{2}$ | $\frac{I_0}{\sqrt{2}}$ | 0 |
|  | *Half-wave rectified*<br>$\langle p \rangle : \frac{P_0}{4}$ | $\frac{I_0}{2}$ | $\frac{I_0}{\pi}$ |
|  | *Full-wave rectified*<br>$\langle p \rangle \ \frac{P_0}{2}$ | $\frac{I_0}{\sqrt{2}}$ | $\frac{2I_0}{\pi}$ |
|  | *Rectangle / Square*<br>$\langle p \rangle : P_0$ | $I_0$ | 0 |