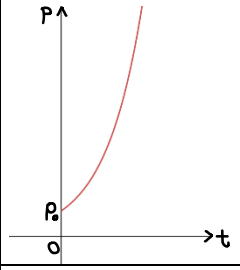
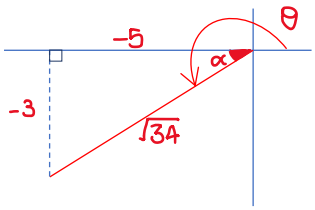
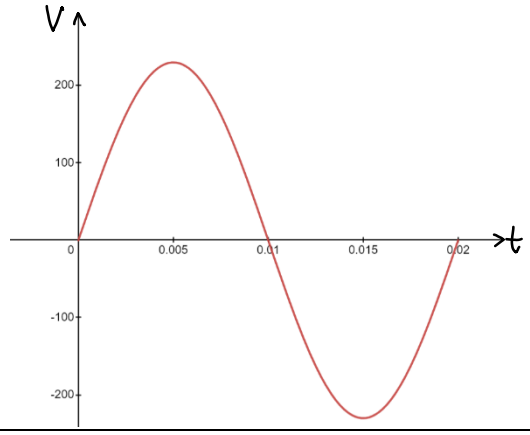
	2023 Yr 3 EXP End Year Exam P1 Mark Scheme	Markers' Comments
1	Making y the subject the linear equation: $3x+y=5 \Rightarrow y=5-3x$ ---- (1) $2x^2+xy-y^2=5$ ----------(2) Sub Eqn (1) into Eqn (2), $2x^2+x(5-3x)-(5-3x)^2=5$ $2x^2+5x-3x^2-(25-30x+9x^2)=5$ $-10x^2+35y-30=0$ $-2x^2+7x-6=0$ $(x-2)(-2x+3)=0$ $\therefore x=2$ or 1.5 $\therefore y=-1$ or $y=0.5$	• Quite well attempted. • Some students forgot to find the other variable to complete the solutions
2	$m_{AB}=\frac{5-8}{-2-4}=\frac{1}{2}$ $Y-8=\frac{1}{2}(X-4)$ OR $8=\frac{1}{2}(4)+c \ \therefore c=6$ $\therefore Y=\frac{1}{2}X+6$ $\frac{x}{y}=\frac{1}{2}x^2+6$ $\frac{x}{y}=\frac{x^2+12}{2} \Rightarrow \frac{y}{x}=\frac{2}{x^2+12}$ $\therefore y=\frac{2x}{x^2+12}$	• Some students input the given points as (x, y) instead of (X, Y): $\frac{4}{8}=\frac{1}{2}(4)^2+c$ $\therefore c=-\frac{15}{2}$
3	$\frac{x^2+1}{x^2-1}=1+\frac{2}{x^2-1}$ $\frac{2}{(x+1)(x-1)}=\frac{A}{x+1}+\frac{B}{x-1}$ $2=A(x-1)+B(x+1)$ Let $x=1$: $B=1$ Let $x=-1$: $A=-1$ $\therefore \frac{x^2+1}{x^2-1}=1-\frac{1}{x+1}+\frac{1}{x-1}$	• Many students did not express use long division to express given algebraic fraction in proper form
4(i)	$100P_0=P_0(2^t)$ $t=\frac{\ln 100}{\ln 2}=6.64$ $\therefore$ 7 hours	

4(ii)		
5	$\log_3 x - 3\log_x 9 = 1$ $\log_3 x - 3\dfrac{\log_3 9}{\log_3 x} = 1$ $\log_3 x - \dfrac{6}{\log_3 x} = 1$ Let $\log_3 x$ be a. $a - \dfrac{6}{a} = 1 \quad \Rightarrow \quad a^2 - a - 6 = 0$ $(a-3)(a+2) = 0$ $a = 3$ or -2 $x = 3^3$ or 3^{-2} $\therefore x = 27$ or $\dfrac{1}{9}$	
6(i)	$\cot\theta + \sec^2\theta$ $= \dfrac{1}{\tan\theta} + 1 + \tan^2\theta$ $= \dfrac{5}{3} + 1 + \left(\dfrac{3}{5}\right)^2$ $= \dfrac{8}{3} + \dfrac{9}{25}$ $= \dfrac{227}{75}$	
6(ii)	$\sin\theta = -\dfrac{3}{\sqrt{34}}$ 	
6(iii)	$\cos(180° - \theta) = -\cos\theta$ $= \dfrac{5}{\sqrt{34}}$	

6(iv)	$\dfrac{\text{cosec}(-\theta)}{\sec(\theta)} = \dfrac{\cos\theta}{\sin(-\theta)} = \dfrac{\cos\theta}{-\sin\theta}$ $= \dfrac{-\dfrac{5}{\sqrt{34}}}{\dfrac{3}{\sqrt{34}}} = -\dfrac{5}{3}$	
7(a)	Comparing coefficient of x^3: $B = 5$ Sub $x = 1$: $A = -33$ Sub $x = 0$: $-33 = C(-3) \quad \therefore C = 11$	
7(b)	$f(x) = (2x^2 + 3x - 2)Q(x) + R$ $= (2x-1)(x+2)Q(x) + ax + b$ $\dfrac{1}{2}a + b = 4 \quad ---(1)$ $-2a + b = -7 \quad ---(2)$ Taking (2) – (1): $-2.5a = -11 \quad \therefore a = 4.4$ $\Rightarrow b = 1.8$	
8(a)	Given $\dfrac{x^2 + 2x + 2}{x^2 - 5x + 6} \geq 0.$ Since $x^2 + 2x + 2 = (x+1)^2 + 1 > 0$ for all real values of x $\Rightarrow x^2 - 5x + 6 > 0$ $(x-2)(x-3) > 0$ $\therefore 2 < x < 3$	
8(b)	$y = x + k$ ----------(1) $xy = y + 2$ ----------(2) Sub (1) into (2): $x(x+k) = x + k + 2$ $x^2 + (k-1)x - k - 2 = 0$ $x^2 + (k-1)x - (k+2) = 0$ $b^2 - 4ac = (k-1)^2 - 4(1)(-k-2)$ $= k^2 - 2k + 1 + 4k + 8$ $= k^2 + 2k + 9$ $= (k+1)^2 + 8$ Since $(k+1)^2 + 8 > 0$ for all real values of k, $\therefore 2$ intersections	

9(i)	$p = \dfrac{2\pi}{100\pi} = \dfrac{1}{50}$ or 0.02	
9(ii)	$\sin(100\pi t) = 1$ $100\pi t = \dfrac{\pi}{2}$ $t = 0.005$	
9(iii)		
9(iv)	$230\sin(100\pi t) = 115$ $\sin(100\pi t) = \dfrac{1}{2}$ $\alpha = \sin^{-1}\dfrac{1}{2} = \dfrac{\pi}{6}$ ✓ ✓ $100\pi t = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$ $t = \dfrac{1}{600}, \dfrac{1}{120}$	
10(i)	Area of triangle ABC $= \dfrac{1}{2}\begin{vmatrix} -2 & 2 & 7 & -2 \\ 3 & -4 & 11 & 3 \end{vmatrix}$ $= 47.5 \text{ units}^2$	
10(ii)	$m_{AB} = -\dfrac{7}{4}$ $m_{AD} = -5$ Since $m_{AB} \neq m_{AD}$, A, B and E are not collinear.	
10(b)	Let the foot of the perpendicular from A to BC be E. $m_{BC} = \dfrac{11-(-4)}{7-2} = 3$ Equation of line BC : $y-(-4) = 3(x-2)$ $\therefore y = 3x - 10$ ---------- (1)	

	$m_{AE} = -\frac{1}{3}$ Equation of line AE : $-3 = -\frac{1}{3}(x+2)$ $\therefore y = -\frac{1}{3}x + \frac{7}{3}$ ---------- (2) (1) = (2): $3x - 10 = -\frac{1}{3}x + \frac{7}{3}$ $9x - 30 = -x + 7$ $10x = 37$ $\therefore x = 3.7$ and $y = 1.1$ $\therefore E(3.7, 1.1)$	