Student's *t* test

1. Introduction

The main reason for carrying out observations and experiments in Biology is to test an idea or hypothesis; Biology, as a science, is an on-going subject, answers to one question, usually leading to further questions. Such work requires data sets. What we want to know is whether the data supports the initial hypothesis or not; does the data enable us to accept our hypothesis as a valid explanation or do we have to modify the hypothesis, or even reject it? You have already been introduced to one statistical test that is used to analyse data sets – the chi-squared test or X^2 test. We will look at another test – the *t* test, in this lecture set.

2. Learning Outcomes

(a) Able to apply *t*-test in Biological situations
(b) Able to calculate standard deviations
(c) Able to test for significant differences between means of two small unpaired samples

Use the knowledge gained in this section in new situations or to solve related problems

3. References

Gavin, J.W., Skills in Advanced Biology – Dealing with Data I
Barnard, C., Gilbert, F. & McGregpr, P. - Asking Questions in Biology, 2nd Edition.
T Test as a Parametric statistic - https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4667138/
Bread and Butter of Statistical Analysis: T Test - https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4744321/
Statistics: A Brief Overview - https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3096219/

Contents

4. Tests of Significance

Notes to self

Biological data is variable. It consists of small samples taken from large populations. Even though these samples are randomly chosen it is very unlikely that the data will conform exactly to the hypothesis. To overcome this problem, tests of significance are used to determine whether the data conform to the hypothesis sufficiently closely for a decision to be made, not only about the samples but also about the general populations from which they were drawn.

Often the hypothesis requires a comparison between two or more samples. The degree of difference between samples may be very small, very large, or anything in between. The real purpose of the exercise is to find out, from the difference between the samples, whether they were drawn from the same (statistical) population or from different (statistical) populations. We cannot be certain about any statistical conclusion – **a statistical test does not prove anything**. *It does, however, give us some degree of confidence in our conclusions.*

5. Null Hypothesis, H_0

This is a **'negative'** hypothesis: for example, if we were comparing two samples, the null hypothesis would be stated in the form **"there is NO significant difference between the two samples"**. It assumes that any differences that do occur are due to chance. **The significance test then confirms whether to not reject or to reject, the null hypothesis.**

If the null hypothesis is not rejected – then there is NO significant difference between the two samples.

If the null hypothesis is rejected – then there is a significant difference between the two samples.

6. X^2 test vs *t* test

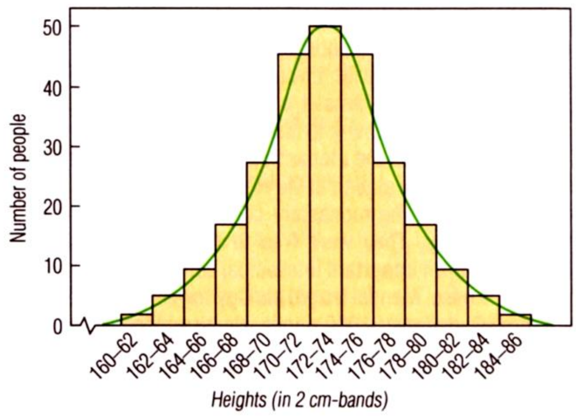
A X^2 test can be used if data are in the form of counts i.e. two groups have been identified and observations classified in terms of the number belonging to each. X^2 can only be used on *raw counts*; it **cannot be used on measurements** (e.g. length, time, weight, volume etc) or **proportions, percentages or any derived values.** The test works by comparing observed counts with those expected by chance or a prior basis. Experiments in Mendelian genetics would be such an example. **Sizes of samples in a X^2 test are generally are over 30**.

The *t* test can be performed on raw counts like X^2 test but it **deals with each contributing data value in the two groups separately instead of as a single total**. It can **deal with data other than counts** e.g. body size, time spent performing a certain behaviour, percentage of patients responding to a drug etc. **Sample sizes can be around 30** (or sometimes less).

7. Normal Distribution Curve

Many if not most characteristics in Biology (e.g. heights of people, weight of rats, number of flowers in plants, number of peas in a pod, body temperature of mice, breathing rate of locusts etc) fall into a normal distribution (refer to Continuous Variation in Genetics). When drawn as a graph, the normal distribution assumes a characteristic 'bell' shape.

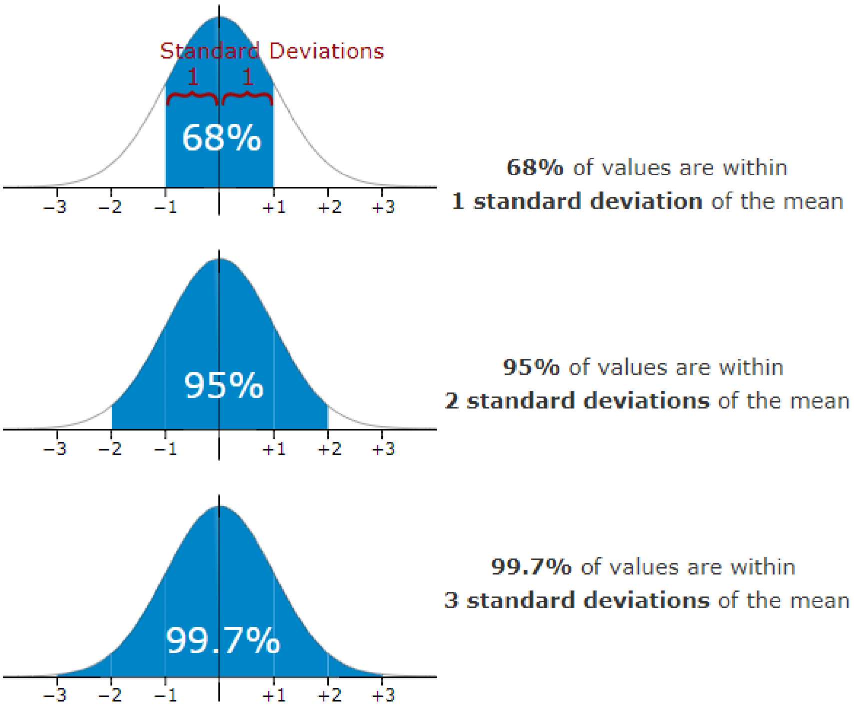
continuous variation
variation in the height of adult male humans; the results cluster quite closely around one value, and show a normal distribution (in this type of distribution the mean, median and mode values coincide); for convenience in plotting, the heights are grouped into 13 arbitrary groups, each covering a height range of 2 cm



A normal distribution is symmetrical about the mean value with 50% of the values less than the mean and 50% of the values more than the mean. The mean = median = mode.

<u>Standard Deviation</u>

The standard deviation (SD) is a measure of the spread of data values from the mean (assuming the data follow normal distribution)**.** It is a measure of the confidence you have that any particular data value will fall within a particular range (the mean + SD and the mean – SD). In a normal distribution, 68% of the values are within 1 standard deviation of the mean, 95% are within 2 standard deviations of the mean and 99.7% are within 3 standard deviations of the mean.



Not in syllabus (but you will see this in data presented in many scientific papers)

The standard error of the mean (SE) measures the spread of multiple sample means around the true population mean. The assumption is that when we take a very large number of random samples from a population, the means from each sample will form a normal distribution. The distribution of sample means will have a mean which is close to actual population mean. Normally we can only take data from a few samples and not a large number of samples, so the SE is an expression of the confidence we have that our sample means falls within a particular range (mean ± SE) of the true population mean. Since sample means can be considered to be normally distributed, we can cite the standard error with sample mean.

Calculating the Standard Deviation (SD)

Suppose we have a set of data where we measured the body lengths of ten grasshoppers caught in a geographical area and obtained following results:

Length, x (cm)
6.3
7.1
6.2
6.5
7.0
6.7
6.5
7.0
6.8
7.1
Mean $\bar{x}$ = 6.7
n (sample size) = 10

Standard deviation has the formula:

$$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$$

Note: *s* may be denoted as σ in calculators

Let's work out the SD from the data above:

Length, x (cm)	x^2	$(x - \bar{x})$	$(x - \bar{x})^2$
6.3	39.69	- 0.4	0.16
7.1	50.41	0.4	0.16
6.2	38.44	- 0.5	0.25
6.5	42.25	- 0.2	0.04
7.0	49.00	0.3	0.09
6.7	44.89	0.0	0.00
6.5	42.25	- 0.2	0.04
7.0	49.00	0.3	0.09
6.8	46.24	0.1	0.01
7.1	50.41	0.4	0.16
Mean, $\bar{x}$ = 6.72	Σx^2 = 452.58		$\Sigma(x - \bar{x})^2$ = 1
Σx = 67.2 $(\Sigma x)^2/n$ = 451.58		n (sample size) = 10	

1. Calculate **$\Sigma(x - \bar{x})^2$**. This **is equivalent to $\Sigma x^2 - (\Sigma x)^2/n$**. $\Sigma x^2 = 452.58$, and $(\Sigma x)^2/n = 451.58$. Therefore $\Sigma(x - \bar{x})^2 = 452.58 - 451.58 = 1$.
2. Divide $\Sigma(x - \bar{x})^2$ by (n-1) = 1/9 = 0.11
3. Square root of 0.11 = 0.33

The standard deviation = 0.33 cm

About 68% of grasshoppers would have body lengths between 6.39 – 7.05 cm (6.72 ± 0.33). These would be -1 SD and +1 SD respectively of the mean in the sample taken.

Standard error calculations are not required by the syllabus (SE = $\frac{s}{\sqrt{n}}$). In the above example, the SE can be worked out as 0.11. Therefore, the confidence of the measurements taken above can be written as: 6.72 ± 0.11 cm.

8. History of Student's t test

The *t* statistic/value was first published in 1908 by William Sealy Gosset, who just joined Guiness Brewery in 1899 from Oxford University as a scientist brewer (he was not a mathematician). He needed to assess the potential of different barley varieties. Since it was not possible to rely on large volumes of data due to the limited number of experimental plots and the growing season, Gosset could only do one experiment each year. Gosset observed that the mean and standard deviation of a large set of samples could be very accurately determined using the statistical techniques available at the time. But for smaller sets of samples, typically below 30, the error progressively increased. Gosset developed the *t* statistic/value to take this error into account when drawing conclusions from the data. From his observations, Gosset was able to put numerical values to the variation associated with small samples.

Small samples are not representative of the larger population and hence the actual nature of the mean and standard deviation of a large (and therefore, normally distributed population) could not be accurately determined using the mean and standard deviation of small samples. Basically, Gosset's *t* value took into account this error from small samples.

As Guiness did not want any work from its staff to be revealed to competitors, staff could not publish their work using their real names. Gosset was given the pseudonyms 'Pupil' and 'Student' to choose from. He chose 'Student'. Hence the *t* statistic became known as Student's *t* or more commonly as Student's *t* test.

9. Calculating the *t* value

The *t* test compares the means of two samples being tested to determine if there is any significant difference between them. The formula for the *t* test is given as:

$$t = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$\bar{x}_1$ is the mean of sample 1 and $\bar{x}_2$ is the mean of sample 2

s_1 is the standard deviation of sample 1 and s_2 is the standard deviation of sample 2

n_1 is the size of sample 1 and n_2 is the size of sample 2

degree of freedom (df) for *t* test = $(n_1 + n_2) - 2$ (compare with chi-squared test)

Worked Example

A parasitologist wanted to compare the number of lymphocytes (a type of white blood cell) present in the blood of patients infected by a blood parasite with the numbers in uninfected individuals. To save time and resources, he limited the investigation to five individuals in each group.

Using prepared blood films, the lymphocyte counts obtained from the equal areas of blood were obtained:

Group A (infected patients)	150, 155, 152, 146, 152
Group B (uninfected individuals)	165, 170, 151, 164, 160

Are these two groups statistically different?

The means and standard deviations were computed as shown:

Group A		Group B	
Lymphocyte Count, x_1	x_1^2	Lymphocyte Count, x_2	x_2^2
150	22500	165	27225
155	24025	170	28900
152	23104	151	22801
146	21316	164	26896
152	23104	160	25600
$\bar{x}_1 = 151$	$\sum x_1^2 = 114049$	$\bar{x}_2 = 162$	$\sum x_2^2 = 131422$
$\Sigma x_1 = 755$, $n_1 = 5$ $(\Sigma x_1)^2/n = 114005.00$ $\sum x_1^2 - (\Sigma x)^2/n = 114049.00 - 114005.00 = 44$ $s = \sqrt{\frac{44}{5-1}} = 3.32$		$\Sigma x_2 = 810$, $n_2 = 5$ $(\Sigma x_2)^2/n = 131220.00$ $\sum x_2^2 - (\Sigma x)^2/n = 131422.00 - 131220.00 = 202$ $s = \sqrt{\frac{202}{5-1}} = 7.11$	

Step 1

State the Null Hypothesis, H_0 – There is no significant difference between the lymphocyte counts in Group A and Group B.

Step 2

Calculate the mean of Group A ($\bar{x}_1$) and Group B ($\bar{x}_2$).

$\bar{x}_1 = 151$ $\bar{x}_2 = 162$

Step 3

Calculate the standard deviation of Group A (s_1) and Group B data (s_2).

$$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$$

Since $\Sigma(x - \bar{x})^2$ is equivalent to $\Sigma x^2 – (\Sigma x)^2/n$ or

$$s_1 = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}} = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}} = \sqrt{\frac{44}{5-1}} = 3.32$$

$$s_2 = \sqrt{\frac{202}{5-1}} = 7.11$$

	Mean	Standard Deviation
Group A	151	3.32
Group B	162	7.11

Step 4

Using the formula:

$$t = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

(a) **calculate the numerator,** we have: $|151 - 162|$ = **11** (taking the absolute value)

(b) **calculate the denominator,** we have $\sqrt{\frac{(3.32)^2}{5} + \frac{(7.11)^2}{5}}$ = $\sqrt{\frac{11.02}{5} + \frac{50.55}{5}} = \sqrt{12.314} = 3.51$

(c) **Therefore the *t* value** = $\frac{11}{3.51}$ = **3.13**

Step 5

Consult a table of *t* distribution (see Appendix A for full table):

	Decreasing value of *p* →			
Degrees of freedom (df)	*P values*			
	0.10	*0.05*	*0.01*	*0.001*
1	6.31	12.71	63.66	636.60
2	2.92	4.30	9.92	31.60
3	2.35	3.18	5.84	12.92
4	2.13	2.78	4.60	8.61
5	2.02	2.57	4.03	6.87
6	1.94	2.45	3.71	5.96
7	1.89	2.36	3.50	5.41
8	1.86	2.31	3.36	5.04
9	1.83	2.26	3.25	4.78
10	1.81	2.23	3.17	4.59

Step 6

Determine the degrees of freedom for *t* test = $(n_1 + n_2) - 2$ = **(5 + 5) – 2** = **8**

*If there were 20 observations in Group A and 28 observations in Group B, what would the degree of freedom be? Ans: ________ **46***

Step 7

Determine "p", using the *df* and *t* value to check against the *t* table.

With df = 8 and *t* value of 3.31, $0.01 < p < 0.05$.

Step 8

Reject or do not reject H_0 based on 5% significance level. Draw your conclusion.

Since $p < 0.05$ at 5% significance level (probability that *any difference is due to chance is less than 5%),* reject H_0.

There is a significant difference between the lymphocyte counts of Group A and Group B. This difference may be due to the infection of the parasite in the Group A patients, causing a decrease in lymphocyte count.

(Note: Lack of statistical evidence against the null hypothesis is not evidence for your experimental hypothesis being true. Therefore, **we cannot say that we accept H_0**. We can only conclude that there is not have enough evidence to reject the null hypothesis – i.e. we do not reject H_0).

If the df = 8, and calculated t value is 1.90, how would your conclusion be?

With df = 8 and *t* value of 1.90, $0.10 > p > 0.05$.

Since $p > 0.05$, at 5% significance level, do not reject H_0.

There is no significant difference between the lymphocyte counts of Group A and Group B, any difference is due to chance alone.

The t test in your current A level syllabus requires that you to perform an **unpaired *t* test** (also known as a **two-tailed *t* test**). A two-tailed *t* test is performed to determine whether the difference between samples/groups is statistically significant in either positive or negative direction. We do not know in what direction the means are heading i.e. are they bigger or smaller than the other.

Additional Information (not in current syllabus)

A paired t test is done when there is only one sample that has been treated twice, each time differently. For example, a group of heart test subjects could be monitored for their heart rates and then this same group was given a drug that can decrease their heart rate. A paired t test would be used on this group to show significance of the effect of the drug on the means of group after the drug was administered. In this case, we know that the administering of the drug would have an effect on the patients, we know that the mean value of the patients' heart rates after taking the drug would be lower. The calculation for a paired t test is the same, except that a different set of p values is consulted in the table of t distribution.

<u>Tutorial Exercise 1</u>

Ten leaves were picked at random from a beech tree – five from the top of the branches ('sun' leaves) and five from the lowest branches ('shade' leaves). It was hypothesised that the sun leaves at the highest branches would be larger than the lower shade leaves since they would be better adapted to trapping the light where it is brightest.

The lengths of the leaves in the two samples were measured carefully in cm and the mean and standard deviation of each sample were computed. The results are as follow:

	Position on Tree	
	Top (A)	Bottom (B)
Mean	6.2	7.6
Standard Deviation	0.8	1.4

<u>Step 1</u>

State the Null Hypothesis, H_0 - ______________________________

There is no difference in the lengths of beech leaves from the top branches and the bottom branches.

<u>Step 2</u>

Calculate the mean of Group A ($\bar{x}_1$) and Group B ($\bar{x}_2$) – already given.

Group A mean = 6.2, Group B mean = 7.6

Step 3

Calculate the standard deviation of Group A (s_1) and Group B data (s_2) – given.

$$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$$

Since $\Sigma(x - \bar{x})^2$ is equivalent to $\Sigma x^2 - (\Sigma x)^2/n$ or

$$s_1 = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}} = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}} = \underline{\qquad\qquad} \ \mathbf{0.8}$$

s_2 = ____________ **1.4**

Step 4

Using the formula:

$$t = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

(a) **calculate the numerator,** we have**: I6.2 – 7.6I = 1.4** (taking the absolute value)

(b) **calculate the denominator,** we have $\sqrt{\frac{(0.8)^2}{5} + \frac{(1.4)^2}{5}} = \sqrt{\frac{(0.64)}{5} + \frac{(1.96)}{5}}$

= ______________________ **0.76**

(c) **Therefore the *t* value** = ____________ $\frac{1.4}{0.76}$ = ________________ **1.94**

Step 5

Consult a table of *t* distribution.

Step 6

Determine the degrees of freedom for *t* test = $(n_1 + n_2) - 2$ = ________**(5 + 5) -2**

= _________**8**

Step 7

Determine "p", using the *df* and *t* value to check against the chi-square table

df = ___________ **8,** *t* value = _________ **1.94**, ______ **p** _______ **$0.05 < p < 0.10$**

Step 8

Reject or do not reject H_0 based on 5% significance level. Draw your conclusion.

Since $p > 0.05$ at 5% significance level, do not reject H_0.

(The probability is very high that any difference is due to chance).

There is no significant difference between the lengths of the leaves at the top branches of the tree and those at the bottom branches, any differences is due to chance alone.

Why was this a badly designed experiment?

There should be more than 5 samples per group (at least 30 per group).

Tutorial Exercise 2

If the two samples now contained 15 leaves each instead of 5, and the same values of mean and standard deviation are used, what would be the result of the t test?

Using the formula for t value:

$$t = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}}$$

Numerator = 1.4

Denominator = $\sqrt{\frac{(0.64)}{15} + \frac{(1.96)}{15}}$ = ***0.173***

t = 1.4/0.173 = 8.09

df = (15 + 15) – 2 = 28

$p < 0.001$, therefore $p < 0.05$ at 5% significance level, reject H_0

There is a significant difference between the two samples.

Looking at the data, the mean length of leaves from the bottom branches is longer than that of the top branches. We still have to reject our original proposed hypothesis (pg 9, question stem) that the topmost leaves are bigger due to adaptation for greater exposure to sunlight. There must be some other reason(s) for the bottom leaves being larger than the top leaves.

So we must propose another hypothesis which we can then test.

Tutorial Exercise 3

A new fertiliser for potatoes was advertised claiming higher yields. To test this claim, potatoes of the same variety were grown in 36 plots of a fixed size. 18 of the plots being treated with the usual fertiliser (A) and 18 plots with the new fertiliser (B).

The results are given as the yield in kilograms.

Fertiliser A	27	20	16	18	22	19	23	21	17	28	19	17	26	22	19	23	22	20
Fertiliser B	28	19	19	21	24	20	25	19	27	25	29	23	21	17	23	27	25	24

Using the above data, determine if the advertisement's claim is justified by using the *t* test. If your degree of freedom is not listed on the table of *t* distribution, round down to the next smallest degree of freedom in the table (e.g. 34 degrees of freedom, use df = 30 from *t* table).

1. **State null hypothesis – There is no significant difference between the yield of potatoes on Fertiliser A compared to Fertiliser B.**

2. **Calculate sample means.**

3. **Calculate sample SD.**

	Fertiliser A	Fertiliser B
Mean	**21**	**23**
SD	**3.4**	**3.5**

4. **Calculate t value.**

$$t = \frac{21-23}{\sqrt{0.64+0.68}} = \frac{2}{1.15} = 1.74$$

5. **Consult t table.**

6. **df = (18 + 18) – 2 = 34 (use df = 30 in *t* table)**

7. **0.05 < p < 0.10, therefore p > 0.05 at 5% significance level, do not reject H_0.**

8. **There is <u>no significant difference</u> between the yields of potatoes under Fertiliser A and those under Fertiliser B. The advertisement's claim is not justified.**

Tutorial Exercise 4

Notes to self

Four varieties of rye-grass (A, B, C and D) were grown under identical conditions after six weeks their heights were measured (using proper sampling techniques).

(a) Fill in the first column of the matrix below to summarise how this experiment will be conducted to compare the four varieties with each other.

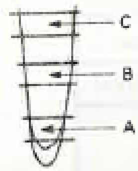
Variety	B	C	D
A			
B			
C			

(b) How many *t* tests would be required to be carried out if each variety was to be compared with the other varieties? List the pairs of tests.

6 *t* tests: A x B; A x C; A x D; B x C; B x D; C x D;

Tutorial Exercise 5

A longitudinal section of the root of a broad bean was mounted on a slide and examined under the microscope. It was observed that the cells seemed to get larger the further they were from the tip.



Longitudinal section of a broad bean root

In order to determine whether this was actually the case, ten cells were chosen at random from each of three areas – A, B and C. The length and width of each cell was measured using an eyepiece graticule.

The results were shown below (in graticule divisions).

	Area A		Area B		Area C	
Cell no.	Length	Width	Length	Width	Length	Width
1	6.5	9.0	18.0	8.5	19.5	14.0
2	7.0	14.0	14.0	8.5	19.0	10.0
3	9.5	9.0	9.5	7.0	21.0	9.5
4	12.0	8.5	11.0	6.0	15.5	13.5
5	7.0	10.0	8.0	6.0	19.5	9.5
6	5.5	8.0	14.5	6.5	14.5	10.0
7	8.0	8.5	8.5	8.0	11.0	11.5
8	12.0	7.5	11.5	6.5	21.5	10.0
9	11.5	9.5	10.0	10.0	14.0	6.5
10	7.5	7.0	8.5	9.0	15.5	7.0

(a) Carry out *t* tests to find out if there is a difference between the lengths of the cells in areas A, B and C. (Hint: Determine how many tests are needed first to compare each pair of areas).
(b) Repeat for the widths of the cells in areas A, B and C.
(c) Suggest reasons for the results you obtain.

Determine the means and standard deviations for both length and width, for each set of data A, B and C. The results are summarised below:

		Length	Width
A	$\bar{x}$	8.65	9.10
	s	2.43	1.94
B	$\bar{x}$	11.35	7.60
	s	3.23	1.39
C	$\bar{x}$	17.10	10.15
	s	3.47	2.40

State the null hypothesis (length of cells) – There is no significant difference in the lengths of cells in areas A and B, in areas B and C and in areas A and C.

State the null hypothesis (width of cells) – There is no significant difference in the widths of cells in areas A and B, in areas B and C and in areas A and C.

Determine the *t* values for each combination pair of A, B and C for both length and width.

Results are summarised below:

Length		
	A	B
B	1.65	
C	4.72	3.83

Width		
	A	B
B	1.30	
C	1.53	2.90

Probabilities of differences due to chance (5% significance level)

Length		
	A	B
B	> 0.05	
C	< 0.05	< 0.05

Width		
	A	B
B	> 0.05	
C	> 0.05	< 0.05

df = 18

A x B: df =18, t value of 1.65, 0.05 < p < 0.10

A x C: df =18, t value of 4.72, p < 0.001

B x C: df = 18, t value pf 3.83, 0.001 < p < 0.01

A x B: df =18, t value of 1.30 p > 0.20

A x C: df = 18, t value of 1.53, 0.10 < p < 0.20

B x C: df =18, t value of 2.90, 0.001 < p < 0.01

Significant differences between the means:

Length		
	A	B
B	NS	
C	S	S

Width		
	A	B
B	NS	
C	NS	S

where NS = non-significant and S = significant

The was no significant difference in the length of cells from areas A and B. There was significant difference between in the length of cells from A and C and B and C.

There was no significant differences in the width of cells from areas A and B and A and C. There was significant difference in the width of cells from areas B and C.

(c) As the root tip grows, the areas furthest away from the root tip i.e. areas B and C compared to area A would show larger cell lengths as these are areas where the cells have elongated and differentiated. This is compared to the cells of the root tip (area A) where the cells are constantly dividing in the growing root tip. These cells tend to be longer compared to the cells in area A.

<u>Appendix A</u> – Table of *t* distribution

Degrees of freedom	Significance level 20% (0.20)	10% (0.10)	5% (0.05)	2% (0.02)	1% (0.01)	0.1% (0.001)
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.598
3	1.638	2.353	3.182	4.541	5.841	12.941
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.859
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.405
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015
17	1.333	1.740	2.110	2.567	2.898	3.965
18	1.330	1.734	2.101	2.552	2.878	3.922
19	1.328	1.729	2.093	2.539	2.861	3.883
20	1.325	1.725	2.086	2.528	2.845	3.850
21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.767
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	2.485	2.787	3.725
26	1.315	1.706	2.056	2.479	2.779	3.707
27	1.314	1.703	2.052	2.473	2.771	3.690
28	1.313	1.701	2.048	2.467	2.763	3.674
29	1.311	1.699	2.043	2.462	2.756	3.659
30	1.310	1.697	2.042	2.457	2.750	3.646
40	1.303	1.684	2.021	2.423	2.704	3.551
60	1.296	1.671	2.000	2.390	2.660	3.460
120	1.289	1.658	1.980	2.158	2.617	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.291