1
12 - 12 - 12 - 12 - 12 - 12 - 12 - 12 -
State of Car

CATHOLIC JUNIOR COLLEGE General Certificate of Education Advanced Level Higher 2 JC2 Preliminary Examination

CANDIDATE NAME			
CLASS	INDEX NUMBER		

MATHEMATICS

Paper 2

9758/02

16 September 2020 3 hours

Candidates answer on the Question Paper. Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.

Write your answers in the spaces provided in the Question Paper.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

Question	1	2	3	4	5	6	7	8	9	10	11	Total
Marks												
Total	5	6	7	12	10	8	8	8	9	13	14	100

This document consists of 28 printed pages.

Section A: Pure Mathematics [40 marks]

- 1. The graph of a curve C is a cubic polynomial and it passes through the origin, (2,0) and (2.55, -0.0631). Given that it has a turning point when x = 0.785, find the equation of C, giving its coefficients correct to 1 decimal place. [5]
- 2. (i) By means of the substitution $x = \csc \theta$, show that $\int_{\frac{2}{\sqrt{3}}}^{2} \frac{1}{x^3\sqrt{x^2-1}} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin^2(\theta) d\theta$. [3]

You are to show all workings clearly.

(ii) Hence evaluate exactly
$$\int_{\frac{2}{\sqrt{3}}}^{2} \frac{1}{x^3\sqrt{x^2-1}} dx$$
. [3]

3. (i) By writing
$$\frac{3}{(3r+1)(3r+4)}$$
 in partial fractions, find $\sum_{r=1}^{n} \frac{3}{(3r+1)(3r+4)}$ in terms of *n*. [4]

- (ii) Hence find the exact value of $\frac{3}{22} \times \frac{1}{25} + \frac{3}{25} \times \frac{1}{28} + \frac{3}{28} \times \frac{1}{31} + \cdots$ [3]
- 4. Botanists are studying how excessive logging is affecting the population of forest trees in a country in South-East Asia. The population of trees in a forest after *t* years is denoted by *x*, in thousands. One simple model proposed is that the population increases at a rate proportional to its population size. It is known that trees are chopped off at a constant rate of 3 thousand trees per year and the population stays constant when x = p.
 - (i) Find a differential equation involving x and t. [3]
 - (ii) The initial population of trees in a forest is 5 thousand trees. Solve the differential equation in part (i) and show that $x = (5-p)e^{\frac{3}{p}t} + p$. [5]
 - (iii) Given that the population of tree decreases in a finite time, state an inequality satisfied by p.

Sketch the graph of x against t for this case, showing clearly the coordinates of the graph's point of intersection with the horizontal axis. Hence explain the meaning of this axial intercept in the context of the question. [4]

5. (a) In the triangle shown below, one vertex is origin *O*, and the two other vertices are *A* and *B* where $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$.

A median of a triangle is a line segment joining a vertex (O, A or B) to the midpoint of the opposite side (e.g. AD is a median of triangle OAB from vertex A). It is given that F is the point of intersection between the medians of triangle OAB from vertices A and B.



- (i) By finding \overrightarrow{AD} and \overrightarrow{BE} , show that $\overrightarrow{OF} = \frac{1}{3}(\mathbf{a} + \mathbf{b})$. [3]
- (ii) Prove that F also lies on OC, the median of triangle OAB from vertex O. [2]
- (b) With reference to the origin O, the points N, P and Q have position vectors **n**, **p** and **q** respectively, and Π has an equation $\mathbf{r} \cdot \mathbf{n} = 0$. It is known that $\mathbf{p} \cdot \mathbf{n} = \mathbf{q} \cdot \mathbf{n} \neq 0$.
 - (i) Show that \overrightarrow{PQ} is perpendicular to **n**. Hence describe the geometrical relationship between the line PQ and the plane Π . [3]
 - (ii) Find a vector equation of the plane that contains points P and Q, and is perpendicular to plane Π , leaving your answer in terms of **n**, **p** and **q**. [2]

Section B: Probability and Statistics [60 marks]

6. A bag contains 4 red counters and 6 blue counters. 4 counters are drawn from the bag at random, without replacement.

(a)

Calculate the probability that(i) all the counters drawn are blue,[1](ii) at least 3 blue counters are drawn,[2](iii) at least 1 counter of each colour is drawn,[2](iv) at least 3 blue counters are drawn, given that at least 1 of each colour is drawn.[2]

- (b) State with a reason whether or not the events "at least 3 blue counters are drawn" and "at least 1 counter of each colour is drawn" are independent. [1]
- 7. Flyers of a tuition centre for General Paper (GP) tuition are given to JC2 students in a particular college with a large population. On average, 8% of the JC2 students will sign up for the GP tuition.
 - (i) State, in context, two assumptions needed for the number of JC2 students signing up for the GP tuition to be well modelled by a binomial distribution. [2]

Assume now that the number of JC2 students signing up for the GP tuition follows a binomial distribution.

- (ii) It is given that 120 JC2 students receive a copy of the flyer.
 - (a) Find the probability that more than 15 students sign up for the GP tuition. [3]
 - (b) Find the variance of the number of students signing up for the GP tuition. [1]

Flyers of another tuition centre for Mathematics tuition are given to JC2 students in the same college. The number of students who sign up for the Mathematics tuition is denoted by M. Assume that M has a distribution B(320, p).

- (iii) It is known that on average the proportion of students who sign up for Mathematics tuition is higher than the proportion of students who sign up for GP tuition. Given that P(M = 30 or 31) = 0.037986 correct to 5 significant figures, find the value of p. [2]
- 8. A box contains 1 red, 1 blue and *n* yellow discs, where $n \ge 2$. Two discs are randomly drawn from the box, without replacement. The random variable *Y* is the number of yellow discs drawn from the box.
 - (i) Show that $P(Y=1) = \frac{4n}{(n+1)(n+2)}$. Hence find the probability distribution of Y. [3]
 - (ii) Show that $E(Y) = \frac{An}{n+2}$, where A is a constant to be determined. [2]
 - (iii) Hence, find Var(Y), giving your answer in the form $\frac{f(n)}{(n+2)^2(n+1)}$ where f(n) is a

quadratic polynomial to be determined.

9. Legox bricks come in different colours and sizes. Bricks of the same size and colour are identical. Jamie has 9 Legox bricks. The following table shows the various colours and sizes of Legox bricks that Jamie has.

	Small	Medium	Large
Green	0	0	2
Blue	1	1	1
Red	2	2	0

- (a) Jamie arranges these 9 bricks in a row. Find the number of different possible arrangements if
 - (i) each of the green bricks are at the ends of the row,

[3]

[2]

[4]

10. In this question you should state clearly all the distributions that you use, together with the values of the appropriate parameters.

possible arrangements if all 3 colours are used.

A petrol station sells two different grades of petrol. The volume, in litres, of the two different grades of petrol sold in an hour are normally distributed with means and standard deviations as shown in the table below.

	Mean	Standard deviation
Grade X	μ	σ
Grade Y	200	5

- By considering the properties of the normal curve, explain why it may not be appropriate for the volume of a randomly chosen grade of petrol sold in an hour to be taken from the volume of a combined grades of petrol sold in an hour at the petrol station to be modelled by a normal distribution.
- (ii) Given that there is a 2.3% chance that the volume of Grade X petrol sold in an hour does not exceed 170 litres and a 16% chance that the volume of Grade X petrol sold in an hour is more than 180 litres, find the values of μ and σ . [4]
- (iii) Find the probability that the volume of Grade X petrol sold in three hours lies within 72 litres of three times the volume of Grade Y petrol sold in 1 hour, giving your answer to 5 significant figures.
- (iv) State an assumption for the calculations in part (iii) to be valid. [1]

The petrol station carries out a promotion in the month of June resulting in an increase in the volume of Grade Y petrol sold in an hour by 10%.

- (v) It is given that there is a probability of at most 0.2 that the mean hourly volume of Grade *Y* petrol sold in a particular day in June is more than the mean hourly volume of Grade *Y* petrol sold in a particular day in May by at least *k* litres. Find the minimum value of *k*, giving your answer as an integer. Assume that the daily operational hours of the petrol station is 24 hours.
- 11. The Coronavirus Disease 2019 (COVID-19) is an acute infectious disease caused by the strain of coronavirus named Severe Acute Respiratory Syndrome Coronavirus 2 (SARS-CoV-2).

An epidemiological study conducted to determine the incubation period of the virus from the time of infection to the onset of the disease, involved a dataset of 59 officially confirmed COVID-19 patients in an Asian country, each with clear history of exposure to the infection as well as onset of the illness.

The incubation period, x days, in the random sample of 59 patients is summarized by

$$n = 59$$
, $\Sigma x = 344.5$,

and the sample variance is 8.46136 days^2 .

(Data adapted from: <u>https://www.medrxiv.org/content/10.1101/2020.02.24.20027474v1</u> on Estimate the incubation period of coronavirus 2019 (COVID-19).

- (a) Based on this dataset, find unbiased estimates of the population mean and variance of the incubation period of the virus. [3]
- (b) An epidemiology official intends to ascertain whether the population mean incubation period of SARS-CoV-2 is different from the known mean incubation period for H7N9, Human Avian Influenza A, which is 6.5 days.
 - (i) Explain whether the official should carry out a 1-tail test or a 2-tail test.
 State the hypotheses for the test, defining any symbols used. [2]
 - (ii) Explain why the hypothesis test could be carried out without knowledge of the population distribution of the SARS-CoV-2's incubation period. [1]
 - (iii) Carry out the test, at the 5% level of significance, stating the *p*-value obtained from the taken sample.
 - (iv) Explain what the *p*-value found in part (iii) means in context. [1]

[3]

(v) Determine the least value of the sample size for a new random sample that has the same mean and same sample variance as that of the earlier taken sample, for which the null hypothesis would be rejected at 2% level of significance in the above test.

End