RAFFLES INSTITUTION
H2 Mathematics (9758)
2016 Year 5

Tutorial 7C : Graphing Techniques

Section A: Basic Questions

1 Sketch, on separate diagrams, the graphs of

(a) $y=\dfrac{2+x}{1-x}$, (b) $y=2x-1-\dfrac{6}{x}$,

giving the equations of any asymptotes and the coordinates of any points of intersection with the x- and y-axes.

2 Sketch, on separate diagrams, the graphs of

(a) $(x+3)^2+(y-4)^2=5$, (b) $\dfrac{x^2}{2^2}+\dfrac{y^2}{3^2}=1$,

(c) $\dfrac{y^2}{4}-\dfrac{x^2}{9}=1$, (d) $x^2+4x+y^2-2y-20=0$.

Section B: Standard Questions

3 **9740/2013/01/Q3**

(i) Sketch the curve with equation $y=\dfrac{x+1}{2x-1}$, stating the equations of any asymptotes and the coordinates of the points where the curve crosses the axes.

(ii) Solve the inequality $\dfrac{x+1}{2x-1}<1$.

[(ii) $x<\dfrac{1}{2}$ or $x>2$]

4 **9740/2008/01/Q9 (Modified)**

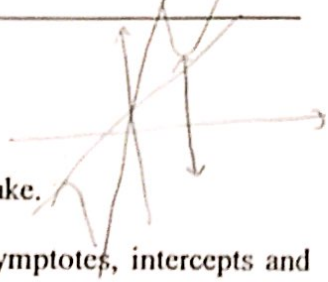
It is given that $\mathrm{f}(x)=\dfrac{ax+b}{cx+d}$, for non-zero constants a, b, c and d.

(i) Given that $ad-bc\neq 0$, show by differentiation that the graph of $y=\mathrm{f}(x)$ has no turning points.

(ii) What can be said about the graph of $y=\mathrm{f}(x)$ when $ad-bc=0$?

(iii) Deduce from part **(i)** that the graph of $y=\dfrac{3x-7}{2x+1}$ has a positive gradient at all points of the graph.

(iv) Sketch the graph of $y=\dfrac{3x-7}{2x+1}$, including the coordinates of the points where the graph cross the axes and the equations of any asymptotes.



5 9740/2013/01/Q2 (modified)

It is given that $y=\dfrac{x^2+x+1}{x-1}$, $x\in\mathbb{R}, x\neq 1$.

(i) Without using a calculator, find the set of values that y can take.

(ii) Sketch the graph of $y=\dfrac{x^2+x+1}{x-1}$, showing clearly any asymptotes, intercepts and turning points.

[(i) $\left(-\infty, 3-2\sqrt{3}\right]\cup\left[3+2\sqrt{3},\infty\right)$]

6 The function f is defined by $\mathrm{f}(x)=\dfrac{ax^2+bx-4}{x-c}$, $x\in\mathbb{R}$, $x\neq c$ for some real constants a, b and c. Given that $x=3$ and $y=x+1$ are asymptotes to the curve $y=\mathrm{f}(x)$, find the values of a, b and c. Draw a sketch of the curve $y=\mathrm{f}(x)$, indicating clearly the asymptotes, intersections with the axes and state the range of f.

[$a=1, b=-2, c=3$, range of f $=\mathbb{R}$]

7 9740/2009/01/Q6

The curve C_1 has equation $y=\dfrac{x-2}{x+2}$. The curve C_2 has equation $\dfrac{x^2}{6}+\dfrac{y^2}{3}=1$.

(i) Sketch C_1 and C_2 on the same diagram, stating the exact coordinates of any points of intersection with the axes and the equations of any asymptotes.

(ii) Show algebraically that the x-coordinates of the points of intersection of C_1 and C_2 satisfy the equation $2(x-2)^2=(x+2)^2(6-x^2)$.

(iii) Use your calculator to find these x-coordinates.

[(iii) -0.515, 2.45]

8 9740/2013/01/Q5(i)

It is given that

$$\mathrm{f}(x)=\begin{cases}\sqrt{\left(1-\dfrac{x^2}{a^2}\right)} & \text{for } -a\le x\le a,\\ 0 & \text{for } a<x<2a,\end{cases}$$

and that $\mathrm{f}(x+3a)=\mathrm{f}(x)$ for all real values of x, where a is a real constant.

Sketch the graph of $y=\mathrm{f}(x)$ for $-4a\le x\le 6a$.

9 Sketch, on separate diagrams, the curves defined by the following pair of parametric equations

(a) $x = t^2, \quad y = t^3$ for $t \geq 0$,

(b) $x = \cos^3 t, \quad y = \sin^3 t, \quad$ for $0 \leq t \leq \dfrac{\pi}{2}$,

(c) $x = (t+1)^2, \quad y = t^2 - 1$ for $t \in \mathbb{R}$,

(d) $x = \dfrac{t}{1-t}, \quad y = \dfrac{t^2}{1-t}$ for $t \in \mathbb{R}, t \neq 1$,

(e) $x = t + \dfrac{1}{t}, \quad y = t - \dfrac{1}{t}$ for $t \in \mathbb{R}, t \neq 0$

Find the equations of the curves in **(b)**, **(d)** and **(e)** in Cartesian form

State the equations of any asymptotes on your sketch of the curves for **(b)**, **(d)** and **(e)**.

[(b) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$ (d) $y = x - 1 + \dfrac{1}{x+1}$ (e) $\dfrac{x^2}{4} - \dfrac{y^2}{4} = 1$]

10 **9740/2012/01/Q11(i), (ii), (iv)**

A curve C has parametric equations

$$x = \theta - \sin\theta, \qquad y = 1 - \cos\theta,$$

where $0 \leq \theta \leq 2\pi$.

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \cot\dfrac{1}{2}\theta$ and find the gradient of C at the point where $\theta = \pi$.

What can be said about the tangents to C as $\theta \to 0$ and $\theta \to 2\pi$?

(ii) Sketch C, showing clearly the features of the curve at the points where $\theta = 0$, π and 2π.

(iv) A point P on C has parameter p, where $0 < p < \dfrac{1}{2}\pi$. Show that the normal to C at P crosses the x-axis at the point with coordinates $(p, 0)$.

Section C:Extension/Challenging Questions

11 Sketch without the use of a graphing calculator, the graphs of

(a) $y = \dfrac{x^2 + 2x - 3}{x - 1}$, $x \neq 1$; **(b)** $y = \dfrac{ax - b}{cx - d}$, $ad = bc$ and $x \neq \dfrac{d}{c}$;

(c) $y = \dfrac{x}{x^2 - a^2}$, $a > 0$, $x \neq \pm a$.

12 Find the equation of the circle that passes through the points :

$$(-1, 2),\ (3, 4) \text{ and } (2, -1).$$

$$\left[\left(x-\frac{5}{3}\right)^2+\left(y-\frac{5}{3}\right)^2=\frac{65}{9}\right]$$

13 A curve C has parametric equations

$$x=\frac{t}{1-t},\quad y=\frac{t^2}{1-t} \text{ and } t \neq 1.$$

(i) For what values of t does y becomes arbitrarily large? For the values of t found what does x tend towards ? Deduce the equation of the vertical asymptote.

(ii) Show that $\frac{y}{x}$ approaches 1 as t approaches 1. By considering $y-x$, show that as t approaches 1, C approaches a straight line, the equation of which is to be determined.

[(i) $x=-1$ (ii) $y=x-1$]

<u>Section D: Self-Practice Questions</u>

y = x − a asymptote

y = kx − a

x < 1

1 **SRJC Prelims 9740/2009/01/Q7**

The curve C has equation $y=\dfrac{x^2+b}{x+a}$, where $a>0$ and $b>0$.

(i) State the coordinates of the intersection(s) of the curve C with the axes in terms of a and b. [1]

(ii) Find the equation(s) of the asymptote(s). [2]

(iii) Draw a sketch of the curve C, labeling the equation(s) of its asymptote(s) and coordinates of any intersection with the axes. [2]

(iv) Hence find the range of values of k, where k is a positive constant, for which the equation $x^2+b=(x+a)(kx-a)$ has no real root. [2]

k < 1

$\frac{x^2+b}{x+a} = y = kx-a$

[(i) $\left(0,\dfrac{b}{a}\right)$, (ii) $x=-a$, $y=x-a$, (iv) $0<k\leq 1$]

2 **VJC Prelim 9740/2013/02/Q2(modified)**

The curve C has equation $y=\dfrac{ax^2+bx+c}{x-1}$, where a, b and c are non-zero constants.

(a) It is given that C passes through the point $\left(3,\dfrac{23}{2}\right)$ and has a minimum point at $(2, 10)$

(i) Find the values of a, b and c. [3]

(ii) Sketch C, giving the coordinates of any turning points, points of intersection with the axes and the equations of any asymptotes. [3]

$-2 < m < 2$ k ≠ 0.

(iii) Find the range of values m and k such that the curve with equation $\frac{x^2}{k^2}+\frac{y^2}{m^2}=1$ does not intersect C. [2]

(b) In the case when $a=-1$ and $b=-1$, find the set of values of c, where $c \neq 2$, such that C has no stationary point. [3]

[(a)(i) $a=3$, $b=-2$, $c=2$, (iii) $-2<m<2, k \neq 0$, (b) $\{c \in \mathbb{R} : c>2\}$]

3 NJC Promo 9740/2006/01/Q3b

A curve is defined parametrically by the equations

$$x=\frac{t}{1+t}, \quad y=\frac{t^2}{1+t}, \quad t \in \mathbb{R}, t \neq -1.$$

(i) Find the Cartesian equation of the curve. [2]

(ii) Sketch the curve. Label your graph clearly, indicating any asymptote(s) and stationary point(s). [3]

[$y=\frac{x^2}{1-x}$]

4 CJC Mid-yr 9740/2006/Q1(modified)

Sketch the graph of $y^2-4x^2=4$, giving the equations of the asymptotes if any.

5 TJC Promo 9740/2006/01/Q10

A curve C is defined by the parametric equations $x=\sqrt{t}$, $y=t-\frac{1}{\sqrt{t}}$, where $t>0$.

(i) Find the coordinates of the x-intercept of C. [2]

(ii) By considering the behaviour of C when $t \to 0$, find the linear asymptote of C. [1]

(iii) Sketch C. [1]

[(i) $(1,0)$ (ii) y-axis]

6 Sketch, on separate diagrams, the curves defined by the following pair of parametric equations, indicating any axial intercepts.

(a) $x=t^2(t+1)$, $y=4t-5$, $t \geq 0$;

(b) $x=\theta-\cos\theta$, $y=\sin\theta$, $-\pi \leq \theta \leq \pi$.

END OF TUTORIAL