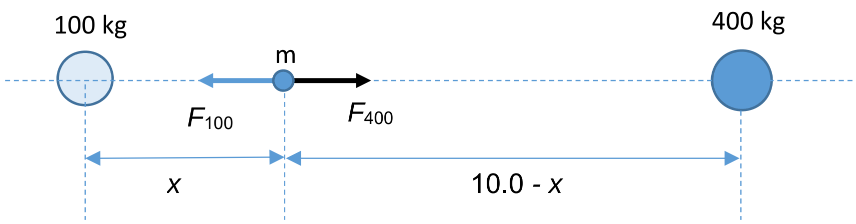
## Gravitation Tutorial 7A Discussion Questions Suggested Solutions

**D1**




$F_{100} = F_{400}$

$$G\frac{(100)m}{x^2} = G\frac{(400)m}{(10.0-x)^2}$$

$x$ = 3.33 m

OR simply $g_{100} = g_{400}$

$$G\frac{(100)}{x^2} = G\frac{(400)}{(10.0-x)^2}$$

NOTE: The concept of finding the point where the resultant force acting on a mass due to the two masses equals zero is equivalent to finding the point where the resultant gravitational field strength due to the two masses equals zero.

**D2** **B**

Vector sum of gravitational field strengths at B due to each of the 3 bodies = 0 at the neutral point

Gravitational field strength at a point due to a mass is proportional to the mass and inversely proportional to the square of the distance between the mass and the point.

Consider Earth and Moon's interaction first, the point where the resultant gravitational field strength is zero is nearer to the smaller mass (refer to D1 where g = 0 is nearer the 100 kg mass). Then consider Moon and Sun's interaction, which the neutral point will be shifted nearer to the Sun, but it will be still nearer to the smaller mass.

Good to know:

Mass of the Earth = 5.97 x $10^{24}$ kg,

mass of the Moon = 7.35 x $10^{22}$ kg,

mass of the Sun = 1.99 x $10^{30}$ kg

**D3** $$g_E = G\frac{M_E}{R_E^2} = 9.81\,m s^{-2}$$

The concept of using ratio is a common method to solve many physics problems.

(a) $$g \propto \frac{1}{R_E^2} \Rightarrow \frac{g_{new}}{g_E} = \frac{R_E^2}{R_{new}^2}$$

$$g_{new} = \frac{R_E^2}{(2R_E)^2} g_E = \frac{1}{4} g_E = 2.45\ m s^{-2}$$

(b) $g=\frac{GM}{R^2}=\frac{G\left(\frac{4}{3}\pi R^3\rho\right)}{R^2}=\frac{4}{3}\pi G\rho R$

$g\propto\rho R$

Same radius but twice the density → g' = 19.6 m s$^{-2}$

(c) Half radius but twice the density → g' = 9.81 m s$^{-2}$

**D4** **B**

$$g=\frac{GM}{r^2}=\frac{G\left(\frac{4}{3}\pi r^3\rho\right)}{r^2}=\frac{4}{3}\pi G\rho r \Rightarrow g\propto\rho r$$

The concept of using ratio again similar to D3.

$$\frac{g_E}{g_M}=\frac{\rho_E r_E}{\rho_M r_M} \Rightarrow \frac{6}{1}=\frac{5}{3}\frac{r_E}{r_M} \Rightarrow \frac{r_E}{r_M}=\frac{18}{5}=3.6$$

**D5** Apparent weight of zero means no force on its support → normal force by ground on it = 0

The entire gravitational force provides the centripetal force.

$W-N=ma_c$ (since $N$ = 0)

$W=ma_c$

$$mg=mr\left(\frac{2\pi}{T}\right)^2$$

Taking $g$ at the surface to be 9.81 m s$^{-2}$, and the given radius of Earth, $T$ = 5100 s (2 s.f.)

**D6 (2009 P3 Q5)**

(a)(i) The gravitational field strength at a point is the **gravitational force per unit mass** acting on a small test mass placed at that point.

(ii) Newton's Law of Gravitation states that every point mass (or particle) attracts every other point mass (or particle) with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

Thus $F=\frac{GMm}{R^2}$

Since $F=mg$,

$mg=\frac{GMm}{R^2}$

$\therefore g=\frac{GM}{R^2}$

(b)(i) Average density = $\frac{\text{total mass}}{\text{total volume}}=\frac{5.2\times10^{30}}{\frac{4}{3}\pi\left(1.7\times10^4\right)^3}=2.53\times10^{17}\ kg\,m^{-3}$

(ii) Gravitational force acts toward the centre of the neutron star; all the particles are pulled inwards. The outer layers compress the inner layers, resulting in increased density towards the centre.

(c)(i) $g = \dfrac{GM}{R^2} = \dfrac{\left(6.67\times10^{-11}\right)\left(5.2\times10^{30}\right)}{\left(1.7\times10^{4}\right)^2} = 1.20\times10^{12}\ ms^{-2}$

(ii) $a = r\omega^2 = (1.7 \times 10^4)\left(\dfrac{2\pi}{0.21}\right)^2 = 1.52 \times 10^7\ \text{m s}^{-2}$

(iii) Suppose there is normal contact force $N$ acting on the particle by the surface of the star.
Applying Newton's 2nd Law, centripetal force = gravitational force – normal contact force,
i.e. $ma = mg - N$,
Based on the computed values of $a$ and $g$, $\boldsymbol{N} = m\,(g - a)$ **> 0**.
Thus the particle does not leave the surface, but <u>remain in contact with the surface</u>. The gravitational force is more than enough to provide the required centripetal force.

**D7 CIE J96/III/2(part)**

(c)(i) $F_g = \dfrac{GMm}{R^2} = \dfrac{\left(6.67\times10^{-11}\right)\left(5.98\times10^{24}\right)(1.00)}{\left(6370\times10^{3}\right)^2} = 9.83\ N$

(ii) $F_c = mr\omega^2 = (1.00)(6370\times10^3)\dfrac{4\pi^2}{(24\times60\times60)^2} = 0.0337\,N$

(iii) $F_g - T = F_c = 0.0337$
$T = 9.80$ N

(d)(i) 9.83 m $s^{-2}$

(ii) 9.80 m $s^{-2}$

(e) The acceleration due to gravity is actually larger than the measured acceleration by the amount of centripetal acceleration due to the Earth's rotation. What the student measured should be called the acceleration of free fall.

**D8 2018 P1 Q11**

**C**
Without firing its rocket, gravity or gravitational force is the only force acting on the spacecraft.
Under the influence of gravity alone, it is possible for paths A, B (spacecraft in circular motion) and D (spacecraft projected away from Earth).

**D9 2021 P2 Q4(b)**

Gravitational force provides for centripetal force.

$$\frac{GMm}{r^2} = mr\omega^2$$

$$GM = r^3\omega^2$$

$$\left(6.67\times10^{-11}\right)\left(6.0\times10^{24}\right) = r^3\left(\frac{2\pi}{110\times60}\right)^2$$

$$r = 7.615\times10^{6}\ \text{m}$$

$$g = \frac{GM}{r^2} = \frac{\left(6.67\times10^{-11}\right)\left(6.0\times10^{24}\right)}{\left(7.615\times10^{6}\right)^2} = 6.9\ \text{N kg}^{-1}$$

**D10 2017 P2 Q2 (Part)**

**2 (a)(i)**

$$T = \frac{2\pi r}{v} = \frac{2\pi\left(1.75\times10^{7}\right)}{200} = 549778\ \text{s} = \frac{549778}{24\times3600} = 6.3632 = 6.36\ \text{days}$$

Note: the distance and velocity must be converted to S.I. units.

Since this is a "show" question, students should express the answer in terms of seconds first then show the steps to convert to days. Should also show that they have hit the calculator by writing the more exact value of 6.3632.

**2 (a)(ii) 1.** Charon will be above the same place on the surface of Pluto at all times.

**2 (a)(ii) 2.** The same face of Charon will be seen from Pluto at all times.

**2 (b)**

$$g_P = \frac{GM_{Pluto}}{r_{Pluto}^2} = \frac{\left(6.67\times10^{-11}\right)\left(1.31\times10^{22}\right)}{\left(1.20\times10^{6}\right)^2} = 0.607\ \text{N kg}^{-1}$$

**D11 2016 P3 Q9 (part)**

(i) The gravitational forces of attraction between the two stars provide the centripetal forces necessary for the stars to go in circular motion. This pair for forces constitute a Newton's 3rd law action-reaction pair, which are always equal in magnitude (and opposite in direction).

(ii) Assuming it is Earth year, $\omega = \frac{2\pi}{T} = \frac{2\pi}{4.0\times365\times24\times60\times60} = 4.98\times10^{-8}$ rad s$^{-1}$

(iii) Since the centripetal forces are equal in magnitude,

$M_A r_A \omega^2 = M_B r_B \omega^2$

Since they also have common $\omega$, $r_B = d - r_A$ , we can re-arrange the above equation:

$$r_A = \frac{M_B}{M_A}(d - r_A)$$

$$\frac{M_A}{M_B} = \frac{(d - r_A)}{r_A} = \frac{d}{r_A} - 1$$

$$\frac{M_A}{M_B} + 1 = \frac{d}{r_A}$$

$r_A = \frac{d}{\left(\frac{M_A}{M_B}+1\right)} = \frac{3.0\times10^{11}}{(3.0+1)} = 7.5 \times 10^{10}$ m

(iv) By Newton's 2nd Law, $F_{net} = ma$;

$$\frac{GM_AM_B}{d^2} = M_A r_A \omega^2$$

$$M_B = \frac{d^2 r_A \omega^2}{G} = \frac{(3.0\times10^{11})^2(7.5\times10^{10})(4.98\times10^{-8})^2}{6.67\times10^{-11}}$$

$M_B = 2.51 \times 10^{29}$ kg

Since $\frac{M_A}{M_B} = 3.0$,

$M_A = 3.0\,(2.51 \times 10^{29}) = 7.53 \times 10^{29}$ kg

Comments: students should distinguish clearly between the symbols for quantities related to each star.

(v) In each period, there will be two instants when the two stars form a straight line with Earth. At these instances, the intensity will dip because one star is behind the other. Therefore, the fluctuation in intensity is half the period of the orbit of the stars.