NATIONAL JUNIOR COLLEGE

PRELIMINARY EXAMINATIONS

Higher 2

MATHEMATICS — 9740/01

Paper 1

14 September 2012

3 hours

Additional Materials: Answer Paper
List of Formulae (MF15)
Cover Sheet

0815 – 1115 hours

READ THESE INSTRUCTIONS FIRST

Write your name, registration number, subject tutorial group, on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for diagrams or graphs.

Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise. Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

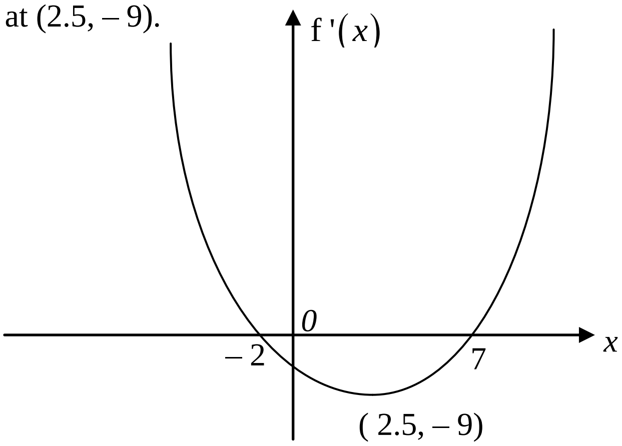
At the end of the examination, fasten all your work securely together.

The number of marks is given in the brackets [ ] at the end of each question or part question.

This document consists of **9** printed pages and **2** blank pages.

National Junior College

1 The diagram shows the graph of $y = \mathrm{f}'(x)$, where $\mathrm{f}(x)$ is a cubic polynomial, has a minimum point at (2.5, – 9).



Given that $\mathrm{f}(0) = 1$, find $\mathrm{f}(x)$. **[4]**

State the range of values of *x* for which $\mathrm{f}''(x) < 0$. **[1]**

2 Relative to the origin *O*, the position vectors of three points, *A*, *B* and *P* are, $-\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$, $5\mathbf{i} + 2\mathbf{k}$ and $(1 + 2\lambda)\mathbf{i} + (\lambda - 2)\mathbf{j} + 2\mathbf{k}$, where λ is a real parameter, $\lambda \neq -1$.

(i) Show that *A*, *B* and *P* are collinear. **[2]**

(ii) Find the value of λ such that *P* is on the line *BA* produced and area of triangle *OAP* is $162\sqrt{5}$ square units. Give a reason for your choice. **[4]**

3 Use the standard series for e^x and $(1+x)^n$ to find the Maclaurin series for $f(x)$, where $f(x) = e^{x^2}\left(\sqrt{1+2x}\right)$, up to and including the term in x^3. **[3]**

(a) By substituting $x = \frac{1}{3}$, find an approximation for $\left(\sqrt{135}\right)e^{\frac{1}{9}}$. **[2]**

(b) Find the series for $f'(x)$ up to and including the term in x^2. Hence or otherwise, find the Maclaurin series for $\frac{e^{x^2}}{\sqrt{1+2x}}$, up to and including the term in x^2. **[3]**

4 It is known that a particular type of bacteria grows very well under certain controlled conditions in a specially prepared Petri dish. The researcher believes that the growth rate of such bacteria can be modeled by $t^2\frac{dx}{dt} - 2xt = -x^2$, where x milligrams is the amount of bacteria grown in the dish after t hours.

(i) Using the substitution $x = ut^2$, show that the differential equation can be reduced to $\frac{du}{dt} = -u^2$. **[2]**

(ii) Find x in terms of t, given that there was 0.2 milligrams of bacteria after 15 minutes. Hence find the amount of this particular type of bacteria after 4 hours. **[4]**

(iii) Explain if this mathematical model is a realistic one. **[2]**

5 A curve C has parametric equations

$$x = \cos 2t, \quad y = \tan t, \quad \text{for } -\frac{\pi}{2} < t < \frac{\pi}{2}.$$

(i) Sketch the curve C, indicating clearly any asymptotes, and axial intercept(s). **[2]**

(ii) The point P on the curve has parameter $t = \frac{\pi}{3}$. If the normal to the curve at P passes the through the point $(b, 0)$, find the exact value of b. **[3]**

(iii) Show that the area bounded by the curve C, y-axis and $y = 4$, is $3 + \frac{\pi}{2} - 2\tan^{-1} 4$. **[4]**

6 The planes p_1 , p_2 and p_3 have equations $x = 1$, $2x + y + az = 5$ and $x + 2y + z = b$, where a and b are real constants. Given that p_1 and p_2 intersect at the line l, show that the vector equation of l, in terms of a, is

$$\mathbf{r} = \mathbf{i} + (3 - \lambda a)\mathbf{j} + \lambda\mathbf{k}, \text{ where } \lambda \text{ is a real constant.}$$ **[2]**

(a) The acute angle between l and p_3 is $60°$. Find the possible values of a. **[3]**

(b) Given that the shortest distance from origin to p_3 is $\frac{\sqrt{6}}{3}$ and without solving for the value of b, determine the possible position vectors of the foot of perpendicular from the origin to p_3. **[2]**

(c) What can be said about a and b if p_1 , p_2 and p_3 do not have any points in common? **[4]**

7(a) A geometric series has common ratio r, and an arithmetic series has first term a and common difference d, where a and d are non-zero. The first three terms of the geometric series are equal to the ninth, fourth and second terms respectively of the arithmetic series.

(i) Show that $d = 3a$. **[2]**

(ii) Deduce that the geometric series is convergent, and find in terms of a, the sum to infinity. **[3]**

(b) A mountaineer climbs a mountain of height x metres. The amount of distance he climbs for the first hour is 300 metres. For each subsequent hour, he climbs 10 metres less than the previous hour. The number of whole hours that has passed just before he reaches the summit is n.

(i) Write down an expression for the total distance climbed by the mountaineer and hence show that $pn^2 + qn \le x$, where p and q are constants to be determined. **[2]**

(ii) Deduce the value of n if $x = 2500$. **[2]**

8 **A calculator is not to be used in answering this question.**

The polynomial $\mathrm{p}(z)$ is defined by

$$\mathrm{p}(z) = z^3 + mz^2 - 7z + 15,$$

where m is a real constant. It is given that $(z+3)$ is a factor of $\mathrm{p}(z)$, find the value of m.

[1]

Given that z_1 , z_2 and z_3 are the roots of the equation $\mathrm{p}(z) = 0$, where $-\pi < \arg(z_1) < \arg(z_2) < \arg(z_3) \leq \pi$,

(i) find z_1, z_2 and z_3 in exact Cartesian form $x + \mathrm{i}y$ where $x, y \in \mathbb{R}$. **[3]**

(ii) show that $z_1^{\ n} - z_2^{\ n}$ is purely imaginary for $n \in \mathbf{Z}^+$. **[3]**

(iii) explain whether the locus of the complex number w satisfying the equation $|w| = a$, for some positive constant a, passes through all the points representing the complex numbers z_1, z_2 and z_3. **[2]**

9 A sequence u_1, u_2, u_3, … is such that $u_1 = \frac{1}{2}$ and

$$u_{n+1} = u_n - \frac{2}{n(n+1)(n+2)}, \qquad \text{for all positive integers } n.$$

(i) Prove by mathematical induction that $u_n = \frac{1}{n(n+1)}$. **[4]**

(ii) Hence, find $\sum_{r=1}^{N} \frac{1}{r(r+1)(r+2)}$. **[3]**

(iii) Deduce from your results that the series

$$\frac{1}{3^3} + \frac{1}{4^3} + \frac{1}{5^3} + \dots$$

is less than $\frac{1}{4}$. **[2]**

(iv) Using the result in (ii), find $\sum_{r=10}^{N} \frac{1}{r(r-1)(r-2)}$, leaving your answer in the form $b + \mathrm{f}(N)$ where b is a constant and $\mathrm{f}(N)$ is a function involving N. **[3]**

10(a) The curve C has equation

$$y=\frac{2x^2-a^2x+4a^2}{x^2-a^2},\ x\neq \pm a,$$

where a is a positive constant.

(i) Find the equations of the asymptotes, leaving your answers in terms of a. **[2]**

(ii) Show that if C has two turning points, then $0<a<6$. **[3]**

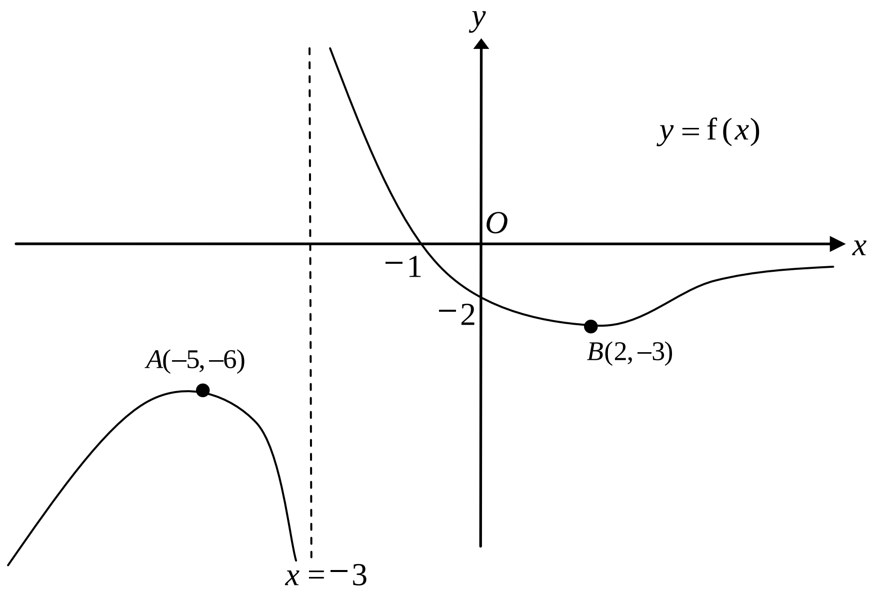
For $a=4$, sketch the curve C, stating the equations of any asymptotes, the coordinates of the turning points and axial intercept(s) if any. **[2]**

Hence, find the range of values of h such that the graph of $h^2\left(x-6+2\sqrt{5}\right)^2+\left(y+1\right)^2=h^2$,

where h is a positive integer, intersects C more than once. **[2]**

(b) The diagram shows the graph of $y=\mathrm{f}(x)$, which has turning points at $A(-5,-6)$ and $B(2,-3)$. The horizontal and vertical asymptotes are $y = 0$ and $x = -3$ respectively.

Sketch the graph of $y=\dfrac{1}{\mathrm{f}(x)}$, showing clearly all relevant asymptotes, axial intercepts and turning point(s), where possible. **[3]**



11 The functions f, g and h are defined by

$$\mathrm{f} : x \mapsto x - 2x^2 + 1, \ x \in \mathbb{R}, \ \frac{1}{4} \le x < 3,$$

$$\mathrm{g} : x \mapsto \ln(1-2x), \ x \in \mathbb{R}, \ k \le x < \frac{1}{4},$$

$$\mathrm{h} : x \mapsto 18|x-2| - 18, \ x \in \mathbb{R}.$$

(i) Sketch, on the same diagram, the graphs of f, f^{-1} and $\mathrm{f}^{-1}\mathrm{f}$, showing clearly the relationship between the graphs and indicating the axial intercepts. Find the exact value of x for which $\mathrm{f}^2(x) = x$. **[5]**

(ii) Find the minimum value of k such that $\mathrm{f}^{-1}\mathrm{g}$ exists. **[2]**

(iii) Using a non-graphical approach, find the set of exact values of x for which $\mathrm{f}(x) \le \mathrm{h}(x)$. **[4]**

End of Paper

NATIONAL JUNIOR COLLEGE

Preliminary Examinations

Higher 2

MATHEMATICS **9740/01**
Higher 2 **Paper 1**

14 September 2012, Friday 0815 – 1115 hours

Candidate Name: ______________________ **Registration No.:_____________**

Subject Class: 2ma______/ 2IPma2_______ **Subject Tutor: _______________**

Cover Page

INSTRUCTIONS TO CANDIDATES

Write your name, registration number, subject tutorial group, subject tutor's name and calculator model in the spaces provided on the cover sheet and attached it on top of your answer paper.

Circle the questions you have attempted and arrange your answers in **NUMERICAL ORDER**.

Write your calculator's model number(s) in the box below.

Scientific Calculator Model:

Graphic Calculator Model:

For office use

Question No.	Marks Obtained	TOTAL MARKS
1		*5*
2		*6*
3		*8*
4		*8*
5		*9*
6		*11*
7		*9*
8		*9*
9		*12*
10		*12*
11		*11*
Presentation	– 1 / –2	
TOTAL		100
GRADE		