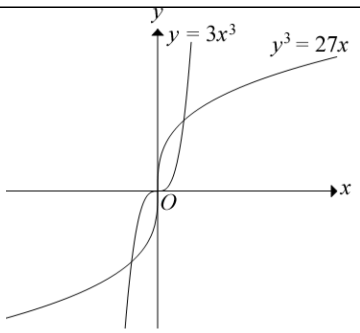
Paper 1

1. Find the value of k for which the line $y+2x=k$ and the curve $y^2=x-2$ do not intersect. [4]

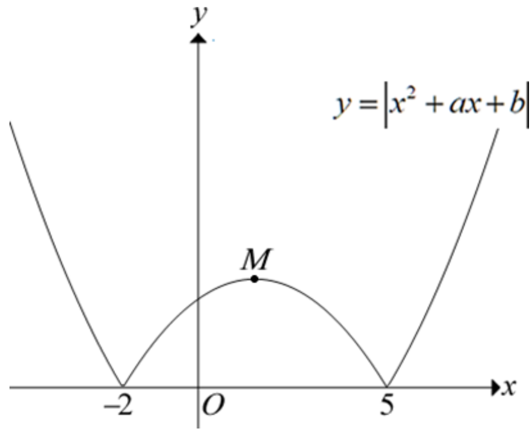
1.	$y=k-2x$ $y^2=x-2$ $(k-2x)^2=x-2$ $k^2-4kx+4x^2-x+2=0$ $4x^2-x(4k+1)+(k^2+2)=0$ $(4k+1)^2-4(4)(k^2+2)<0$ $16k^2+8k+1-16k^2-32<0$ $8k-31<0$ $k<\frac{31}{8}$	**(4 marks)**

2. **(i)** On the same axes sketch the curves of $y^3=27x$ and $y=3x^3$. [3]

(ii) Find the length of the line segment which joins all the points of intersection of the two curves. [3]

2. (i)		
(ii)	At point of intersection, $27x^9=27x$ $x^9-x=0$ $x(x^8-1)=0$ $x=0,\ -1,\ 1$ $y=0,\ -3,\ 3$ Length $=2\sqrt{1^2+3^2}$ or $\sqrt{2^2+6^2}$ $=2\sqrt{10}$ units	**(6 marks)**

3. The diagram shows part of the graph of $y = |x^2 + ax + b|$. The curve touches the x-axis at (–2, 0) and at (5, 0) and has a maximum point at $M(p, q)$.

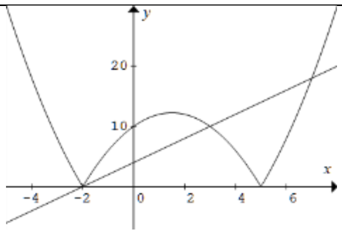


(i) Find the value of a and of b. [2]

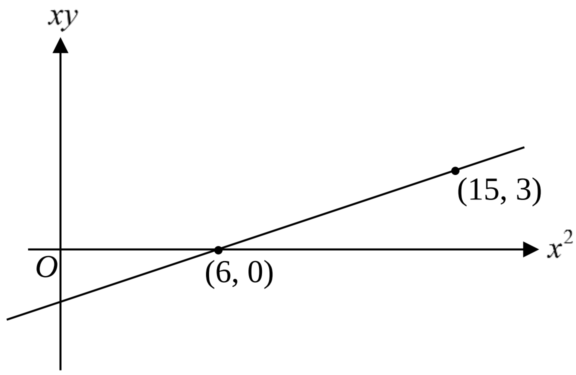
(ii) Find the coordinates of M. [2]

(iii) Solve the equation $|x^2 + ax + b| = 2x + 4$.

Hence, solve the inequality $|x^2 + ax + b| < 2x + 4$. [3]

3. (i)	$a = -(-2+5) = -3$ $b = -2(5) = -10$	
(ii)	At M, $x = \frac{-2+5}{2} = \frac{3}{2}$ $y = \left\lvert \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) - 10 \right\rvert = \frac{49}{4}$ $M\left(\frac{3}{2}, \frac{49}{4}\right)$ or (1.5, 12.25)	
(iii)	$\lvert x^2 - 3x - 10 \rvert = 2x + 4$ $x^2 - 3x - 10 = 2x + 4$ or $x^2 - 3x - 10 = -(2x+4)$ $x^2 - 5x - 14 = 0$ $\quad$ $x^2 - x - 6 = 0$ $(x+2)(x-7) = 0$ $\quad$ $(x+2)(x-3) = 0$ $x = -2,\ 7$ $\quad$ $x = -2,\ 3$	
	 $\lvert x^2 - 3x - 10 \rvert < 2x + 4 \Rightarrow 3 < x < 7$	**(7 marks)**

4. The diagram shows part of a straight line drawn to represent the equation $x + \frac{p}{x} = qy$.



Calculate the value of p and of q. [4]

4.	Gradient $= \frac{3}{9} = \frac{1}{3}$ Equation of straight line, $xy = \frac{1}{3}(x^2 - 6)$ $3xy = x^2 - 6$ $3y = x - \frac{6}{x}$ $p = -6$ $q = 3$	**(4 marks)**

5. **(a)** **Without using a calculator**, show that $\cos^4 15^\circ - \sin^4 15^\circ = \frac{\sqrt{3}}{2}$. [2]

(b) Given that $0 \le 2x \le 2\pi$ and $\cos 2x = -\frac{23}{49}$, calculate the exact value of $\sin x$. [2]

5. (a)	$\cos^4 15^\circ - \sin^4 15^\circ$ $= (\cos^2 15^\circ - \sin^2 15^\circ)(\cos^2 15^\circ + \sin^2 15^\circ)$ $= (\cos^2 15^\circ - \sin^2 15^\circ)$ $= \cos 30^\circ$ $= \frac{\sqrt{3}}{2}$	
(b)	$\cos 2x = -\frac{23}{49}$ $1 - 2\sin^2 x = -\frac{23}{49}$ $2\sin^2 x = \frac{72}{49}$ $\sin^2 x = \frac{36}{49}$ $\sin x = \pm\frac{6}{7}$ $\frac{\pi}{2} \le 2x \le \frac{3\pi}{2}$ $\frac{\pi}{4} \le x \le \frac{3\pi}{4}$ Hence, $\sin x = \frac{6}{7}$	**(4 marks)**

6. Without the use of a calculator, find the values of the integers p and q for which the solution of the equation $x\sqrt{24}+\sqrt{96}=\sqrt{108}+x\sqrt{12}$ is $\sqrt{p}+q$. [4]

6.	$x\sqrt{24}+\sqrt{96}=\sqrt{108}+x\sqrt{12}$ $x\sqrt{2}+\sqrt{8}=\sqrt{9}+x$ $x\sqrt{2}-x=3-2\sqrt{2}$ $x(\sqrt{2}-1)=3-2\sqrt{2}$ $x \ = \dfrac{3-2\sqrt{2}}{\sqrt{2}-1}$ $= \left(\dfrac{3-2\sqrt{2}}{\sqrt{2}-1}\right)\left(\dfrac{\sqrt{2}+1}{\sqrt{2}+1}\right)$ $=3\sqrt{2}+3-4-2\sqrt{2}$ $=\sqrt{2}-1$ $p=2$ $q=-1$	**(4 marks)**

7. **(a)** Find the term independent of x in the expansion of $\left(x-\frac{1}{5x^2}\right)^9$. [3]

(b) Obtain the first four terms in the expansion, in ascending order of x, of $\left(2-\frac{x}{3}\right)^6$. [2]

Hence, find the coefficient of x^3 in the expansion of $\left(2-\frac{x}{3}\right)^6(3+x)^2$. [3]

7.	**(a)**	$T_{r+1}=\binom{9}{r}x^{9-r}\left(-\frac{1}{5x^2}\right)^r$ $=\binom{9}{r}\left(-\frac{1}{5}\right)^r x^{9-3r}$ When $9-3r=0$, $r=3$ The term independent of x $=\binom{9}{3}\left(-\frac{1}{5}\right)^3$ $=-\frac{84}{125}$ or -0.672	
	(b)	$\left(2-\frac{x}{3}\right)^6$ $=2^6-\binom{6}{1}(2^5)\left(\frac{x}{3}\right)+\binom{6}{2}(2^4)\left(\frac{x}{3}\right)^2-\binom{6}{3}(2^3)\left(\frac{x}{3}\right)^3+...$ $=64-64x+\frac{80x^2}{3}-\frac{160x^3}{27}+...$ $\left(2-\frac{x}{3}\right)^6(3+x)^2$ $=\left(64-64x+\frac{80}{3}x^2-\frac{160}{27}x^3+...\right)(9+6x+x^2)$ Coefficient of x^3 $=(-64\times 1)+\left(\frac{80}{3}\times 6\right)+\left(-\frac{160}{27}\times 9\right)$ $=\frac{128}{3}$ or $42\frac{2}{3}$	**(8 marks)**

8. **(i)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left[(x-1)\sqrt{2x-1}\right]$ can be expressed in the form $\frac{ax+b}{\sqrt{2x-1}}$ where a and b are integers. [4]

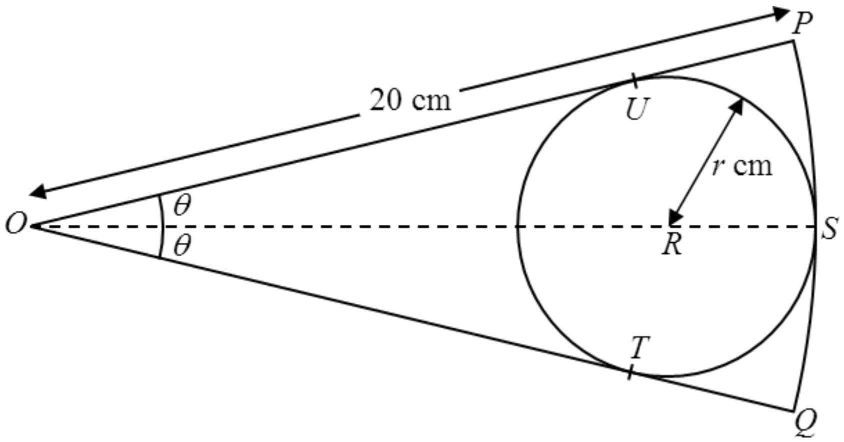
(ii) Integrate $\frac{3x}{\sqrt{2x-1}}$ with respect to x. [3]

(iii) Given that the curve $y = \mathrm{f}(x)$ passes through the point $\left(\frac{5}{2},\ 8\right)$ and is such that $\mathrm{f}'(x) = \frac{3x}{\sqrt{2x-1}}$, find $\mathrm{f}(x)$. [2]

8.	**(i)**	$\frac{\mathrm{d}}{\mathrm{d}x}\left[(x-1)\sqrt{2x-1}\right] = (2x-1)^{\frac{1}{2}}(1) + (x-1)\times\frac{1}{2}(2x-1)^{-\frac{1}{2}}(2)$ $= \frac{2x-1+x-1}{\sqrt{2x-1}}$ $= \frac{3x-2}{\sqrt{2x-1}}$	
	(ii)	$\int\frac{3x}{\sqrt{2x-1}}\mathrm{d}x = (x-1)\sqrt{2x-1} + \int\frac{2}{\sqrt{2x-1}}\mathrm{d}x$ $= (x-1)\sqrt{2x-1} + \frac{2(2x-1)^{\frac{1}{2}}}{\frac{1}{2}\times 2} + C$ $= (x-1)\sqrt{2x-1} + 2\sqrt{2x-1} + C$ $= (x+1)\sqrt{2x-1} + C$	
	(iii)	$\mathrm{f}(x) = (x+1)\sqrt{2x-1} + C$ At $\left(\frac{5}{2},\ 8\right)$, $8 = \left(\frac{5}{2}+1\right)\sqrt{2\left(\frac{5}{2}\right)-1} + C$ $8 = 7 + C$ $C = 1$ $\therefore \mathrm{f}(x) = (x+1)\sqrt{2x-1} + 1$	**(9 marks)**

[Turn over

9. The figure shows a sector *OPQ* of a circle, centre *O*, radius 20 cm. Angle *POQ* = 2θ radians where $0 < \theta < \frac{\pi}{2}$. A circle centre *R*, radius *r* cm, touches the arc *PQ* at the point *S*. The lines *OP* and *OQ* are tangents to the circle at the points *U* and *T* respectively.



(i) Write down, in terms of *r*, the length of *OR*. [1]

(ii) Hence show that $r = \dfrac{20\sin\theta}{1+\sin\theta}$. [2]

(iii) Given that *r* is increasing at 2 cm s^{-1}, find the rate at which θ is increasing when $\theta = \frac{\pi}{6}$. [4]

9.	**(i)**	$OR = (20 - r)$ cm	
	(ii)	$\dfrac{UR}{OR} = \sin\theta$ $\dfrac{r}{20-r} = \sin\theta$ $r = \sin\theta(20 - r)$ $r(1+\sin\theta) = 20\sin\theta$ $r = \dfrac{20\sin\theta}{1+\sin\theta}$	
	(iii)	$\dfrac{dr}{d\theta} = \dfrac{20\cos\theta(1+\sin\theta) - 20\sin\theta\cos\theta}{(1+\sin\theta)^2}$ $= \dfrac{20\cos\theta}{(1+\sin\theta)^2}$ When $\dfrac{dr}{dt} = 2$, $\theta = \dfrac{\pi}{6}$, $\dfrac{d\theta}{dt} = \dfrac{dr}{dt} \div \dfrac{dr}{d\theta}$ $= 2 \times \dfrac{\left(1+\sin\frac{\pi}{6}\right)^2}{20\cos\frac{\pi}{6}}$ $= 0.260$ rad per second	**(7 marks)**

10. The points A and B lie on a circle with centre C. The coordinates of A and B are (1, 7) and (– 3, 9) respectively. The line $y = 8x + 4$ passes through the centre of the circle.

(i) Find the coordinates of C and the radius of the circle. [5]

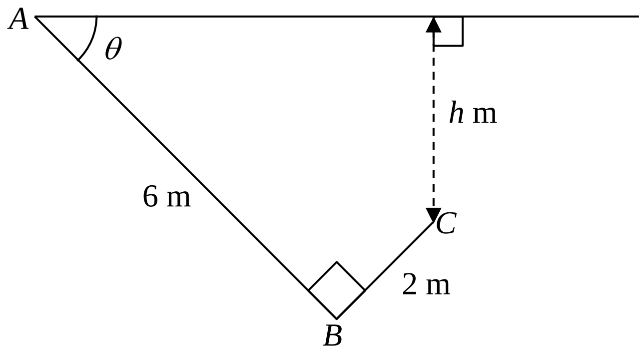
(ii) Hence find the equation of the circle. [1]

Another circle, with centre D(–3, 6), has a radius of 6 units.

(iii) Do the two circles intersect? Support your answer with working. [2]

10. (i)	Gradient of $AB = \frac{9-7}{-3-1} = -\frac{1}{2}$ Gradient of perpendicular bisector of AB = 2 Midpoint of AB = (–1, 8) Equation of perpendicular bisector of AB, $y - 8 = 2(x+1)$ $y = 2x + 10$ At C, $8x + 4 = 2x + 10$ $6x = 6$ $x = 1$ $y = 12$ Centre (1, 12) Radius $= \sqrt{4^2 + 3^2}$ cm $= 5$	
(ii)	$(x-1)^2 + (y-12)^2 = 25$	
(iii)	Sum of radius = (5+6) cm =11 cm Distance between the 2 centres $= \sqrt{4^2 + 6^2}$ $= 2\sqrt{13}$ units < Sum of radius The 2 circles intersect.	**(8 marks)**

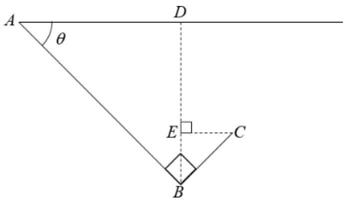
11. The diagram shows two rods, *AB* and *BC*, of length 6 m and 2 m respectively. The rods are fixed at *B* such that angle $ABC = 90^\circ$ and hinged at the ceiling, at *A* so that they can rotate in a vertical plane. The rod *AB* makes an acute angle θ with the ceiling.



(i) Obtain an expression, in terms of θ, for *h*, where *h* m is the vertical distance from the ceiling to *C*. [2]

(ii) Express h in terms of $R\sin(\theta-\alpha)$ where $R>0$ and $0^\circ < \alpha < 90^\circ$. [4]

(iii) Find the value of θ for which *C* is 1.6 m below the ceiling. [2]

11.	**(i)**	In ΔABD, $\sin\theta = \dfrac{BD}{6}$ $BD = 6\sin\theta$ m In ΔBCE, $\cos\theta = \dfrac{BE}{2}$ $BE = 2\cos\theta$ $\therefore h = BD - BE$ $= 6\sin\theta - 2\cos\theta$  	
	(ii)	Let $6\sin\theta - 2\cos\theta = R\sin(\theta-\alpha)$ $R = \sqrt{6^2+2^2}$ $= 2\sqrt{10}$ or $\sqrt{40}$ or 6.32 $\tan\alpha = \dfrac{1}{3}$ $\alpha = 18.434^\circ$ $\therefore h = 2\sqrt{10}\sin(\theta-18.4^\circ)$ or $\sqrt{40}\sin(\theta-18.4^\circ)$ or $6.32\sin(\theta-18.4^\circ)$	
	(iii)	When $h=1.6$, $\sqrt{40}\sin(\theta-18.434^\circ)=1.6$ $\sin(\theta-18.434^\circ) = \dfrac{1.6}{\sqrt{40}}$ Basic $\angle = 14.654^\circ$ $\theta - 18.434^\circ = 14.654^\circ$ $\theta = 33.1^\circ$	**(8 marks)**

12. The velocity, v ms^{-1}, of a particle P, travelling in a straight line, at time t seconds after leaving a fixed point O, is given by

$$v = 2e^{2t} - 19e^{t} + 35, \text{ where } t \geq 0.$$

Find

(i) the value of t when the particle is instantaneously at rest, [3]

(ii) the acceleration of the particle when $t = \ln 7$. [2]

The displacement of the particle P, at time t seconds after leaving a fixed point O, is denoted by s metres.

(iii) Find an expression, in terms of t, for s. [3]

Hence,

(iv) find the distance travelled by the particle in the first 1.5 seconds. [3]

12. (i)	When $v=0$, $2e^{2t}-19e^{t}+35=0$ $(2e^{t}-5)(e^{t}-7)=0$ $e^{t}=\frac{5}{2},\ 7$ $t=\ln\left(\frac{5}{2}\right),\ \ln 7$ $=0.916,\ 1.95$	
(ii)	$\frac{dv}{dt}=4e^{2t}-19e^{t}$ When $t=\ln 7$, $\frac{dv}{dt}=4e^{2\ln 7}-19e^{\ln 7}$ $=63$ ms^{-2}	
(iii)	$s=\int v\,dt$ $=\int(2e^{2t}-19e^{t}+35)dt$ $=e^{2t}-19e^{t}+35t+C$ When $t=0$, $s=0$, $0=1-19+C$ $C=18$ $\therefore s=e^{2t}-19e^{t}+35t+18$	
(iv)	When $t=0$, $s=0$ When $t=\ln\left(\frac{5}{2}\right)$, $s=\left(\frac{5}{2}\right)^{2}-19\left(\frac{5}{2}\right)+35\ln\left(\frac{5}{2}\right)+18$ $=8.8201$ When $t=1.5$, $s=e^{3}-19e^{1.5}+35(1.5)+18$ $=5.4334$ Total distance travelled $=8.8201+(8.8201-5.4334)$ $=8.8202+3.3867$ $=12.2$ m	**(11 marks)**