Sec 4 Add Math Preliminary Exam 2023 P2 Marking Scheme

Qn. No.	Solution	Marks	AO
1(a)	$\begin{array}{r} 2x^2+1 \\ x+3\overline{)2x^3+6x^2+x+3} \\ -(2x^3+6x^2) \\ \hline x+3 \\ -(x+3) \\ \hline 0 \end{array}$ Or $2x^3+6x^2+x+3 \div x+3 = 2x^2+1$	B1 B1(either presentation)	AO1
1(b)	$\frac{9x^2-10x-16}{2x^3+6x^2+x+3} = \frac{9x^2-10x-16}{(x+3)(2x^2+1)} = \frac{A}{(x+3)}+\frac{Bx+C}{(2x^2+1)}$ $9x^2-10x-16 = A(2x^2+1)+(Bx+C)(x+3)$ Let $x=-3$, $81+30-16=19A$ $95=19A$ $A=5$ Let $x=0$, $-16=5+3C$ $-21=3C$ $C=-7$ Let $x=1$, $-17=15+4(B-7)$ $-17=-13+4B$ $B=-1$ $\frac{9x^2-10x-16}{2x^3+6x^2+x+3} = \frac{5}{(x+3)}+\frac{(-x-7)}{(2x^2+1)}$ or $\frac{5}{(x+3)}-\frac{x+7}{(2x^2+1)}$	M1 M1(substitution or comparison method) A1(1st unknown) Allow FT2 A1(for the remaining unknown) B1	AO1
2(a)	$\frac{d}{dx}\left(e^{2x}(5x-4)\right) = (5x-4)2e^{2x}+5e^{2x}$ $= e^{2x}(10x-8+5)$ $= e^{2x}(10x-3)$ (shown)	M2(M1 for each part of using product rule) AG1	AO3

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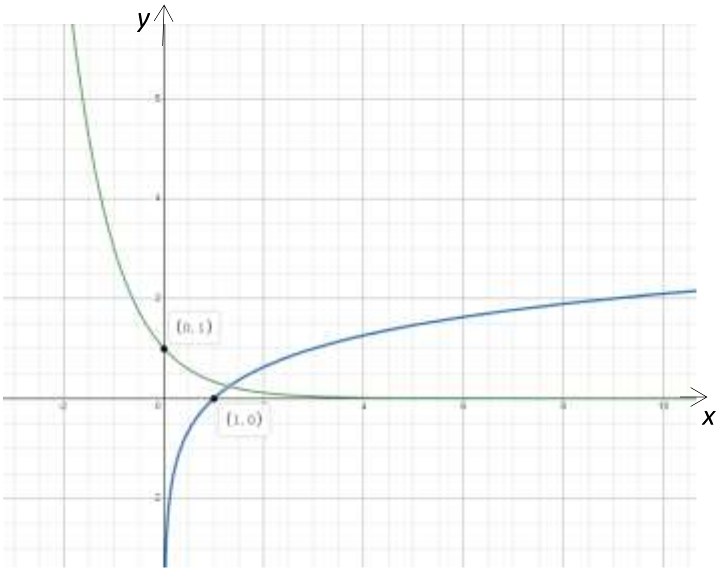
2(b)	$\int\left[e^{2x}(10x-3)\right]dx = e^{2x}(5x-4)+C$ $\int 10xe^{2x}-3e^{2x}\ dx = e^{2x}(5x-4)+C$ $10\int xe^{2x}dx - 3\int e^{2x}\ dx = e^{2x}(5x-4)+C$ $10\int xe^{2x}dx\ = e^{2x}(5x-4)+3\int e^{2x}dx + C$ $10\int xe^{2x}dx\ = e^{2x}(5x-4)+3\left(\frac{e^{2x}}{2}\right)+C$ $\frac{10}{4}\int 4xe^{2x}dx\ = e^{2x}(5x-4)+3\left(\frac{e^{2x}}{2}\right)+C$ $= e^{2x}\left(5x-4+\frac{3}{2}\right)+C$ $= e^{2x}\left(5x-\frac{5}{2}\right)+C$ $\int 4xe^{2x}dx\ = \frac{2}{5}e^{2x}\left(5x-\frac{5}{2}\right)+C$ or $= e^{2x}(2x-1)+C$ $\int_0^3 4xe^{2x}\ dx = \left[e^{2x}(2x-1)\right]_0^3$ $= \left[5e^{6}-(-1)\right]$ $= 5e^{6}+1$ or 2020 (3sf)	M1 M1(correct integration) M1(manipulation) M1(simplify and obtain $\int 4xe^{2x}dx$) A1	AO2
3(a)	$3=\log_p 27$ $p^3=27$ $p=3$ $-2=\log_3 q$ $3^{-2}=q$ $q=\frac{1}{9}$	 B1 B1	AO1

Qn. No.	Solution	Marks	AO
3(b)		C2(for sketch of 2 curves correctly) P1(the x-intercept and y-intercept clearly indicated) Minus 1 mark if axes not labelled	AO1
3(c)	1 solution	B1	AO2
4(a)	$\log_6(2^y+1)-\log_6(2^y-4)=1$ $\log_6\frac{(2^y+1)}{(2^y-4)}=1 \text{ or } \log_6 6$ $\frac{(2^y+1)}{(2^y-4)}=6^1$ $2^y+1=6(2^y-4)$ $2^y+1=6(2^y)-24$ $5(2^y)=25$ $2^y=5$ $y\ln 2=\ln 5$ $y=\frac{\ln 5}{\ln 2}$ $y=2.32$ (3 sf)	M1(quotient law) M1(simplify) M1(ln on both sides and power law) A1	AO1

Qn. No.	Solution	Marks	AO
4(b)	$\left(\log_x x+\log_x y\right)\left(\frac{\log_x x^6}{\log_x y}\right)=8$ $\left(1+\log_x y\right)\left(\frac{6}{\log_x y}\right)=8$ $\frac{6}{\log_x y}+6=8$ $\frac{6}{\log_x y}=2$ $\log_x y=\frac{6}{2}$ $y=x^3$	M1(product law) M1(change of base) M1(change log. to exponential form) A1	AO1
5(a)	$f'(x)=\int\left(4\cos 4x+2\sin 2x\right)\,dx$ $=\sin 4x-\cos 2x+C_1$ $f(x)=\int\left(\sin 4x-\cos 2x+C_1\right)\,dx$ $=\frac{-\cos 4x}{4}-\frac{\sin 2x}{2}+C_1x+C_2$ $f(0)=\frac{-1}{4}+C_2=0$ $C_2=\frac{1}{4}$ $f(\frac{\pi}{4})=\frac{1}{4}-\frac{1}{2}+C_1\left(\frac{\pi}{4}\right)+\frac{1}{4}=\frac{3}{4}$ $C_1\left(\frac{\pi}{4}\right)=\frac{3}{4}$ $C_1=\frac{3}{\pi}$ $f(x)=\frac{-\cos 4x}{4}-\frac{\sin 2x}{2}+\frac{3}{\pi}x+\frac{1}{4}$	M1(award marks even without C_1) M1(award marks even without C_2) M1(for substitution) A1(for either C_1 or C_2 correct) A1	AO2

Qn. No.	Solution	Marks	AO
5(b)	$f\left(\frac{\pi}{6}\right) = \frac{-\cos\frac{2\pi}{3}}{4} - \frac{\sin\left(\frac{\pi}{3}\right)}{2} + \frac{1}{2} + \frac{1}{4}$ $= \frac{\frac{1}{2}}{4} - \frac{\frac{\sqrt{3}}{2}}{2} + \frac{3}{4}$ $= \frac{1}{8} - \frac{\sqrt{3}}{4} + \frac{3}{4}$ $= \frac{7}{8} - \frac{\sqrt{3}}{4}$ $= \frac{7-2\sqrt{3}}{8}$ (shown)	M1(for basic angles) M1 AG1(depends on $\frac{7}{8} - \frac{\sqrt{3}}{4}$)	AO3
6(a)	(a) 2g = –4 and 2f = 6 g = –2 and f = 3 Centre (2, -3) Sub $x = 2$ and $y = -3$, $3(-3) - 4(2) = k$. $k = -17$ Radius = $\sqrt{4+9+12}$ = 5 units	B1(for centre) M1(substitution) A1 B1	AO2
6(b)	Length between centre of C_1 to centre (14, 2) $\sqrt{(14-2)^2 + (2+3)^2}$ = 13 units Radius of C_2 = 8 units Eqn. of C_2 ----- $(x-14)^2 + (y-2)^2 = 64$ Or $x^2 + y^2 - 28x - 4y + 136 = 0$	M1 A1 A1	AO2

Qn. No.	Solution	Marks	AO
7(a)	LHS $= \dfrac{2\tan x + 1 + \tan^2 x}{1-\tan^2 x}$ $= \dfrac{(1+\tan x)^2}{(1-\tan x)(1+\tan x)}$ $= \dfrac{(1+\tan x)}{(1-\tan x)}$ $= \dfrac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}}$ $= \dfrac{\frac{\cos x+\sin x}{\cos x}}{\frac{\cos x-\sin x}{\cos x}}$ $= \dfrac{\cos x+\sin x}{\cos x-\sin x}$ = RHS (proved)	M1(change $\sec^2 x$) M1(either factorization of numerator or denominator) M1(change tan x) AG1	AO3
Or 7(a)	LHS $= \dfrac{2\frac{\sin x}{\cos x}+\frac{1}{\cos^2 x}}{1-\frac{\sin^2 x}{\cos^2 x}}$ $= \dfrac{\frac{2\sin x\cos x+1}{\cos^2 x}}{\frac{\cos^2 x-\sin^2 x}{\cos^2 x}}$ $= \dfrac{2\sin x\cos x+1}{\cos^2 x-\sin^2 x}$ $= \dfrac{2\sin x\cos x+\sin^2 x+\cos^2 x}{(\cos x+\sin x)(\cos x-\sin x)}$ $= \dfrac{(\cos x+\sin x)(\cos x+\sin x)}{(\cos x+\sin x)(\cos x-\sin x)} = \dfrac{\cos x+\sin x}{\cos x-\sin x}$ = RHS(proved)	M1(change tan x & $\sec^2 x$) M1(simplify fractions) M1(factorization of either numerator or denominator	

Qn. No.	Solution	Marks	AO
7(b)	$\operatorname{cosec}^2 x - 5\cot x = -5$ $\left(1+\cot^2 x\right) - 5\cot x + 5 = 0$ $\cot^2 x - 5\cot x + 6 = 0$ $\left(\cot x - 2\right)\left(\cot x - 3\right) = 0$ $\cot x = 2$ or $\cot x = 3$ $\tan x = \frac{1}{2}$ or $\tan x = \frac{1}{3}$ $\alpha = 26.56°$ or $18.43°$ $x = 18.4°,\ 26.6°,\ 198.4°,\ 206.6°$ (1 dp)	M1(sub. $1+\cot^2 x$) M1(factorization or use quadratic formula) A2(all values of *x*) Or A1(for 2 angles)	AO1
8(a)	Area of triangle OAB = $\frac{1}{2}\times 50\times 50\times \sin(90^o - \theta)$ = $1250\cos\theta$ Area of triangle ODC = $\frac{1}{2}\times 80\times 80\times \sin\theta$ = $3200\sin\theta$ Total area *S* = 3200 sin θ + 1250 cos θ (shown)	M1 AG1	AO3

8(b)	(a) Let $3200\sin\theta + 1250\cos\theta = R\sin(\theta+\alpha)$ $= R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$ By comparing, $R\cos\alpha = 3200$ (1), $R\sin\alpha = 1250$ (2) (2)/(1) $\frac{R\sin\alpha}{R\cos\alpha} = \frac{1250}{3200}$ $\tan\alpha = \frac{25}{64}$ $\alpha = 21.3^o$ (3 sf) $(1)^2 + (2)^2$, $R^2 = 10240000 + 1562500$ $R = \sqrt{11802500} = 50\sqrt{4721}$ or 3440 (3sf) $S = \sqrt{11802500}\sin(\theta + 21.3^o)$ or $50\sqrt{4721}\sin(\theta+21.3^o)$ or $3435.5\sin(\theta+21.3^o)$ or $3440\sin(\theta+21.3^o)$	M1(for two eqns) B1(for α) B1(for R) A1	AO1
8(c)	<u>Max. value of *S*</u> When $\sin(\theta+21.34^o) = 1$ $\theta + 21.34^o = 90^o$ $\theta = 68.66^o$ *or* 68.7^o	M1 A1	AO1
9(a)	$2x + y = -5$ ----- (1) From (1), $y = -5 - 2x$ Sub, into $x(-5-2x) + 3 = 0$ $-2x^2 - 5x + 3 = 0$ $2x^2 + 5x - 3 = 0$ $(x+3)(2x-1) = 0$ $x = -3$ (NA) or $x = \frac{1}{2}$ When $x = \frac{1}{2}$, $y = -6$ B($\frac{1}{2}$, -6)	M1(substitution) M1(factorization or using quad. formula) A1	AO1

9(b)	Area of triangle = $\frac{1}{2}\times\left(\frac{5}{2}+\frac{1}{2}\right)\times 6 = 9$ $\int_{\frac{1}{2}}^{1} -\frac{3}{x}\,dx = -3\left[\ln x\right]_{\frac{1}{2}}^{1}$ $= 3\ln\frac{1}{2}$ $= -3\ln 2$ or $\ln\frac{1}{8}$ $= -\ln 8$ Area above curve = ln 8 Total area = 9 + $\ln 8$	M1(allow FT using their values) M1 M1(correct application of limits) M1(apply law of log. get $-\ln 8$) A1(total area)	AO2
9(c)	$y = \frac{-3}{x}$ $\frac{dy}{dx} = -3\left(-x^{-2}\right)$ $= 3x^{-2}$ At x = 1, $\frac{dy}{dx} = 3$ Gradient of normal = $-\frac{1}{3}$ When x =1 , $y = -3$ <u>Equation of normal</u> $y+3 = -\frac{1}{3}(x-1)$ or $y = -\frac{1}{3}x - \frac{8}{3}$	 B1 B1 B1(find y coordinate of pt. C) A1	AO2

<table>
<tr><td>10(a)</td><td>$x^2+2x-3 = (x-1)(x+3)$
f(1) = $1+6+2a+b-3a=0$
$b-a=-7$ -------- (1)

f(−3) = $(-3)^4+6(-3)^3+2a(-3)^2+b(-3)-3a=0$
$15a-3b-81=0$
$5a-b=27$ ----------- (2)

(1) + (2), $4a=20$
$a=5$
Sub. $a=5$ into (1), $b=-2$</td><td>M1(factorization to obtain 2 factors)

M1(sub x =1 and obtain eqn)

M1(sub x =-3 and obtain eqn)

M1(solve simultaneous eqns)

A2</td><td>AO1</td></tr>
<tr><td>10(b)</td><td>f(x) = $x^4+6x^3+10x^2-2x-15$
$= (x^2+2x-3)(x^2+kx+5)$
By comparing coeff. of x^3, $6=k+2$
$k=4$
The other quadratic factor is x^2+4x+5.</td><td>M1(comparison or long division method)

A1</td><td>AO1</td></tr>
<tr><td>10(c)</td><td>$(x^2+2x-3)(x^2+4x+5)=0$
$(x-1)(x+3)(x^2+4x+5)=0$
$x=1$ or $x=-3$ $b^2-4ac = 4^2-4(1)(5)$
$= -4<0$
No real roots
There is only 2 real distinct roots.</td><td>M1(two real roots)

M1(discriminant)

AG1</td><td>AO3</td></tr>
<tr><td>11(a)</td><td>See attached graph.
<table><tr><td>x</td><td>15</td><td>20</td><td>25</td><td>30</td><td>35</td><td>40</td></tr><tr><td>$lg\ y$</td><td>−0.82</td><td>−0.42</td><td>−0.022</td><td>0.37</td><td>0.77</td><td>1.17</td></tr></table></td><td>P1
L1</td><td>AO1</td></tr>
</table>

11(b) (i)	$\lg y = \lg\left(10^{-M} n^x\right)$ $= \lg 10^{-M} + \lg n^x$ $= -M \lg 10 + x \lg n$ $= -M + x \lg n$ y-intercept = $-2.025 \Rightarrow -M = -2.025$ $M = 2.025 \pm 0.05$ Gradient = $\dfrac{1.175 - 0.775}{40 - 35} = 0.08 \pm 0.01$ $\lg n = 0.08 \Rightarrow n = 10^{0.08} = 1.20 \pm 0.03$	M1(product law) B1(for M) M1(gradient) A1	AO2
11(b) (ii)	$y = 10$ $\lg y = \lg 10 = 1$ When $\lg y = 1$, from the graph, $x = 37.75 \pm 1$	M1(find lg y) A1	AO1