Answer all the questions.

1i) Sub $(0,-1)$ into $x-py-2=0 \Rightarrow p=2$

1ii) Sub $x=2y+2$ into $x^2-2xy+y^2=1$

$y^2+4y+3=0$

$(y+1)(y+3)=0$

$y=-1\ (N.A)\ \ or\ \ y=-3$

The other point is $(-4,-3)$

2. Let $f(x)=bx^3+cx^2-7x+c$

$f(-1)=b(-1)^3+c(-1)^2-7(-1)+c$

$-b+c-7(-1)+c=7$

$b=2c$ -------------------------(1)

$f(3)=b(3)^3+c(3)^2-7(3)+c$

$27b+10c-21=43$

$27b+10c=64$ ---------------(2)

Substitute (1) into (2):

$27(2c)+10c=64$

$c=1,\quad b=2$

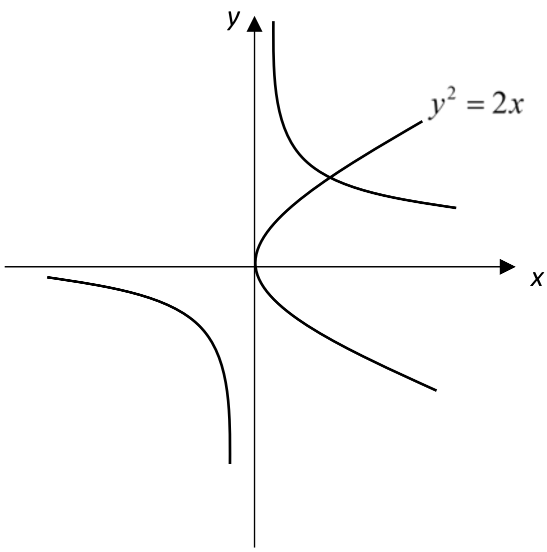
3. $\left(\frac{1}{x}\right)^2=2x$

$x^3=\frac{1}{2}$

$x=0.794$

$y=1.26$

The point of intersection is $(0.794, 1.26)$



4. Base area of cuboid = $\left(1+\sqrt{3}\right)^2$

$= \left(4+2\sqrt{3}\right)cm^2$

Height of cuboid = $\dfrac{14+12\sqrt{3}}{4+2\sqrt{3}} \times \dfrac{4-2\sqrt{3}}{4-2\sqrt{3}}$

$= \dfrac{56-28\sqrt{3}+48\sqrt{3}-72}{16-4(3)}$

$= \left(-4+5\sqrt{3}\right)$cm

5. Let $\dfrac{x^2+5x+4}{x\left(x^2+4\right)} = \dfrac{a}{x} + \dfrac{bx+c}{x^2+4}$,

$x^2+5x+4 = a\left(x^2+4\right)+x\left(bx+c\right)$

By comparing constants, $a=1$

By comparing x terms, $c=5$

By comparing x^2 terms, $1=1+b$

$b=0$

$\dfrac{2x^3-x^2+3x-4}{x\left(x^2+4\right)} = \dfrac{1}{x} + \dfrac{5}{x^2+4}$

6 (i) Let $f(y) = 2y^3 + (m-2)y^2 + (m-7)y - 3$

$$f(-1) = 2(-1)^3 + (m-2)(-1)^2 + (m-7)(-1) - 3$$
$$= -2 + m - 2 - m + 7 - 3$$
$$= 0$$

(ii) $$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + (m-2)\left(\frac{1}{2}\right)^2 + (m-7)\left(\frac{1}{2}\right) - 3 = 0$$

$$\frac{1}{4} + \frac{m-2}{4} + \frac{m-7}{2} - 3 = 0$$

$$1 + m - 2 + 2m - 14 - 12 = 0$$

$$m = 9$$

(iii) $$f(y) = 2y^3 + 7y^2 + 2y - 3$$

$$2y^3 + 7y^2 + 2y - 3 = (y+1)(2y-1)(ay+b)$$

Comparing coefficient of x^3, $a = 1$

Comparing constants, $b = 3$

$$\Rightarrow (y+1)(2y-1)(y-3) = 0$$

$$y = -1, y = \frac{1}{2} or\ y = 3$$

7a) $$mx^2 + m + 6 > -8x$$

$$mx^2 + 8x + m - 6 > 0$$

$$b^2 - 4ac < 0$$

$$8^2 - 4m(m-6) < 0$$

$$64 - 4m^2 + 24m < 0$$

$$m^2 - 6m - 16 > 0$$

$$(m+2)(m-8) > 0$$

$m < -2$ *or* $m > 8$ and $m > 0$

$m > 8$

7b) $$2x^2 + (2k+1)x = 2 - k$$

$$2x^2 + (2k+1)x + k - 2 = 0$$

$$b^2 - 4ac = (2k+1)^2 - 4(2)(k-2)$$
$$= 4k^2 + 4k + 1 - 8k + 16$$
$$= 4k^2 - 4k + 17$$

$$= 4\left(k^2 - k + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right) + 17$$

$$= 4\left(k - \frac{1}{2}\right)^2 + 16$$

Since discriminant > 0, the equation has real and distinct roots for all real values of *k*.

Alternatively,

$$2x^2 + (2k+1)x = 2 - k$$

$$2x^2 + (2k+1)x + k - 2 = 0$$

$$b^2 - 4ac = (2k+1)^2 - 4(2)(k-2)$$

$$= 4k^2 + 4k + 1 - 8k + 16$$

$$= 4k^2 - 4k + 17$$

$b^2 - 4ac = (-4)^2 - 4(4)(17) < 0$ and $a > 0 \Rightarrow 4k^2 - 4k + 17 > 0$

Since discriminant > 0, the equation has real and distinct roots for all real values of *k*.

8 (a) $\frac{2^{x-3}}{4^{-x}} = \frac{16}{\sqrt{8^x}}$

$$\frac{2^{x-3}}{2^{-2x}} = \frac{2^4}{2^{\frac{3x}{2}}}$$

$$2^{x-3-(-2x)} = 2^{4-\frac{3x}{2}}$$

$$3x - 3 = 4 - \frac{3x}{2}$$

$$x = \frac{14}{9}$$

(b) $3^{2x} + 5\left(3^x\right) - 6 = 0$

Let $y = 3^x$

$$y^2 + 5y - 6 = 0$$

$$(y+6)(y-1) = 0$$

$3^x = -6$ $(N.A)$ *or* $3^x = 1$

$x = 0$

(c) $7^{2-x} = 28^{x+3}$

$$7^{2-x} = 7^{x+3}4^{x+3}$$

$$7^{-2x-1} = 2^{2x+6}$$

$$\frac{1}{7^{2x+1}} = 2^{2x+6}$$

$$\frac{1}{7\left(2^6\right)} = 14^{2x}$$

$$14^{2x} = \frac{1}{448}$$

$$\lg 14^{2x} = \lg \frac{1}{448}$$

$$x = -1.16$$

9 i) find the value of *a* and of *b*,

$$y = x + ax^b$$

$$y - x = ax^b$$

$$\lg\left(y - x\right) = b\log x + \lg a$$

Gradient = $b = \frac{0.2}{0.52} = \frac{5}{13}$

Intercept = lg *a* = 0.62

$a = 4.17(3s.f.)$

ii) find the value of *y* when *x* = 15, and

When *x* = 15, lg *x* = 1.176

$$\lg\left(y - x\right) = 1.07$$

$$y - 15 = 11.75$$

$$y = 26.75$$

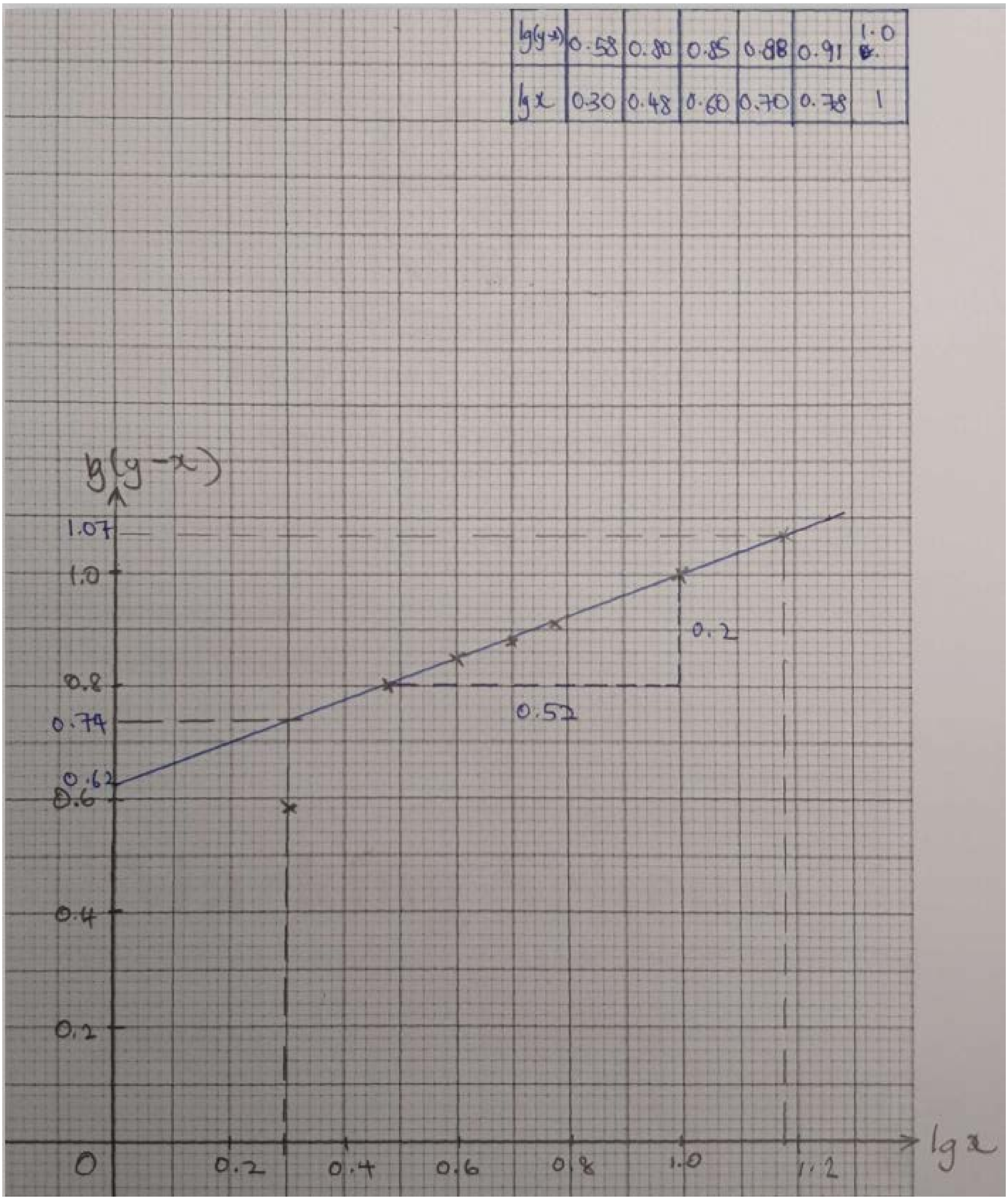
iii) identify the abnormal reading of *y*, and estimate its correct value.

Abnormal $y = 5.80$

$$\lg\left(y - x\right) = 0.74$$

$$y = 10^{0.74} + 2$$

$$y = 7.50$$


lg(y-x) 0.58 0.80 0.85 0.88 0.91 1.0
lg x 0.30 0.48 0.60 0.70 0.78 1
lg(y-x)
1.07
1.0
0.8
0.74
0.62
0.6
0.4
0.2
0.2
0.52
O
0.2
0.4
0.6
0.8
1.0
1.2
lg x