Class	Index Number

Name : Answers

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2023
Secondary 4

Thursday	**MATHEMATICS**	**4052/02**
17 August 2023	**Paper 2**	2 h 15 min

Candidates answer on the Question Paper.

INSTRUCTIONS TO CANDIDATES

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For p, use either your calculator value or 3.142, unless the question requires the answer in terms of p

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90**.**

90

This question paper consists of 21 printed pages and 3 blank pages.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{\text{r}}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f}-\left(\frac{\sum fx}{\sum f}\right)^2}$$

1 **(a)** It is given that $y = \frac{1}{4}z(w^2 - x^2)$.

(i) Find z such that $y = 4, w = -1 \; and \; x = 3$.

$4 = \frac{1}{4}z[(-1)^2 - 3^2]$
$16 = z\,[\,-8\,]$
$z = -2$

Answer ……- 2…….[2]

(ii) Make x the subject of the formula $y = \frac{1}{4}z(w^2 - x^2)$.

$y = \frac{1}{4}z(w^2 - x^2)$
$4y = z(w^2 - x^2)$
$w^2 - x^2 = \frac{4y}{z}$
$x^2 = w^2 - \frac{4y}{z}$
$x = \pm\sqrt{w^2 - \frac{4y}{z}}$

Answer $x = \pm\sqrt{w^2 - \frac{4y}{z}}$ [2]

(b) Simplify $\frac{3}{x+2}+\frac{6x}{4-x^2}$.

$$\frac{3}{x+2}+\frac{6x}{4-x^2}$$
$$= \frac{3}{x+2}+\frac{6x}{(2-x)(2+x)}$$
$$= \frac{3(2-x)+6x}{(2-x)(2+x)}$$
$$= \frac{6-3x+6x}{(2-x)(2+x)}$$
$$= \frac{3(2+x)}{(2-x)(2+x)}$$
$$= \frac{3}{(2-x)}$$

Answer $\frac{3}{(2-x)}$ ……..……[3]

(c) In 2022, Singapore has a population of 5.64 million.
The land area of Singapore is 728.6km².
Find the population of Singapore per km², leaving your answer as standard form.

Population = $\frac{5.64\times10^6}{728.6}$
= 7.74 x 10^3 **(3sf)**

Answer 7.74 x 10^3 [2]

2 **(a)** A closed cone, of height h cm, is filled with water to half its height as shown in Diagram A. It is then inverted as shown in Diagram B.
Find the height of the water level, x cm, in terms of h, in Diagram B.

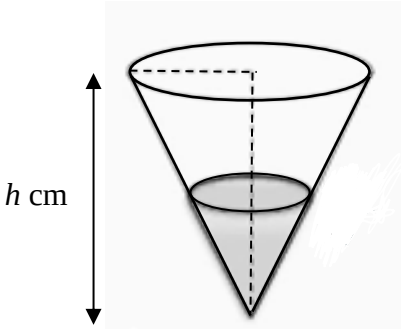


Diagram A

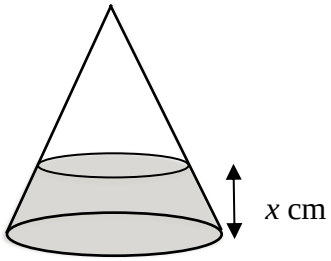


Diagram B

$$\frac{V_{water}}{V_{cone}} = \left(\frac{1}{2}\right)^3$$
$$= \frac{1}{8}$$
$$\frac{V_{air\ in\ \boldsymbol{B}}}{V_{cone}} = \frac{7}{8}$$
$$\left(\frac{h-x}{h}\right)^3 = \frac{7}{8}$$
$$\frac{h-x}{h} = \sqrt[3]{\frac{7}{8}}$$
$$h - x = \frac{\sqrt[3]{7}}{2}h$$
$$x = h - \frac{\sqrt[3]{7}}{2}h$$

Answer …….. [3]

(b) Janet began her round-island 145 km cycling route from Point A at 6 am, cycling at an average speed of x km/h.

(i) Write down the expression, in terms of x, for the time taken by Janet to complete the cycling route.

$$\frac{145}{x}$$

Answer $\frac{145}{x}$ h [1]

Angel started on the same route 10 minutes later. She cycled at a speed of 2 km/h faster than Janet for the first 20 km and cycled x km/h for the remaining route.

(ii) Write down the expression, in terms of x, for the time taken by Angel to complete the cycling route.

Answer $\frac{20}{x+2}+\frac{125}{x}$ h [1]

Angel reached the end of the route at the same time as Janet.

(iii) Form an equation in x and show that it reduces to $x^2 + 2x - 240 = 0$.

Answer [3]

$$\frac{20}{x+2}+\frac{125}{x}+\frac{1}{6}=\frac{145}{x}$$

$$\frac{20}{x+2}+\frac{1}{6}=\frac{20}{x}$$

$$\frac{120+x+2}{6(x+2)}=\frac{20}{x}$$

$$122x+x^2=120x+240$$

$$x^2+2x-240=0$$

(iv) Solve the equation $x^2 + 2x - 240 = 0$.

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-240)}}{2(1)}$$

= 14.5 or -16.5 (3sf)

Answer x = ……..…….. [3]

(v) What time did Janet reach the ending point?

Time taken = $\frac{145}{14.524}$ = 9 h 59 min

Time arrived = 3.59 pm

Answer …….……. [2]

3 **(a)** In a logistics company, each delivery man is paid \$$x$ for every correct delivery made, \$0 for no delivery but has to pay a penalty of \$1 for each damaged delivery.

Tom and Jerry made the following deliveries on a particular day.

$$\mathbf{A} = \begin{matrix} \\ \text{Tom} \\ \text{Jerry} \end{matrix} \begin{matrix} \text{correct} & \text{no delivery} & \text{damaged} \\ \begin{pmatrix} 13 & 4 & 3 \\ 10 & 8 & y \end{pmatrix} \end{matrix} \qquad \mathbf{B} = \begin{pmatrix} x \\ 0 \\ -1 \end{pmatrix}$$

(i) Evaluate **AB** in terms of x and y.

$$\begin{pmatrix} 13 & 4 & 3 \\ 10 & 8 & y \end{pmatrix} \begin{pmatrix} x \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 13x - 3 \\ 10x - y \end{pmatrix}$$

Answer ……..…… [2]

(ii) Explain what your answer in (a)(i) represents.

Rep amount of money Tom and Kerry will be paid respectively for their deliveries on a particular day……..… [1]

It is given that Tom earned \$18 more than Jerry despite both making the same number of deliveries.

(iii) Find the values of x and y.

$$13 + 3 = 10 + y$$
$$y = 6$$

$$13x - 3 = 10x - 6 + 18$$
$$x = 5$$

Answer x = 5…….. y =6…….. [3]

(iv) By using matrix multiplication, calculate the total amount of money the company needs to pay both Tom and Jerry on that particular day.

$$\begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}13(5)-3\\10(5)-6\end{pmatrix}=\begin{pmatrix}106\end{pmatrix}$$

Answer \$106……[2]

(b) Tom is looking for a new job. He targets to earn an annual salary of \$150 000 at the end of 5 years. The job that he is interviewing for is offering an annual salary increment of 3% per annum.
What is the minimum starting annual salary should Tom request for to meet his target. He needs to set his salary request in multiples of \$1000.

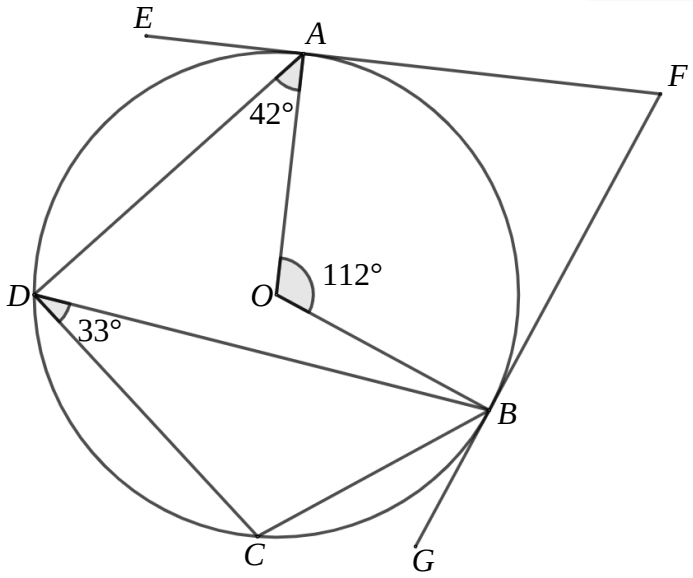
Let P be the annual starting salary.

$$150000 = P\left(1+\frac{3}{100}\right)^5$$

P = 129391.377

Answer \$ 130 000… [3]

4 **(a)** In the diagram, *A*, *B*, *C* and *D* lie on the circle with centre *O*.
EF and *GF* are tangents to circles at *A* and *B* respectively.
It is given that $\angle DAO = 42^o$, $\angle BDC = 33^o$ and $\angle AOB = 112^o$



Find

(i) $\angle CBD$,

$\angle OAB = \dfrac{180° - 112°}{2}$ (base angle of isos Δ)

$= 34°$

$\angle BCD = 180° - 34° - 42°$ (angles in opp segment)

$= 104°$

$\angle CBD = 180° - 104° - 33°$ (angles sum of Δ)

$= 43°$

Answer $\angle CBD$ = ... [3]

(ii) $\angle ABF$.

Method 1

$\angle ABD = 112° \div 2$ (angle at centre = 2 angle at circumference)

$= 56°$

$\angle ABF = 56°$ (alternate segment theorem)

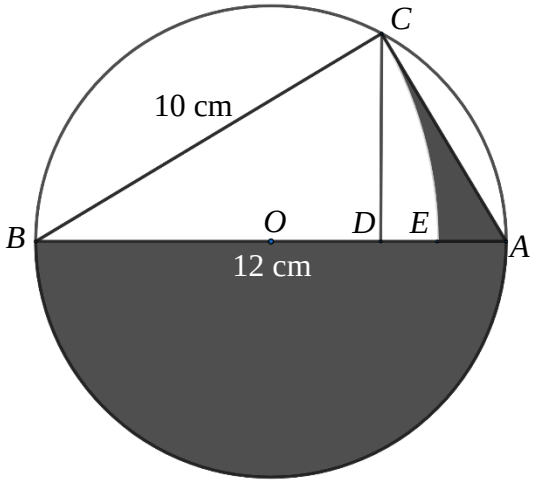
Method 2

$\angle OBF = 90°$ (tan perpendicular to rad)

$\angle ABF = 90° - 34° = 56°$

Answer $\angle ABF$ = ... [2]

(b) The diagram shows a circle, centre O, with a diameter *AB* of 12 cm. *CD* is perpendicular to *AB*. *CE* is an arc of a circle with B as the centre and radius 10 cm.



(i) Show that $\angle CBD = 0.586$ when corrected to 3 significant figures.

Answer [1]

$$\cos \angle CBD = \frac{10}{12}$$
$$\angle CBD = 0.58568..$$
$$\angle CBD = 0.586$$

(ii) Find the arc length *CE*.

10 x 0.586 = 5.86

Answer …….. cm [1]

(iii) Calculate the area of the shaded region.

Shaded area = $\frac{1}{2}\pi(6)^2 + [\frac{1}{2}(10)(12)\sin 0.586 - \frac{1}{2}(10)^2(0.586)]$

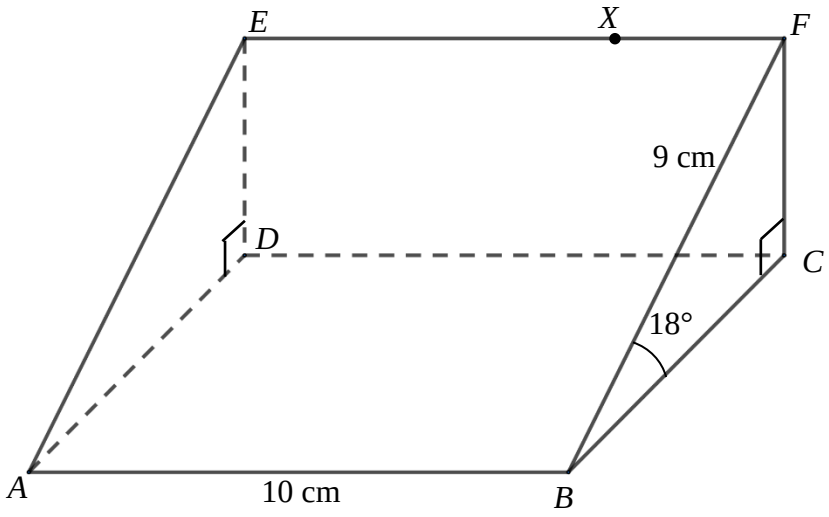
= 60.4 cm²

Answer …… cm² [3]

5 A wooden triangular wedge has a rectangular base as shown in the diagram.

AB = 10 cm and BF = 9 cm and $\angle FBC = 18°$.

X is on EF such that $EX : XF = 3 : 1$.



(i) Find the length of AC.

$$\cos 18° = \frac{BC}{9}$$
$$BC = 9\cos 18°$$
$$AC = \sqrt{10^2 + (9\cos 18°)^2}$$

= 13.163
= 13.2 (3sf)

Answer …….. cm [3]

(ii) Find the length of CX.

$$XF = \frac{1}{4}(10) = 2.5$$
$$\sin 18° = \frac{FC}{9}$$
$$FC = 9\sin 18°$$
$$XC = \sqrt{2.5^2 + (9\sin 18°)^2}$$

= 3.7396
= 3.74 (3 sf)

Answer …….. cm [2]

(iii) Find the greatest angle of elevation of X from BC.

Let the angle of elevation be θ

$$\cos\theta = \frac{2.5}{3.3796}$$
$$= 48.0°$$

Answer …… [2]

(iv) Find the area of triangle BXC.

Method 1

$$BX = \sqrt{(2.5)^2 + 9^2} = \sqrt{\frac{349}{4}}$$

$$(9\cos 18°)^2 = (9\sin 18°)^2 + \left(\sqrt{\frac{349}{4}}\right)^2 - 2(9\sin 18°)\left(\sqrt{\frac{349}{4}}\right)\cos\angle BXC$$

$$\angle BXC = 65.289$$

$$\text{Area} = \frac{1}{2}\left(\sqrt{\frac{349}{4}}\right)(3.7396)\sin 65.289°$$

$$= 15.866 = 15.9 \text{ cm}^2$$

Method 2

$$\text{Area} = \frac{1}{2}(BX)(XC)$$

$$= \frac{1}{2}[9\cos 18°](3.7396)$$

$$= 16.0 \text{ cm}^2$$

Answer ……. cm² [2]

6 A solid cuboid has a square base of x cm, height h cm and a total surface area of 24 cm².

(a) Find h in terms of x.

$$2x^2 + 4xh = 24$$
$$4xh = 24 - 2x^2$$
$$h = \frac{6}{x} - \frac{x}{2}$$

Answer h = ……..[2]

(b) Show that the volume of the cuboid, y cm³, is given by the formula $y = \frac{1}{2}x(12 - x^2)$.

Answer [1]

$$y = x^2 h$$
$$= x^2\left(\frac{6}{x} - \frac{x}{2}\right)$$
$$= 6x - \frac{x^3}{2}$$
$$= \frac{1}{2}x(12 - x^2)$$

The values of x and y for $y = \frac{1}{2}x(12 - x^2)$ are given in the table below:

x	0.5	0.8	1	1.5	2	2.5	3
y	2.9	4.5	5.5	7.3	8	7.2	4.5

(c) On the given grid, draw the graph of $y = \frac{1}{2}x(12 - x^2)$ $f$or $0.5 \leq x \leq 3$. [2]

(d) Use your graph to find the volume of the cuboid when the base area is 4.41 cm².

$$x = \sqrt{4.41} = 2.1$$

Answer 7.9 cm³[2]

(e) (i) By drawing a tangent, find the gradient of the curve when x = 1.7 cm.

Working for gradient

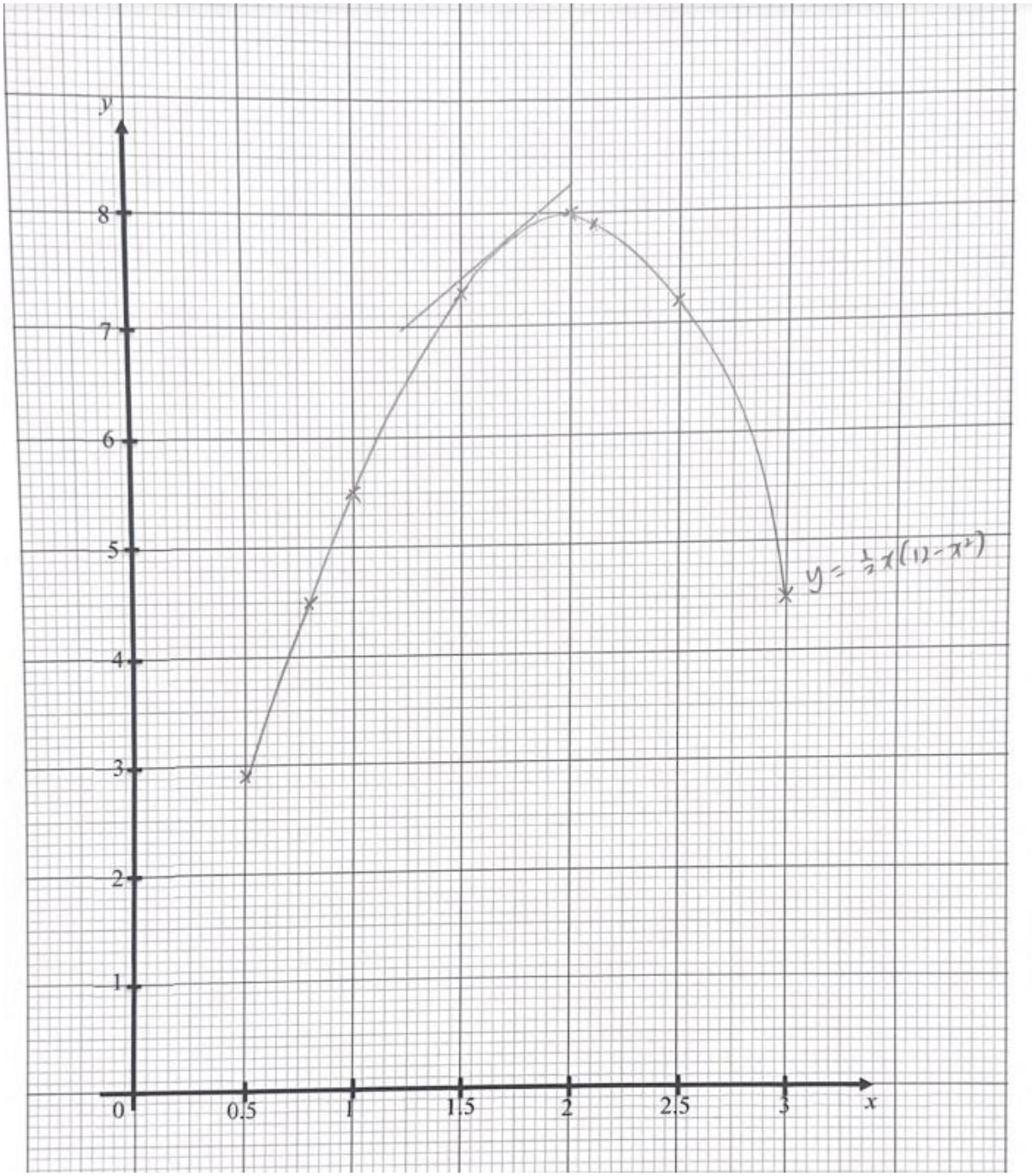
Answer accept 1-1 to 2.2……..[2]

(ii) What does this gradient represent?

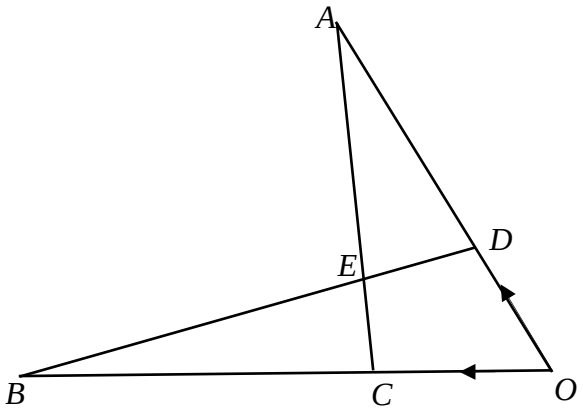
Answer Rep the rate of change of volume of changing. [1]

(f) State the maximum volume of the cuboid.

Answer 8 cm³ [1]


y
8
7
6
5
4
3
2
1
0
0.5
1
1.5
2
2.5
3
x
y = ½x(12 − x²)

7 In the diagram, $\overrightarrow{OD} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{b}$. It is given that $OD = \frac{1}{2}DA$ and $OC = \frac{1}{3}OB$.



(a) Express, as simply as possible, in terms of **a** and/or **b**,

(i) $\overrightarrow{AC}$,

Answer -3**a** + **b**….. [1]

(ii) $\overrightarrow{DB}$.

Answer **-a** + 3**b** .. [1]

(b) Given that $\frac{DE}{DB} = \frac{1}{4}$, express $\overrightarrow{OE}$ in terms of **a** and **b**.

$\overrightarrow{OE} = \overrightarrow{OD} + \overrightarrow{DE}$

$= \mathbf{a} + \frac{1}{4}\overrightarrow{DB}$

$= \mathbf{a} - \frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$

$= \frac{3}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$

Answer $\frac{3}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$ [2]

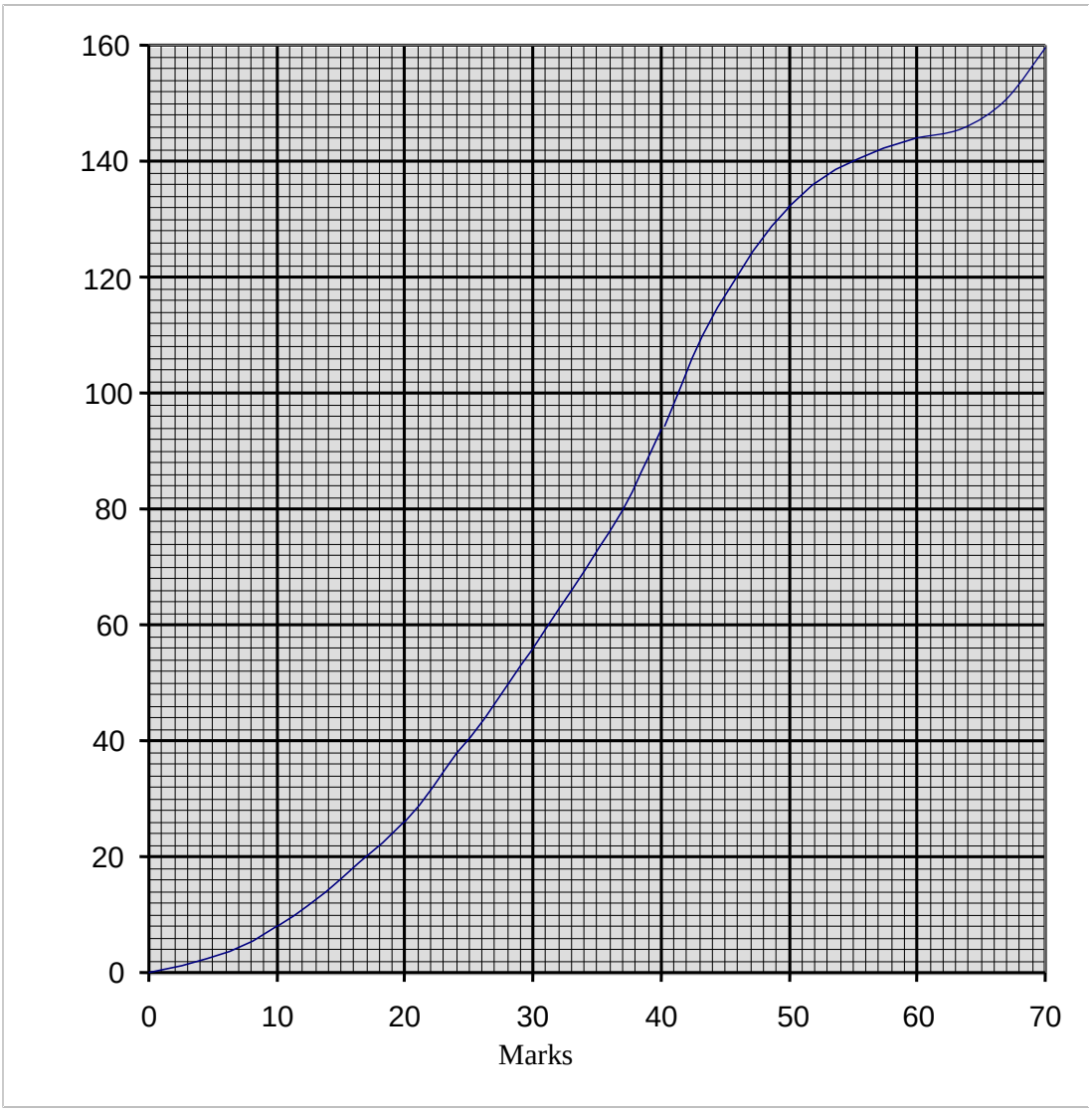
(c) Calculate the numerical value of $\frac{EC}{AE}$.

$$\overrightarrow{AE} = \overrightarrow{AO} + \overrightarrow{OE}$$
$$= -3\mathbf{a} + \frac{3}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$$
$$= -\frac{9}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$$

$$= \frac{3}{4}(\mathbf{b} - 3\mathbf{a})$$
$$= \frac{3}{4}\overrightarrow{AC}$$
$$\frac{AE}{AC} = \frac{3}{4}$$
$$\frac{EC}{AE} = \frac{1}{3}$$

Answer ……[2]

8 The cumulative frequency curve below shows the distribution of the marks obtained by 160 students for Mathematics Test 1.



(a) Estimate from the graph,

(i) the median,

Answer 37……[1]

(iii) the interquartile range,

46 - 25

Answer 21..……[1]

(iii) the percentage of students who obtained more than 50 marks.

$$\frac{28}{160}\times 100\% = 17.5\%$$

Answer 17.5%……[1]

(b) Given that 15% of the students will be awarded a distinction grade, use the graph to find the lowest mark scored by this group of students.

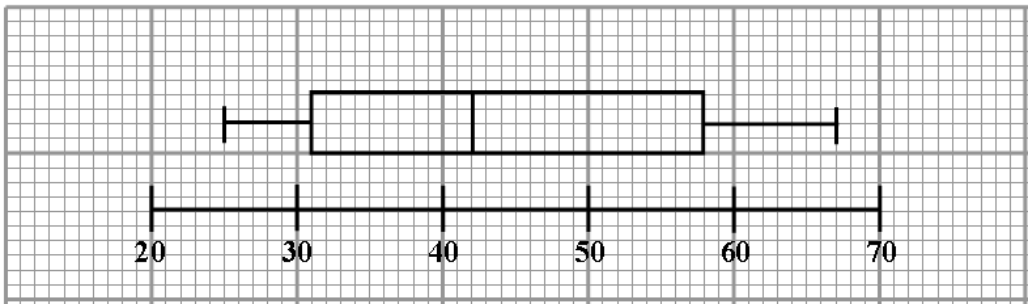
Answer 52……[1]

(c) Find the probability of choosing 2 students whose marks are in the 90th percentile and above.

$$\frac{16}{160}\times\frac{15}{159}=\frac{1}{106}$$

Answer ……[2]

(d) The marks obtained by the same group of 160 students who sat for Mathematics Test 2 are also noted. The results are summarised by a box-and-whisker plot shown below.



(i) State the median mark.

Answer 42…….. [1]

(ii) Find the interquartile range of the marks.

$$58 - 31 = 27$$

Answer 27 ..……[1]

(e) Make 2 comparisons between the marks obtained by the students in the two tests.

1 Students did better in Test 2 as the median mark in Test 2 is higher.

…….…… [1]

2 Students' performance more consistent in Test 1 as the interquartile range is smaller which implies a smaller spread about the median mark . ……. ……..……..…… [1]

9 An event company is holding a 5-day food exhibition for 200 exhibitors to promote their food items. The event company collects a registration fee of $88 and a daily booth rental of $200 from each exhibitor.

(a) Calculate the total amount of money collected from each exhibitor.

88 + 200 (5) = $1088

Answer $…….. [1]

The event company needs to rent **one entire exhibition space** for the food exhibition.
The space required for the exhibition is as follows:

Size of each exhibition booth	$20m^2$
Space for visitors' movement and dining area	60% of the total area occupied by the exhibition booths

The layout of the 5 available exhibition halls is as shown:

Hall A	**Hall B**	**Hall C**	**Hall D**	**Hall E**
2000 m^2	3000 m^2	3500 m^2	4500 m^2	2500 m^2
$6000 per day	$8500 per day	$9000 per day	$10000 per day	$7000 per day

Note: Adjacent exhibition halls have removable partitions to expand into a bigger exhibition hall.

(b) Calculate the **minimum** amount payable by the event company for the daily rental of the exhibition space needed.

Space needed for exhibitor = 200 x 4 x5 = 4000m^2
Total space needed = 4000 x 1.6 = 6400 m^2
Hall D and E cheapest

[A1]
Answer $17 000 [2]

The event company will set up the entire exhibition one day before the actual exhibition starts. They need to include the rental of the entire exhibition space, tables and chairs for this extra day of set up.

50 long tables and 350 chairs will be set up in the dining area.
Each exhibition booth will require 4 square tables and 4 chairs.

Items available for rental for exhibition booths set-up:

Rental of square tables: Per 100 tables at $30 per day
Rental of long tables: $2 each table per day
Rental of chairs: Per 50 chairs at $10 per day

The event company also needs to employ 3 security guards and 2 part-time staff for 12 hours per exhibition day.

Information of other costs:

Security Guards: Company A: \$10 per hour per guard Company B: \$8 per hour for first 8 hours per guard per day and \$12 for subsequent hour per guard per day.
Part-time staff: \$9 per hour per staff

(c) It is estimated that there will be a total of 48 000 visitors for the 5-day exhibition. The event company wishes to earn a total of \$200 000 profit from organizing the event. Using your answers in parts **(a)** and **(b)**, calculate and suggest a suitable amount of entrance fee that the event company should charge for each visitor.
Explain your answer.

Square Tables
Number of tables required: 4 x 200 = 800
Cost of rental = 800/100 x 30 x 6 = \$1440

Long Tables
Cost of rental = 50 x 2 x 6= \$600

Chairs
Number of chairs required = 800 + 350 =1150
Cost of Rental = 1150/50 x 10 x 6 = \$1380

Venue
Cost of rental = 6 x 17000 =\$102 000

Labour
Wages = [8 x 8 + 12 x 4] x 3 x 5 + 9 x 12 x 2 x 5 =\$2760

Total cost = 1440 + 600 + 1380 + 102 000 + 2760
= \$108 180

Collection from exhibitors = 1088 x 200 = \$217 600

Entrance Fee = $\frac{108180 + 200000 - 217600}{48000}$
= \$1.887

Answer \$2 because this amount can cover all costs and profits needed and there is also an excess in case there are less than 48 000 visitors. ……. [8]

End of Paper