VICTORIA JUNIOR COLLEGE

2023 PROMO PRACTICE PAPER B

(Modified from 2019 VJC H2 Math PROMO)

- Exam conditions (one sitting, 3 hours)
- Manage your time well
- Check against solutions and learn from your mistakes before the next practice

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1 **(i)** Sketch the curve with equation $y = 3\ln x + 3$, giving the equation of the asymptote and the coordinates of any points of intersection with the axes. On the same diagram, sketch the curve with equation $y = x^{\frac{5}{3}}$. [3]

(ii) Solve the inequality $3\ln x + 3 \geqslant x^{\frac{5}{3}}$. [2]

2 Given that $\left(3x^2 - y^2\right)\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2xy$, and that $y = 1$ when $x = 0$, find the values of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ when $x = 0$.
Hence write down the first two non-zero terms in the Maclaurin series for y. [4]

3 **(i)** Using integration, show that $\int \mathrm{e}^{2x} \sin x \, \mathrm{d}x = \dfrac{1}{5}\mathrm{e}^{2x}\left(2\sin x - \cos x\right) + C$, where C is an arbitrary constant. [4]

(ii) Hence find the gradient of the curve

$$y = \mathrm{e}^{2x+2}\left[2\sin\left(x+1\right) - \cos\left(x+1\right)\right]$$

at $x = \dfrac{\pi}{2} - 1$, leaving your answer in an exact form. [2]

4 The curve C has equation $\dfrac{\left(y-2\right)^2}{9} - \dfrac{\left(x-3\right)^2}{4} = 1$.

(i) Sketch C, giving the equations of its asymptotes and the coordinates of any turning points. [4]

The curve D has equation $12y^2 - 48y + 48 + ax^2 - 6ax - 3a = 0$, where a is a positive constant.

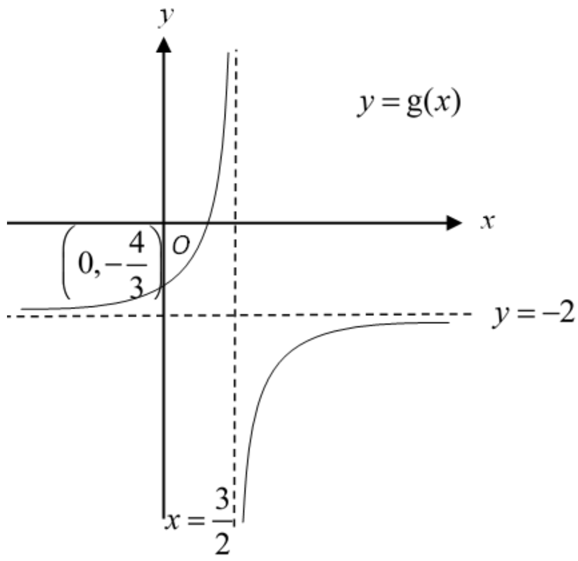
(ii) Find the set of values of a for which C and D do not intersect. [4]

5 **(i)** Find the binomial expansion for $\dfrac{1+x^2}{2-x^2}$, up to and including the term in x^4. Give the coefficients as exact fractions in their simplest form. [3]

(ii) Find the set of values of x for which this expansion is valid. [2]

(iii) Using your answer in part **(i)**, find the series expansion of $\left(2-x^2\right)^{-2}$, up to and including the term in x^2. [3]

6 The diagram below shows the graph of $y = \mathrm{g}(x)$, where $\mathrm{g}(x) = \dfrac{ax+b}{2x+c}$.



Determine the values of *a*, *b* and *c*. [3]

It is also given that $\mathrm{g}(x) = \mathrm{f}\left(\frac{1}{2}x - 1\right)$. State a sequence of 2 transformations that will map the graph of $y = \mathrm{g}(x)$ to the graph of $y = \mathrm{f}(x)$.

Find $\mathrm{f}(x)$. [5]

7 **(a)** Find $\int \sin 2x \cos^6 2x \, \mathrm{d}x$. [2]

(b) **(i)** Find $\int \dfrac{3x}{x^2+2} \, \mathrm{d}x$. [2]

(ii) Show that $\dfrac{2}{x-3} + \dfrac{3x+1}{x^2+2} = \dfrac{Ax^2+Bx+1}{(x-3)(x^2+2)}$, where *A* and *B* are constants to be found. [2]

(iii) Using your answers to parts **(i)** and **(ii)**, find $\displaystyle\int_0^2 \frac{10x^2-16x+2}{(x-3)(x^2+2)} \, \mathrm{d}x$.

Give your answer in the form $a \tan^{-1} b - \ln c$, where *a*, *b* and *c* are constants to be determined. [4]

(c) **JJC Prelim 9758/2018/02/Q4**

By using the substitution $x = \frac{1}{3}\sin^2\theta$, where $0 \le \theta < \frac{\pi}{2}$, find the exact value of

$$\int_0^{\frac{1}{4}} \sqrt{\frac{x}{1-3x}} \, \mathrm{d}x.$$ [5]

8 **2018/NJC/Prelim/2/4**

(a) It is given that $z^* = \dfrac{(-1-\mathrm{i})^3}{1-\mathrm{i}\sqrt{3}}$, where z^* is the conjugate of a complex number z.

(i) Find the exact values of the modulus and argument of $\dfrac{1}{z}$. [5]

(ii) Hence determine the exact values of a and b (where $-\pi < b \le \pi$) in the equation $\mathrm{e}^{2a+ib} = \dfrac{1}{z^4}$. [3]

(b) The complex variables u and v satisfy the equations

$$\mathrm{i}u - v = 3 \quad \text{and} \quad u^* + (1-\mathrm{i})v = 7 + 4\mathrm{i}.$$

Find the values of u and v, giving your answers in the form $x + \mathrm{i}y$. [4]

9 A curve C has equation $y = \dfrac{\alpha x^2 + x + 1}{x+2}$, where α is a real, non-zero constant.

Show that if C has 2 stationary points, then $\alpha < 0$ or $\alpha > k$, where k is a constant to be determined. [4]

Sketch the curve C for $\alpha = -1$, giving the equations of asymptotes, the coordinates of stationary points and points of intersection with the axes. [4]

10 During test drives, a sensor is used to record the number of revolutions per minute made by a particular wheel of a vehicle.

(a) In a test drive, a car is initially travelling at constant speed but starts to slow down due to engine malfunction. The total number of revolutions is recorded on every 1-minute interval after malfunction. In the first n minutes, the total number of revolutions recorded, S_n, is given by $S_n = 54n(29-n)$.

(i) Show that the number of revolutions recorded in each minute after the malfunction occurs, before the car comes to a complete stop, follows an arithmetic progression. [3]

(ii) The diameter of each wheel of the car is measured to be 61 cm. Show that the car travels a distance of 21.7 km, correct to 3 significant figures, from the time the malfunction occurs until it comes to a complete stop. [3]

(b) In another test drive, a truck was travelling at constant speed before it entered the rough terrain.

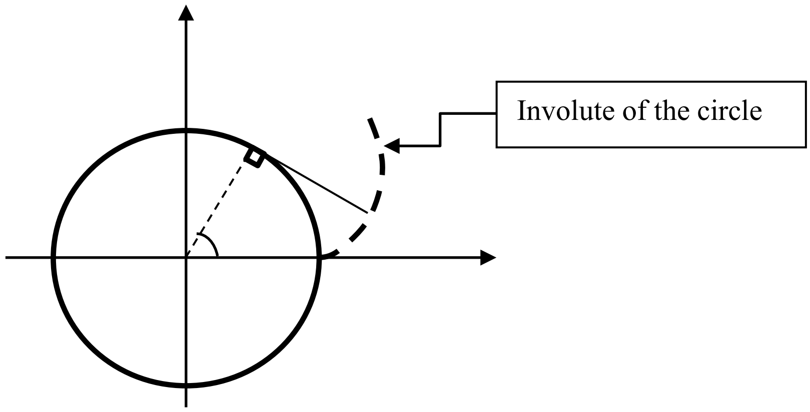
Before entering the rough terrain, the wheel was rotating at 486 revolutions per minute (rpm). After entering the rough terrain, engine power increases the rate of rotation by 20 rpm almost immediately at the beginning of each minute. However, at the end of each minute, friction slows the truck down such that the rate of rotation is $\dfrac{2}{3}$ of that recorded at the beginning of that minute. The rate of rotation of the wheel at the end of the the nth minute after entering the rough terrain is denoted by v_n rpm.

(i) Show that $v_n = 446\left(\dfrac{2}{3}\right)^n + 40$. [4]

(ii) Explain why the wheel always rotates at a rate of more than 40 rpm. [2]

(iii) Given that the rate of rotation of the wheel was less than 45 rpm at the end of m minutes, find the least integer value of m. [2]

11 The diagram shows a string that is unwound from a circle while being held taut. The curve traced by the end point P of the string is called the *involute* of the circle. One of the major applications of involute of circle is in designing of gears for revolving parts where gear tooth follow the shape of involute.



A circle has fixed radius a units and centre O and the initial position of P is at $(a, 0)$.

The parameter θ, $\left(0 \leqslant \theta \leqslant \frac{\pi}{2}\right)$, is the angle measured from the positive x-axis to OT in the anti-clockwise direction, where T is the point on the circle such that PT is tangential to the circle.

Show that the involute has parametric equations

$$x = a(\cos\theta + \theta\sin\theta),\ y = a(\sin\theta - \theta\cos\theta), \text{ for } 0 \leqslant \theta \leqslant \frac{\pi}{2}. \qquad [3]$$

The point W on the involute has parameter $\theta = \frac{\pi}{3}$.

(i) Show that the equation of the normal to the involute at W is $\sqrt{3}y = 2a - x$. [5]

(ii) At W, x increases at a rate of 0.3 units per second. Given that $z = xy$, determine, in terms of a, the rate of change of z at W. [4]