NAME		INDEX NO.		CLASS	

NORTHLAND SECONDARY SCHOOL
PRELIMINARY EXAMINATION
Secondary 4 Express / 5 Normal Academic

ADDITIONAL MATHEMATICS **4049/02**

Paper 2

28 August 2024

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

For Examiner's Use
90

Setter: Mr Chen Weizhong

Vetter: Ms Joanne Yap

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Express $\dfrac{1-8x-3x^2}{(x-1)(2x^2+3)}$ in partial fractions. [5]

2 **(a)** Find the smallest integer c for which the line $y = 7x + c$ intersects the curve $y = 2x^2 - 4$ at two distinct points. [4]

(b) Find the range of values of k for which the curve $y = kx^2 + 2(2k - 5)x + 9k$ lies below the x-axis. [4]

3 **(a)** Prove the identity $\dfrac{1-\cos x}{\sin x-\operatorname{cosec} x+\cot x}=\tan x$. [4]

(b) Hence solve the equation $\dfrac{1-\cos 2x}{\sin 2x - \operatorname{cosec} 2x + \cot 2x} = -3$ for $0° < x < 180°$. [3]

(c) Show that there are no solutions to the equation $\dfrac{1-\cos x}{\sin x - \operatorname{cosec} x + \cot x} = \tan 2x$ for $0° < x < 180°$. [2]

4 **(a)** The equation $\log_2 x + \log_8 x = \log_4 2$ has the solution $x = 2^m$. Find the value of *m*. [4]

(b) Sketch the graph of $y = \log_3 x$. [2]

(c) Explain why the equation $\log_5(2x-11)-\log_5(x-4)=1$ has no real solutions. [4]

5 **(a)** The function f is defined by $\mathrm{f}(x)=\mathrm{e}^{x^2+x}$ where $x>0$. Explain, with working, whether f is an increasing or a decreasing function. [3]

(b) The equation of a curve is $y = \dfrac{x^2}{x+3}$.

(i) Find an expression for $\dfrac{dy}{dx}$ and obtain the coordinates of the stationary points of the curve. [5]

(ii) Find an expression for $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ and hence determine the nature of these stationary points. [3]

6 The points P and Q both lie on a circle and have coordinates $(2,\ 7)$ and $(-6,\ 1)$ respectively. The line with equation $y+2x+4=0$ is a normal to the circle.

(a) Find the equation of the perpendicular bisector of PQ. [4]

(b) Find the equation of the circle. [5]

(c) Find the exact value of the coordinates of the point on the circle which is furthest from the y-axis. [2]

7 A particle moves in a straight line, such that its velocity, v m/s, t seconds after passing a fixed point O, is given by $v = -0.3(4-t)^2 + 1.2$.

(a) Find the acceleration of the particle when it first comes to instantaneous rest. [4]

After 1 second, its displacement, s m, is 2.5 m from O.

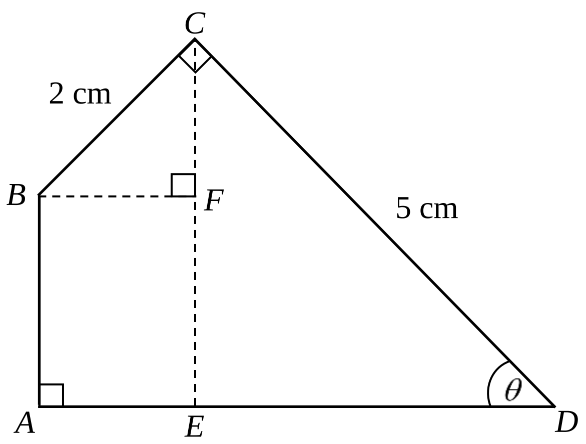
(b) Find the initial displacement of the particle. [3]

(c) Explain clearly why the total distance travelled by the particle in the interval $t = 0$ to $t = 7$ is **not** obtained by finding the value of s when $t = 7$. [2]

(d) Find the total distance travelled by the particle in the interval $t = 0$ to $t = 7$. [3]

8 **(a)** On the same axes, sketch, for $0 \le x \le 2\pi$, the graphs of $y = \sin 2x$ and $y = 1 - 3\cos x$. [3]

(b)



The diagram shows a quadrilateral in which *CFE* and *AED* are straight lines. $BC = 2$ cm and $CD = 5$ cm. Angle $BCD =$ angle $BFC =$ angle $BAE = 90°$. Angle $CDE = \theta$, where θ is an angle in degree.

(i) Show that the perimeter of the quadrilateral is $3\cos\theta + 7\sin\theta + 7$. [3]

(ii) Express $3\cos\theta + 7\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is acute. [3]

The perimeter of the quadrilateral is 13 cm.

(iii) Find the angle θ and hence find the total area of the quadrilateral. [3]

9 **(a)** Express $\dfrac{3x}{3x+1}$ in the form $a+\dfrac{b}{3x+1}$ where a and b are constants, and hence find $\displaystyle\int \frac{3x}{3x+1}\,\mathrm{d}x$. [4]

(b) Given that $y = x\ln(3x+1)$, find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(c) Using the results from parts **(a)** and **(b)**, show that $\int_0^4 \ln(3x+1)\,\mathrm{d}x = a\ln 13 + b$, where a and b are constants to be found. [5]

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