HWA CHONG INSTITUTION
JC2 Preliminary Examination
Higher 3

CANDIDATE NAME	

CENTRE NUMBER					**INDEX NUMBER**				

PHYSICS — 9814/01

Paper 1 — **16 September 2024**

3 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams, graphs or rough working.
Do not use staples, paper clips, glue or correction fluid.

The use of an approved scientific calculator is expected, where appropriate.

Section A
Answer **all** questions.
You are advised to spend about 1 hour and 50 minutes on Section A.

Section B
Answer **two** questions only.
You are advised to spend about 35 minutes on each question in Section B.

The number of marks is given in brackets [ ] at the end of each question or part question.

For Examiner's Use		
Section A		
1		**8**
2		**7**
3		**8**
4		**6**
5		**14**
6		**17**
Section B		
7		**20**
8		**20**
9		**20**
Deductions		
Total		**100**

This document consists of **33** printed pages and **1** blank page.

Data

speed of light in free space	c	=	3.00×10^{8} m s^{-1}
permeability of free space	μ_0	=	$4\pi \times 10^{-7}$ H m^{-1}
permittivity of free space	ε_0	=	8.85×10^{-12} F m^{-1}
			$(1/(36\pi)) \times 10^{-9}$ F m^{-1}
elementary charge	e	=	1.60×10^{-19} C
the Planck constant	h	=	6.63×10^{-34} J s
unified atomic mass constant	u	=	1.66×10^{-27} kg
rest mass of electron	m_e	=	9.11×10^{-31} kg
rest mass of proton	m_p	=	1.67×10^{-27} kg
molar gas constant	R	=	8.31 J K^{-1} mol^{-1}
the Avogadro constant	N_A	=	6.02×10^{23} mol^{-1}
the Boltzmann constant	k	=	1.38×10^{-23} J K^{-1}
gravitational constant	G	=	6.67×10^{-11} N m^{2} kg^{-2}
acceleration of free fall	g	=	9.81 m s^{-2}

Formulae

uniformly accelerated motion	s	=	$ut + \frac{1}{2}at^2$
	v^2	=	$u^2 + 2as$
moment of inertia of rod through one end	I	=	$\frac{1}{3}ML^2$
moment of inertia of hollow cylinder through axis	I	=	$\frac{1}{2}M\left(r_1^2 + r_2^2\right)$
moment of inertia of solid sphere through centre	I	=	$\frac{2}{5}MR^2$
moment of inertia of hollow sphere through centre	I	=	$\frac{2}{3}MR^2$
work done on/by a gas	W	=	$p\Delta V$
hydrostatic pressure	p	=	ρgh
gravitational potential	ϕ	=	$-Gm/r$
Kepler's third law of planetary motion	T^2	=	$\dfrac{4\pi^2 a^3}{GM}$
temperature	T / K	=	T / °C + 273.15

pressure of an ideal gas	p	=	$\frac{1}{3}\frac{Nm}{V}\langle c^2\rangle$
mean translational kinetic energy of an ideal gas molecule	E	=	$\frac{3}{2}kT$
displacement of particle in s.h.m.	x	=	$x_0 \sin\omega t$
velocity of particle in s.h.m.	v	=	$v_0 \cos\omega t$
		=	$\pm\omega\sqrt{\left(x_0^2 - x^2\right)}$
electric current	I	=	$Anvq$
resistors in series	R	=	$R_1 + R_2 + \ldots\ldots$
resistors in parallel	$1/R$	=	$1/R_1 + 1/R_2 + \ldots\ldots$
capacitors in series	$1/C$	=	$1/C_1 + 1/C_2 + \ldots\ldots$
capacitors in parallel	C	=	$C_1 + C_2 + \ldots\ldots$
energy in a capacitor	U	=	$\frac{1}{2}CV^2$
electric potential	V	=	$\frac{Q}{4\pi\varepsilon_0 r}$
electric field strength due to a long straight wire	E	=	$\frac{\lambda}{2\pi\varepsilon_0 r}$
electric field strength due to a large sheet	E	=	$\frac{\sigma}{2\varepsilon_0}$
alternating current/voltage	x	=	$x_0 \sin\omega t$
magnetic flux density due to a long straight wire	B	=	$\frac{\mu_0 I}{2\pi d}$
magnetic flux density due to a flat circular coil	B	=	$\frac{\mu_0 NI}{2r}$
magnetic flux density due to a long solenoid	B	=	$\mu_0 nI$
energy in an inductor	U	=	$\frac{1}{2}LI^2$
RL series circuits	τ	=	$\frac{L}{R}$
RLC series circuits (underdamped)	ω	=	$\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$
radioactive decay	x	=	$x_o \exp(-\lambda t)$
decay constant	λ	=	$\frac{\ln 2}{t_{1/2}}$

Section A

Answer **all** questions in this section.

You are advised to spend about 1 hour 50 minutes on this section.

1 **(a)** The ceiling fan in Fig. 1.1 has blades that are 0.60 m long and is spinning at 80 revolutions in one minute. It is turned off and comes to rest in 40 s.

Fig. 1.1

(i) Calculate the speed of the tip of any blade just before the fan is switched off.

speed = …………… m s^{-1} [2]

(ii) After it is switched off, determine the fan's average angular acceleration.

average angular acceleration = …………….. rad s^{-2} [1]

(iii) Calculate the number of revolutions that the fan completes before coming to rest.

no. of revolutions = …………………….. [2]

(b) A uniform solid cylinder of mass *M,* initially at rest, is mounted on a fixed horizontal axle running through its centre of mass.

A small piece of blu-tac, with mass *m* and velocity *v, is projected towards* the cylinder, as shown in Fig. 1.2. The line of motion of the projectile is perpendicular to the axle and at a distance of *d* from the centre, where $d < R$.

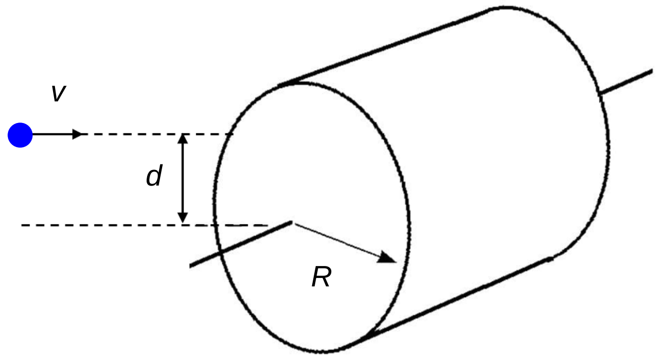


Fig. 1.2

Determine the angular speed ω, of the system just after the blu-tac sticks to the surface of the cylinder in terms of *M*, *m*, *v*, *R* and *d*.

[3]

[Total: 8 marks]

2 (a) Two identical particles of mass *m* collide elastically on a low-friction table as shown in Fig. 2.1.

Fig. 2.1

Show that the total kinetic energy of the system (E_k) after the collision is $\frac{1}{4}mu^2$ in the zero-momentum frame (centre-of-mass frame).

[2]

(b) Two asteroids of equal mass in the asteroid belt between Mars and Jupiter collide with a glancing blow, and move off in different directions, as shown in Fig. 2.2. Asteroid A, which was initially traveling at 40.0 m s^{-1}, is deflected 30° from its original direction. Asteroid B, which was initially at rest, travels at 45° to the original direction of A.

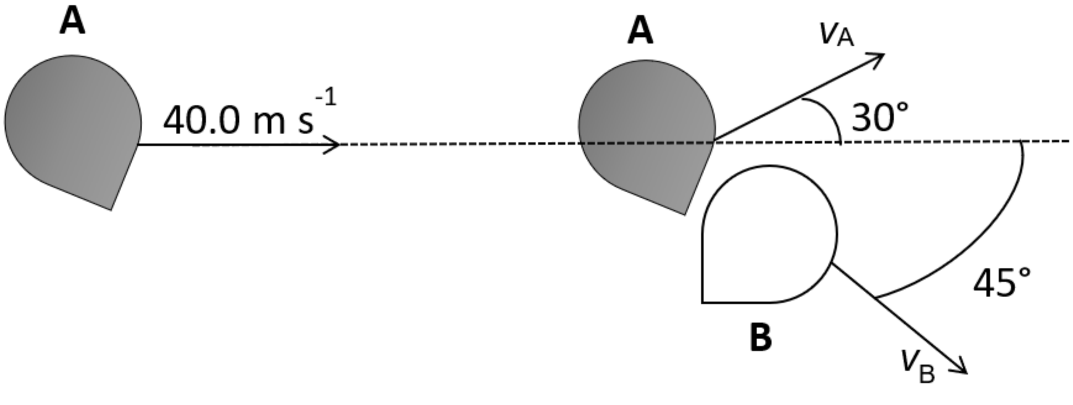


Fig. 2.2

(i) Calculate the speed of each asteroid after the collision.

v_A = ……………….. m s^{-1}

v_B = ……………….. m s^{-1} [3]

(ii) Determine the fraction of the kinetic energy of asteroid A dissipated during this collision.

fraction = ………………….. [2]

[Total: 7 marks]

3 **(a)** Fig. 3.1 shows point sources of waves, S_1 and S_2. P is a point equidistant from the two sources.

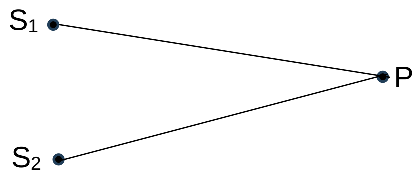


Fig. 3.1

The sources radiate waves of the same wavelength, but of amplitudes A_1 and A_2 respectively. At P, the time variation of the disturbance due to the wave from S_1 may be represented by

$$y_1 = A_1 \cos \omega t$$

(i) There is a phase angle ϕ between the waves from the two sources reaching P. Write down an expression for the disturbance y_2 due to the wave from S_2.

[1]

(ii) Show that the amplitude of the resultant disturbance, A_r at P is given by

$$A_r^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos\phi$$

[2]

(b) Fig. 3.2 shows two slits Q_1 and Q_2 in an opaque screen, a distance d apart.

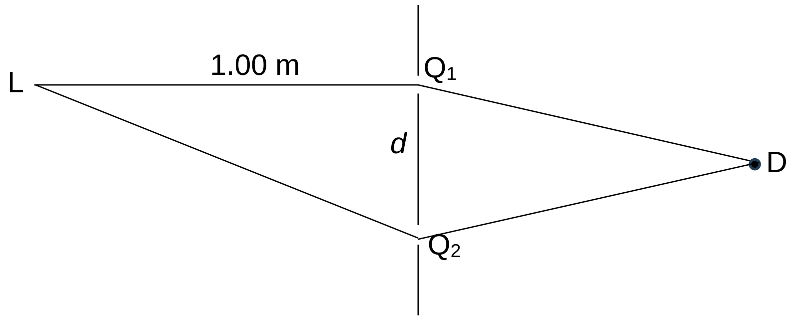


Fig. 3.2

A vertical strip-light source producing monochromatic light of wavelength λ is set up at L, exactly 1.00 m from Q_1 on the normal to the screen, so that both slits are illuminated.

A detector is placed at D, exactly equidistant from the slits on the other side of the screen. Readings of the light intensity at D are taken as follows:

- I_1 with the slit Q_2 covered but Q_1 open,
- I_2 with Q_1 covered but Q_2 open,
- I_r with both slits open.

It is found that I_r is exactly equal to the sum of I_1 and I_2.

(i) By reference to the relationship given in **(a)(ii)**, deduce the phase difference between the waves reaching Q_1 and Q_2.

[2]

(ii) Hence, show that the relationship between the separation between the two slits *d* and the wavelength λ of the monochromatic light is

$$16d^2 = \lambda\left(8 + \lambda\right)$$

[3]

[Total: 8 marks]

4 A square coil of wire of length α , is formed by a fixed length L of insulated wire such that an integral numbers of turns, are obtained when the wire is wound on a square core. The coil is mounted on an axle IJ, passing through the midpoints on opposite sides of the coil as shown in Fig. 4.1. A motor connected to the axle turns it at a constant frequency f.

A region of uniform magnetic field of flux density B is directed perpendicularly through a square area PQRS as shown in Fig. 4.2. The flux density is zero outside this area.

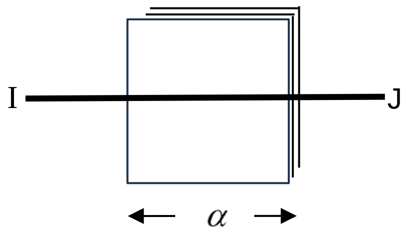


Fig. 4.1

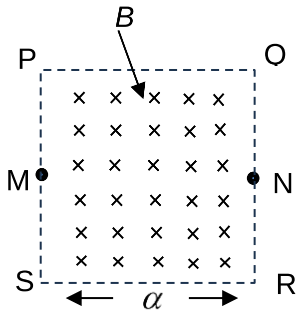


Fig. 4.2

(a) The square coil is placed with its plane initially perpendicular to the magnetic field and with its axle passing through M and N, the mid points of area PQRS as shown in Fig. 4.3.

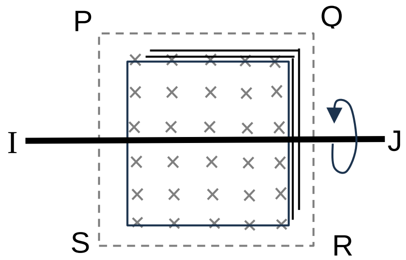


Fig. 4.3

Derive an expression, in terms of B, L, α and f, for the magnitude of the maximum instantaneous induced e.m.f. across the ends of the rotating square coil.

[3]

(b) The axle IJ is then removed from the mid points and mounted on the side of the square coil. The coil is then placed with its plane initially perpendicular to the magnetic field and its axle positioned along the side QR of the magnetic field as shown in Fig. 4.4.

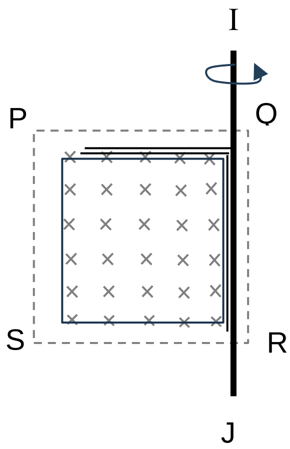


Fig. 4.4

Sketch the induced e.m.f across the ends of the rotating coil within one cycle of its rotation. Mark on the axes, the intervals of time within one period *T*.

[3]

[Total: 6 marks]

5 A satellite of mass 10 kg is orbiting the Earth. The effective radial potential energy of the satellite-Earth system, U_{eff} is given by

$$U_{\text{eff}} = \frac{A}{r^2} - \frac{B}{r}$$

where $A = 8.39 \times 10^{22}\ \text{J m}^2$, $B = 3.98 \times 10^{15}\ \text{J m}$ and r is the distance of the satellite from the center of Earth.

(a) **(i)** Initially, the satellite is in a circular orbit. Show that the radius of this orbit is 4.22×10^7 m.

[2]

(ii) Show that the total energy of the system is - 4.72×10^7 J.

[3]

(iii) Show that the angular speed of the satellite is 7.27×10^{-5} rad s^{-1}.

[2]

(b) **(i)** The rocket engine of this satellite is fired for a while.

The total energy of the system increases by 1.00×10^{7} J and the satellite now moves into an elliptical orbit.

Calculate the closest distance of the satellite from the centre of the Earth.

distance = ………………………… m [3]

(ii) Calculate the period of the elliptical orbit.

period = ………………..…… s [4]

[Total: 14 marks]

6 Carl David Anderson discovered the anti-electron – now called the positron – in 1932, for which he won the Nobel Prize in Physics. The positron was the first evidence of antimatter and was discovered when Anderson allowed cosmic rays to pass through a cloud chamber containing a lead plate. In a cloud chamber, dark tracks are made by streams of droplets condensing on ions produced by a passing alpha-particle or beta-particle, shown in Fig. 6.1.

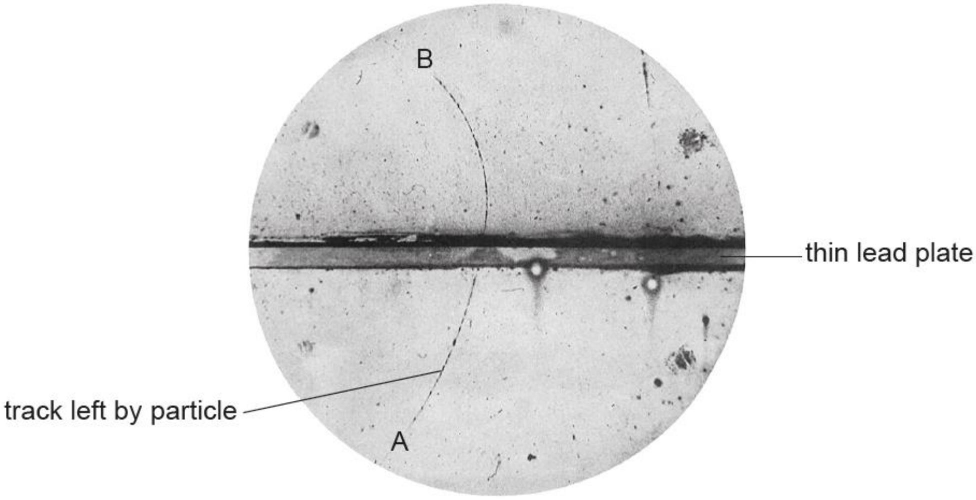


Fig. 6.1

The whole cloud chamber is in a uniform magnetic field at right angles to the plane of the photograph. The track AB is left by a particle passing from A to B through the chamber. The thickness of this track is much less than that produced by an alpha-particle and similar to that produced by a beta-particle.

(a) **(i)** State how the photograph shows that the particle producing the track AB is electrically charged.

....

....

.... [1]

(ii) The magnetic field direction is vertically into the plane of the paper.

State how this shows that the particle producing the track AB is a positron, not an electron.

....

....

.... [1]

(iii) By comparing the track near A with the track near B, explain what happens to the energy of the particle as it passes through the thin lead plate.

……………………………………………………………………………………………………..…

……………………………………………………………………………………………………..…

……………………………………………………………………………………………………..…

……………………………………………………………………………………………………..…

…………………………………………………………………………………………………… [3]

(b) The decay of polonium-210 produces a daughter nucleus lead, and an alpha-particle.

$$^{210}_{84}\text{Po} \rightarrow {}^{206}_{82}\text{Pb} + {}^{4}_{2}\alpha$$

(i) The polonium-210 nucleus is initially stationary. As the alpha-particle is emitted, the lead nucleus recoils.

Explain why the magnitude of the recoil velocity (v_{Pb}) of the lead nucleus is less than 2% of the magnitude of the velocity of the alpha-particle (v_{α}).

……………………………………………………………………………………………………..…

……………………………………………………………………………………………………..…

…………………………………………………………………………………………………… [2]

(ii) Show that the energy of alpha particles from the decay of Polonium-210 is 5.4 MeV.

The nuclear mass of
Polonium-210 = 209.93676u
Lead-206 = 205.92945u
Alpha particle = 4.00151u

[2]

When the kinetic energy of alpha-particles emitted by polonium-210 is measured, the spectrum of Fig. 6.2 is obtained. This is exactly what would be expected: each decay liberates the same amount of energy, and conservation of momentum allows only one way for the sharing of this energy. Nearly all the energy is given to the alpha-particles, which all emerge with the same energy of 5.4 MeV.

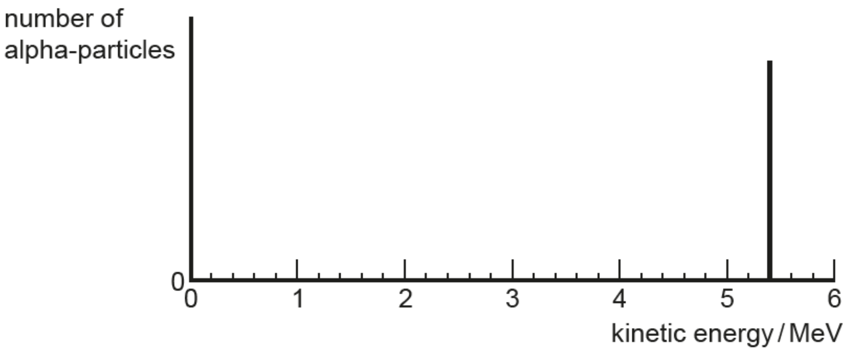


Fig. 6.2

In beta decay, electrons emerge from the nuclei at higher speeds than the alpha-particles produced by alpha decay, but with less kinetic energy. Fig. 6.3 shows the beta-particle energy spectrum obtained when nuclei of bismuth-210 decay. The kinetic energy varies greatly.

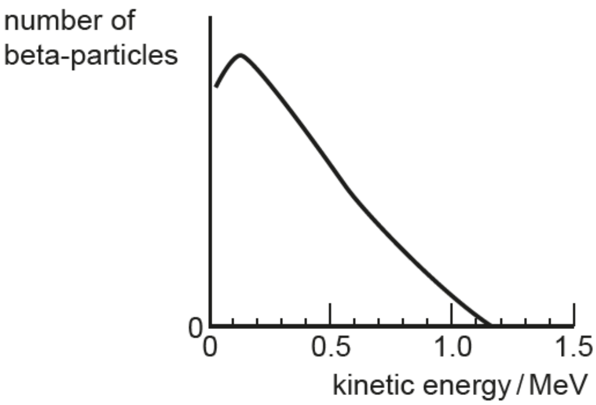


Fig. 6.3

As Fig. 6.3 shows, some beta-particles have an energy of 1.16 MeV. This should always be the energy released by the process, as with alpha decay. What could have happened to the missing energy for the overwhelming majority of beta-particles, which emerge with less energy? Wolfgang Pauli suggested that the results were exactly what you would expect if there was **another** particle released with the beta-particle. This ‘extra’ particle, for which Enrico Fermi later suggested the Italian name ‘neutrino’, or ‘little neutral one’, would carry off the energy that was missing from the beta-particle.

(c) Sketch in Fig. 6.3 how the profile of kinetic energies of beta-particles would be if there was no neutrino released. [1]

(d) The interaction between an antineutrino and a proton produces a positron and a neutron as shown in Fig. 6.4.

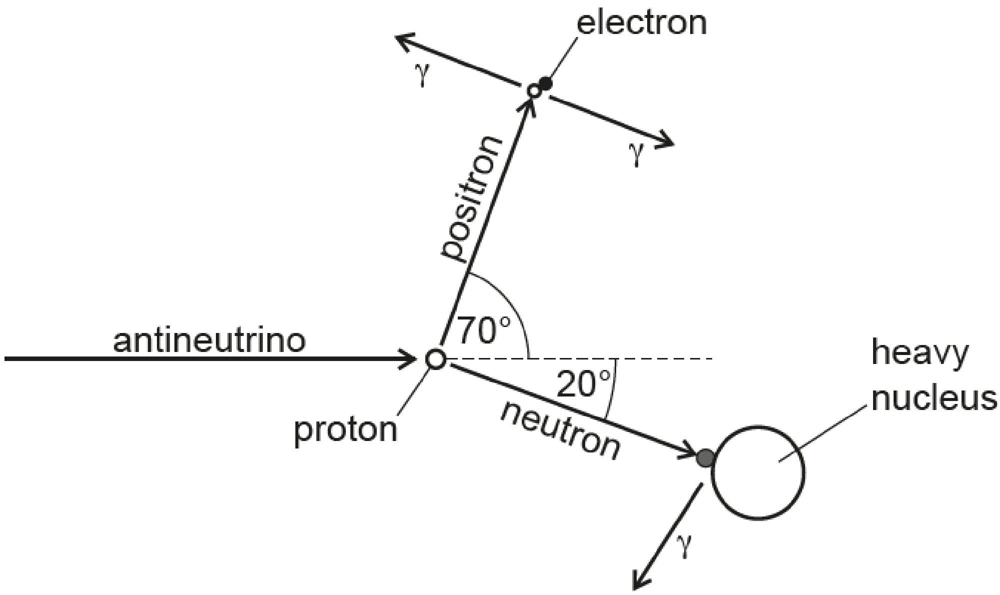


Fig. 6.4

(i) The positron in Fig. 6.4 is travelling at a speed of 2.2×10^{8} m s^{-1}.

mass of positron = 9.11×10^{-31} kg mass of neutron = 1.67×10^{-27} kg

Determine the speed of the neutron.

speed = …………….. m s^{-1} [2]

(ii) When a slow positron meets a slow electron, a process called pair annihilation takes place. In this process, both positron and electron are converted into energy in the form of two photons of electromagnetic radiation, γ .

In this context, “slow” means that the particles are almost at rest.

Calculate the maximum wavelength λ of each photon emitted in the positron–electron annihilation.

maximum wavelength λ = ……………. m [3]

(iii) Explain why the photons move in opposite directions.

…………………………………………………………………………………………………..…

…………………………………………………………………………………………………..…

…………………………………………………………………………………………………..…

…………………………………………………………………………………………………… [2]

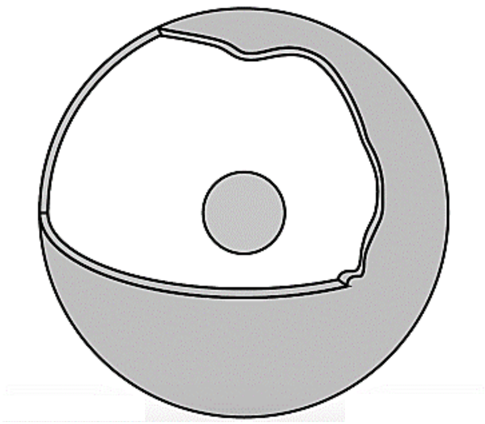
[Total: 17 marks]

Section B

Answer **two** questions from this section.

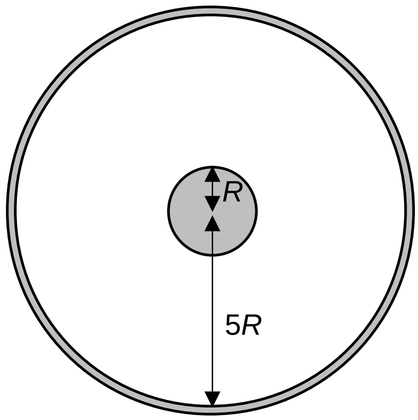
You are advised to spend about 35 minutes on each question.

7 **(a)** Fig. 7.1 shows a cutout view of a nonconducting sphere of uniform volume charge density which is surrounded by a thin concentric conducting spherical shell. As shown in Fig. 7.2, the radii of the inner sphere and spherical shell are R and $5R$, respectively.
The sphere has a charge of $-2Q$ and the shell has a charge of $+4Q$.



cutout view

Fig. 7.1



cross-section view

Fig. 7.2

(i) Calculate the charge on the outer surface of the shell in terms of Q.

charge = …………………. Q [1]

(ii) Using Gauss's law, derive an expression for the electric field strength at distance of r from the center of the sphere for $r < R$. Express your answer in terms of Q, R, r, and permittivity of free space ε_0.

[3]

(iii) The magnitude of the electric field strength at $r = R$ is 10 N C^{-1}. Calculate the value of the electric field strength at $r = 2R$.

electric field strength = ... N C^{-1} [2]

(iv) Derive an expression for the value of the potential difference between the outer surface of the sphere and the inner surface of the shell (between R and $5R$, both inclusive). Express your answer in terms of Q, R, r, and permittivity of free space ε_0.

[3]

(v) **1.** Sketch in Fig. 7.3 how the electric field strength E varies with distance r from the centre of the sphere.

Fig. 7.3

[3]

2. Sketch in Fig. 7.4 how the electric potential V varies with distance r from the centre of the sphere.

Fig. 7.4

[2]

(b) A long cylindrical copper wire of radius R carries a current I. The current density J varies according to radial distance r from the centre of the wire, given by $J = kr^2$ where k is a constant.

(i) Derive an expression for the magnitude of the magnetic flux density B at a radial distance of

1. $r < R$ from the centre of the wire. Express your answer in terms of k, r, and permeability of free space μ_0.

[2]

2. $r > R$ from the centre of the wire. Express your answer in terms of k, R, r, and permeability of free space μ_0.

[2]

(ii) Hence state the expression for the largest possible magnitude of the magnetic flux density B generated by the copper wire.

[2]

[Total: 20 marks]

8 **(a)** A capacitor is made up of two parallel plates, separated by air. A switch S, a battery of e.m.f. E, a capacitor of capacitance C and a resistor of resistance R are connected as shown in Fig. 8.1.

Initially, the switch S touches the contact J till the capacitor is fully charged to Q_0. At time $t = 0$, the switch S touches contact K.

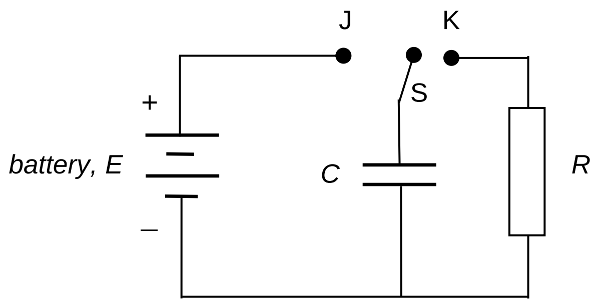


Fig. 8.1

(i) Just after S and K are in contact, derive an expression for the instantaneous voltage V across the capacitor in terms of t, E, R and C.

[3]

(ii) Fig. 8.2 shows a graph of the variation of the voltage across the capacitor with time for **(a)(i)**.

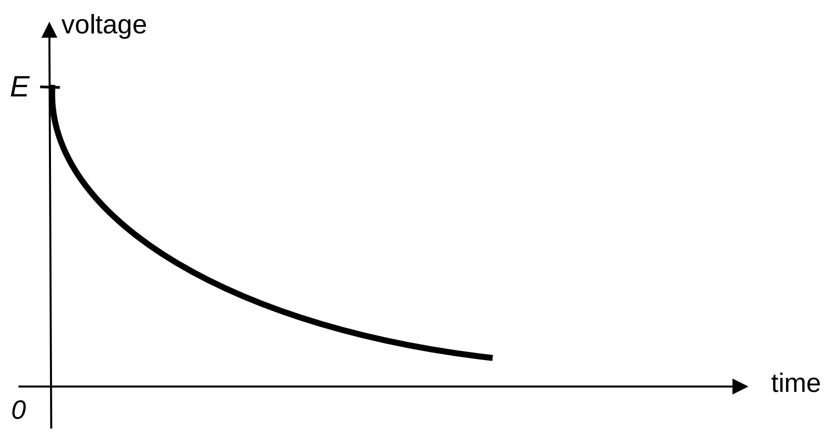


Fig. 8.2

Initially, the switch S touches the contact J till the capacitor is fully charged to Q_0. After which, the switch S touches contact K. Now, the distance between the two parallel plates of the capacitor is increased significantly.

Sketch a graph to show the new variation of the voltage in Fig. 8.2. [2]

(b) An inductor is made up of a long solenoid. A switch S, a battery of e.m.f. *E*, an inductor of inductance *L* and a resistor of resistance *R* are connected as shown in Fig. 8.3.

Initially, the switch S touches the contact J till the current in circuit is constant, I_0. At time t = 0, the switch S touches contact K.

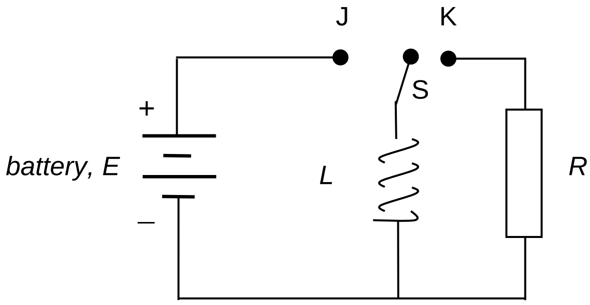


Fig. 8.3

(i) Derive an expression for the inductance *L* of the inductor in terms of *A*, *l*, *N* and μ_o , where μ_o is the permeability of free space, *A* is the cross-sectional area of the solenoid, l is the length of the solenoid and *N* is the number of turns of the solenoid,

[3]

(ii) Fig. 8.4 shows a graph of the variation of the current through the inductor with time for **(b)(i)**.

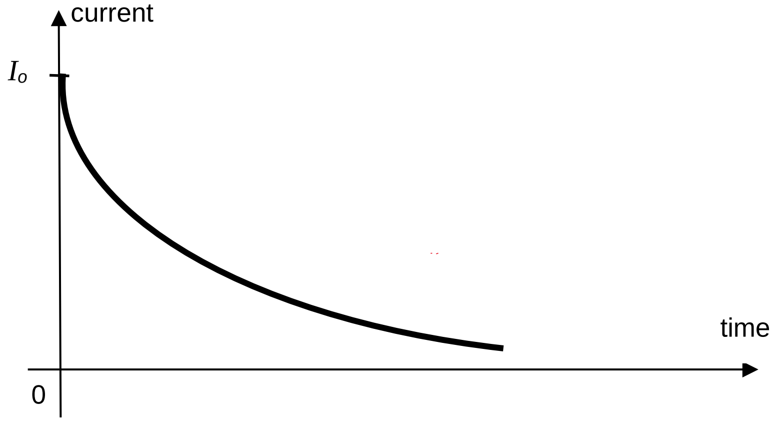


Fig. 8.4

Initially, the switch S touches the contact J till the current in circuit is constant, I_0. After which, the switch S touches contact K. Now, an iron rod is inserted along the axis of and into the solenoid.

Sketch a graph to show the new variation of the current in Fig. 8.4. [2]

(c) A switch S, a battery of e.m.f. E , an inductor of inductance L and a capacitor of capacitance C are connected as shown in Fig. 8.5. Initially, the switch S touches contact J until no current flows in the circuit. At time $t = 0$, switch S touches contact K.

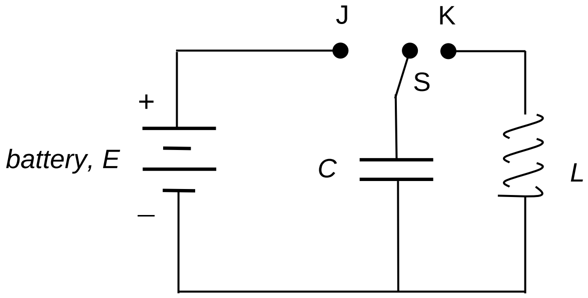


Fig. 8.5

(i) S and K are in contact now. Derive an expression for the voltage across the inductor in terms of t, E, C and L.

[3]

(ii) Sketch a graph of the variation of voltage across the inductor with time for one cycle in Fig. 8.6.

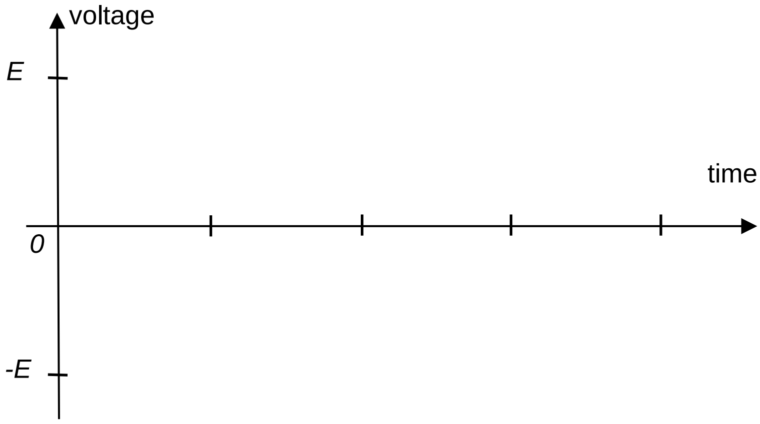


Fig. 8.6

[2]

(iii) Given that E = 16.0 V, L = 2.00 mH, C = 2.00 mF,

1. calculate the voltage across the capacitor when the energy stored in the capacitor is 1/4 of its maximum,

voltage = ………… V [2]

2. calculate the shortest time lapsed from the instance when S and K are placed in contact to when the energy stored in the inductor is 3/4 of its maximum.

time = ………… s [3]

[Total: 20 marks]

9 The diagram in Fig. 9.1 shows two rolls of paper made by winding a length of paper around a cardboard tube. They are both falling to the ground.

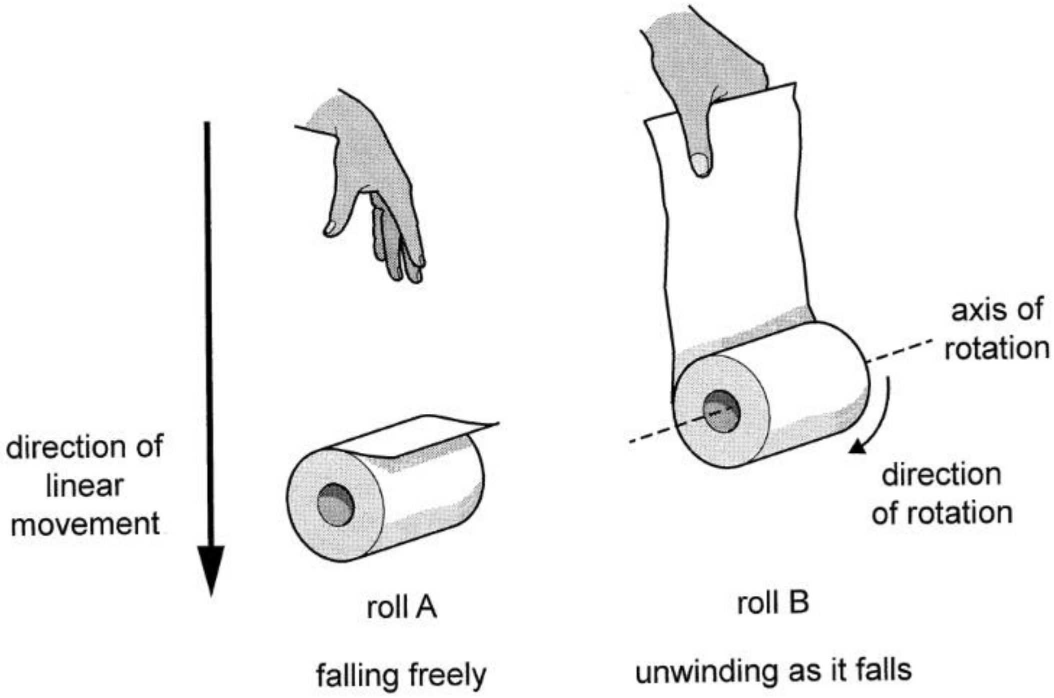


Fig 9.1

Roll A is allowed to fall freely, intact. Roll B is held at one end so that the paper unwinds while the roll is falling. Air resistance may be neglected.

The two initially identical rolls are released simultaneously from rest at a height *h*.

(a) State which roll lands first. Explain your answer.

....

....

....

....

.... [2]

(b) A roll of paper can be modelled as a cylinder of mass M with outer radius R_{out} and inner radius R_{in}.

Show, from first principles, that the moment of inertia about the central axis of the roll is given by

$$I = \frac{1}{2}M\left(R_{in}^2 + R_{out}^2\right)$$

[4]

(c) Fig. 9.2 shows roll B as it unwinds.

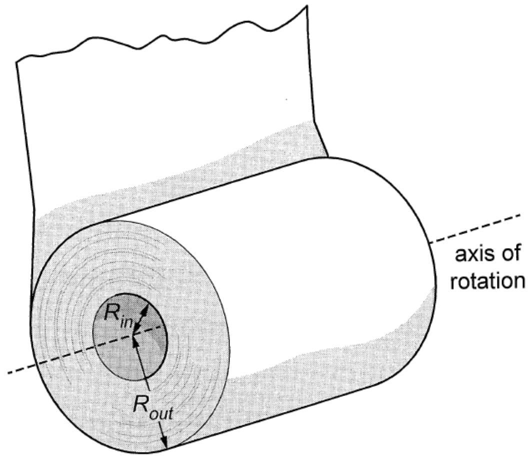


Fig. 9.2

Use conservation of energy arguments to show that the angular acceleration α is given by

$$\alpha = \frac{2gR_{out}}{R_{in}^2 + 3R_{out}^2}$$

[4]

(d) A curious Physics student wishes to demonstrate both the free-falling and unwinding rolls of paper being released simultaneously and landing simultaneously.

The outer radii of the rolls of paper used R_{out} are four times its inner radii R_{in}.

She will release the unwinding roll B from a height h.

Determine the height H from which she should release the free-falling roll A, in terms of h.

Show your working clearly.

H = ……………………… h [4]

(e) A student conducted the experiment, one roll of paper shown in **(a)** unwinds completely, and the cardboard tube contained within falls vertically to the flat ground. Due to inertia, the cardboard tube is still rotating with an angular speed of ω_i . Fig. 9.3 shows a side view of the tube just before it hits the ground.

The cardboard tube slips after establishing contact with the ground. After some time, it starts to roll without slipping.

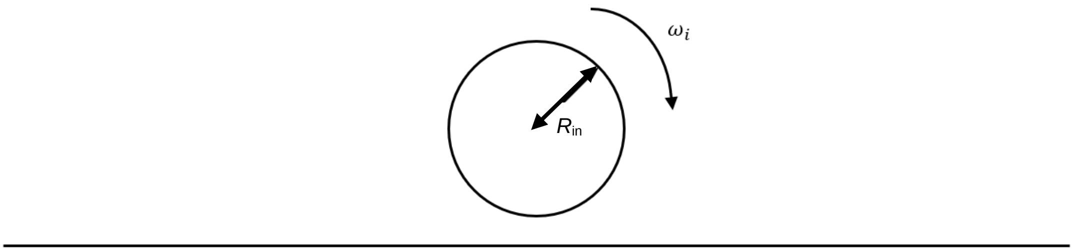


Fig. 9.3

It is known that the frictional force between the tube and the ground is $\mu m_{tube} g$, where μ is a constant and m_{tube} is the mass of the cardboard tube. You may treat the cardboard tube as a **thin cylindrical shell**.

(i) Determine the angular acceleration α of the cardboard tube in terms of μ , R_{in} and g.

[3]

(ii) Determine the angular speed ω of the cardboard tube in terms of ω_i after the cardboard tube starts rolling without slipping.

[3]

[Total: 20 marks]

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