Name: _____ ( ) Class:_____

PRELIMINARY EXAMINATION
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

MATHEMATICS Student copy **4052/01**

Paper 1 **Thursday 17 August 2023**

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, register number, and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue, correction fluid or correction tape.

Answer **all** questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

90

This document consists of **20** printed pages.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 Given that $\frac{4}{128} = 2^a$, find the value of a.

$$\frac{4}{128} = \frac{1}{32} = 2^{-5}$$
$$\therefore a = -5$$

Answer $a =$ ……..…..…… [1]

---

2 Ms Li takes a loan of \$12000 from a bank that charges a simple interest rate of 3.5% per annum for 4 years.
Calculate the total amount she has to pay the bank at the end of 4 years.

$$\text{Simple interest} = \frac{12000 \times 3.5 \times 4}{100}$$
$$= \$1680$$
$$\text{Total amount} = 12000 + 1680$$
$$= \$13680$$

Answer \$..……..…..…… [2]

---

3 Store A sells a spice blender for \$124.90 with a 10% discount.
Store B sells the same blender for \$115.00 with a 12% discount.

(a) By rounding the prices to 2 significant figures and the discounts to 1 significant figure, show that the discounted price in Store A and the discounted price in Store B are equal.

Show the numbers you use.

Answer

Store A $\$120 \times 0.9 = \$108$

Store B $\$120 \times 0.9 = \$108$

[1]

(b) Without doing any further calculation, explain why the actual discounted price in Store B is lower than the estimated discounted price.

Answer ……...

The actual price (\$115) is lower than the estimated price (\$12)). The actual discount percent (12%) is more than the estimated discount percent (10%). This gives a lower actual discounted price than the estimated discounted price.

……

…….. [1]

---

4 The times, in minutes, taken by some students to complete an assignment are shown below.

49 53 45 44 53 49 59 y 60 50

(a) If the modal time is 53 minutes, state the value of y.

44 45 49 49 50 53 53 y 59 60

$Mode = 53$

$y = 53$

Answer y =……..…….…… [1]

(b) Hence, find the median time.

44 45 49 49 50 53 53 53 59 60

Middle position at $\frac{10+1}{2} = 5.5th$

Median $= \frac{50+53}{2}$

$= 51.5$

Answer ……..... minutes [1]

---

5 Simplify $\left(\frac{27x^9}{y^{12}}\right)^{\frac{2}{3}} \div \frac{x^5}{y^7}$.

$= \left(\frac{3^3x^9}{y^{12}}\right)^{\frac{2}{3}} \div \frac{x^5}{y^7}$

$= \frac{9x^6}{y^8} \times \frac{y^7}{x^5}$

$= \frac{9x}{y}$

……. [3]

---

6 Write as a single fraction in its simplest form $\frac{12-4x}{(x-3)^2} - 1$.

$= \frac{4(3-x)}{(x-3)^2} - 1$

$= \frac{-4}{(x-3)} - 1$

$= \frac{-4-(x-3)}{(x-3)}$

$= \frac{-x-1}{x-3} = \frac{x+1}{3-x}$

$= \frac{4(3-x)-(x-3)^2}{(x-3)^2}$

$= \frac{12-4x-x^2+6x-9}{(x-3)^2}$

$= \frac{-x^2+2x+3}{(x-3)^2}$

$= \frac{(3-x)(1+x)}{(x-3)^2}$

$= \frac{-(x-3)(1+x)}{(x-3)^2}$

$= \frac{-(1+x)}{(x-3)}$

Answer ….

---

7 In a school club, 72 of the members are boys and 48 are girls.

Calculate the least number of additional girls that would need to join the club in order to raise the percentage of girls to greater than 45%.

Let x be the number of additional girls needed to join the club.

$\frac{48+x}{48+72+x} > \frac{45}{100}$ or $\frac{48+x}{48+72+x} > \frac{9}{20}$

$9(120+x) < 960+20x$

$1080+9x < 960+20x$

$11x > 120$

$x > 10.91$, least number is 11

Answer …… [2]

---

8 **(a)** Solve the inequalities $-8.5 \le 3x-4 < 11$.

$-8.5 \le 3x-4$ $\quad$ $3x-4<11$

$3x \ge -4.5$ $\quad$ $3x<15$

$x \ge -1.5$ $\quad$ $x<5$

$\therefore -1.5 \le x < 5$

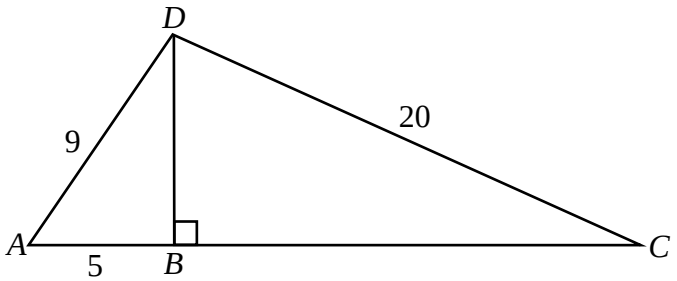
Answer …… [2]

(b) Write down all the integers that satisfy $-8.5 \le 3x-4 < 11$.

$-1, 0, 1, 2, 3, 4$

Answer …… [1]

9



In the triangle ACD, AB = 5 cm, AD = 9 cm, CD = 20 cm and BD is perpendicular to AC.

Calculate BC.

$BD^2 = 9^2 - 5^2$

$BD^2 = 56$

$BD^2 + BC^2 = 20^2$

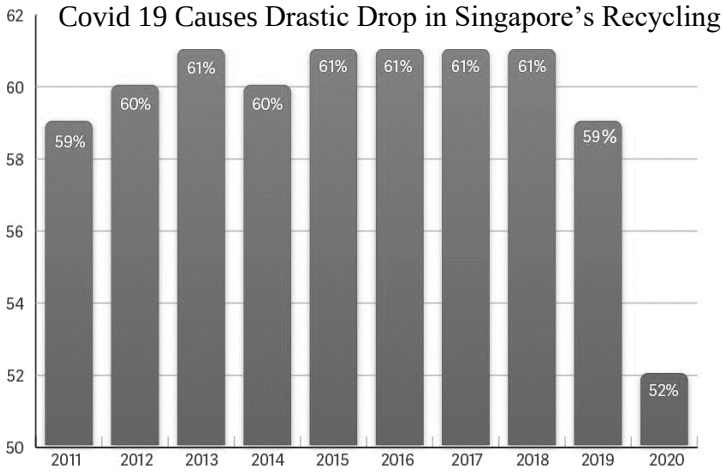
$\therefore BC = \sqrt{20^2 - 56}$

$\therefore BC = \sqrt{344}$

$\therefore BC = 18.5$ cm (3sf)

Answer BC = ……..…..…… cm [2]

10



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Vertical axis starts from 50 instead of 0.

It leads to misinterpretation because it exaggerates the differences between the years.

The title is biased. (Or the title uses loaded words "drastic drop").

It leads to misinterpretation because it does not allow the reader to make own judgement.

---

11 **(a)** Expand and simplify $(5x-14)(5x+14)$.

$(5x-14)(5x+14)$
$=(5x)^2-14^2$
$=25x^2-196$

Answer …… [2]

(b) Given that x is a positive integer and $(5x-14)(5x+14)$ is a prime number between 20 and 30, find this prime number.

$5x-14=1$
$5x=15$
$x=3$
Prime number $1\times(5\times3+14)=29$

Answer …… [1]

---

12 John, Eric and Kate walk from point *A* to point *B* along the same route.

Eric takes 10 minutes 30 seconds to complete this journey.
John takes 10% less than the time taken by Eric.
Kate takes 20% more than the time taken by Eric.

(a) Calculate the time taken by Kate in seconds.

Kate's time $=1.2\times630$ seconds
$=756$ seconds

Answer ……..… seconds [1]

(b) Express the ratio of the times taken by John, Eric and Kate in the form $a:b:c$, where a, b and c are integers in their lowest terms.

John's time $=0.9\times630$ seconds
$=567$ seconds

$567:630:756$

$=9:10:12$

or

John, Eric and Kate

$=90\%:100\%:120\%$
$=9:10:12$

Answer ……..… **:** ……..… **:** …… [2]

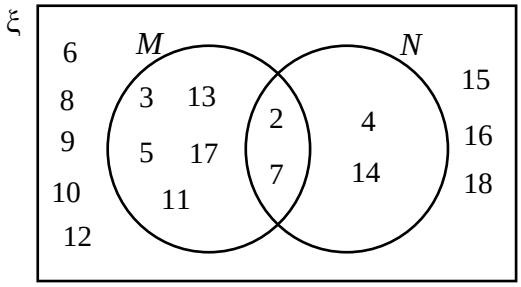
---

13 **(a)** $\xi = \{\text{integers } x : 2 \le x \le 18\}$

$M = \{\text{prime numbers}\}$

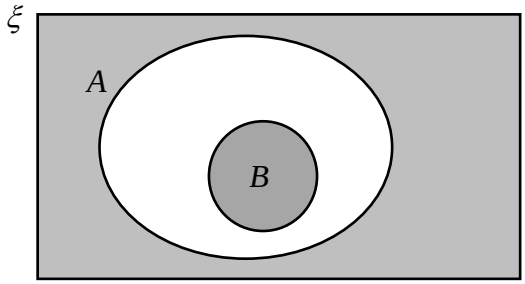
$N = \{\text{factors of } 28\}$

Draw a Venn diagram to illustrate this information.
Answer



[2]

(b) Use set notation to describe the shaded region.

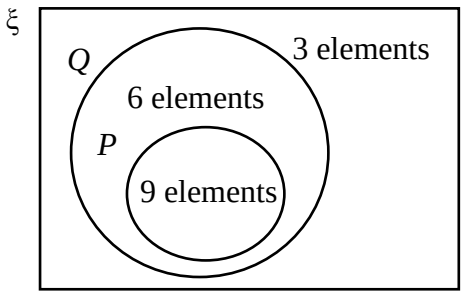


B ∪ A'

Answer ……...…… [1]

(c) The sets ξ, P and Q satisfy the conditions $n(\xi) = 18$, $n(P) = 9$ and $n(Q) = 15$.

Find the largest possible value of $n(P \cup Q)'$.



Largest possible value of $n(P \cup Q)' = 3$

Answer ……..……...…… [1]

14 **(a)** Written as a product of its prime factors, $504 = 2^3 \times 3^2 \times 7$.

p and q are both prime numbers where $p < q$ and the number $\frac{504p}{q}$ is a perfect square.

Find the value of p and the value of q

$\frac{504p}{q} = 2^3 \times 3^2 \times 7 \times \frac{2}{7}, \quad p = 2,\ q = 7$

Answer $p =$ ……

$q =$ …….. [2]

(b) A rectangle has an area of 200 cm^2. The length is x cm and the breadth is y cm.

Given that x and y are integers and $x > y \geq 10$, find the value of x and the value of y.

$200 = 2^3 \times 5^2$

$x = 2^2 \times 5 = 20$

$y = 2 \times 5 = 10$

200 x 1
100 x 2
50 x 4
40 x 5
25 x 8
20 x 10

Answer $x =$ ……

$y =$ …….. [2]

15 (a) Factorise completely $7xy-10-5x+14y$.

$=7xy+14y-5x-10$

$=7y(x+2)-5(x+2)$

$=(x+2)(7y-5)$

$=7xy-5x-10+14y$

$=x(7y-5)-2(5-7y)$

$x(7y-5)+2(7y-5)$

$=(7y-5)(x+2)$

Answer …… [2]

(b) Using factorization, solve $63x^2-4x-4=0$.

$63x^2-4x-4=0$

$(7x-2)(9x+2)=0$

$x=\frac{2}{7}$ or $x=-\frac{2}{9}$

Answer x =…… or ……. [3]

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16 Two geometrically similar models of a house are made using the same material.
The mass of the smaller model is 750 g while the mass of the larger model is 6 kg.

(a) Find the ratio of the height of the smaller model to the height of the larger model in the form 1 : n.

$\frac{h_{\text{smaller}}}{h_{\text{larger}}}=\sqrt[3]{\frac{750}{6000}}$ or $\left(\frac{h_{\text{smaller}}}{h_{\text{larger}}}\right)^3=\frac{750}{6000}$

$=\frac{1}{2}$

$h_{\text{smaller}}:h_{\text{larger}}=1:2$

$\frac{m_{\text{smaller}}}{m_{\text{larger}}}=\frac{density\times V_{\text{smaller}}}{density\times V_{\text{larger}}}$

$=\left(\frac{h_{\text{smaller}}}{h_{\text{larger}}}\right)^3$

Answer …… [2]

(b) The larger model is made to a scale of 3 : 4000.
The length of the actual house is 120 m.

Find the length of the house in the smaller model.

$\frac{l_{\text{larger}}}{12000}=\frac{3}{4000}$

$l_{\text{larger}}=\frac{3}{4000}\times 12000$

$=9$ cm

$\frac{l_{\text{larger}}}{l_{\text{smaller}}}=\frac{h_{\text{larger}}}{h_{\text{smaller}}}=\frac{2}{1}$

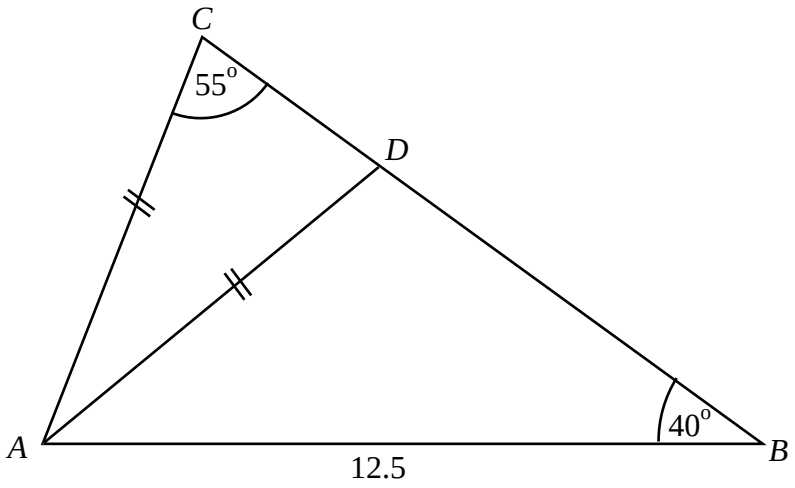
$\frac{9}{l_{\text{smaller}}}=\frac{2}{1}$

$\therefore l_{\text{smaller}}=4.5$ cm

Actual length 120m,
Large model length 9cm
Small model length 4.5 cm

Answer [2]

17



In the triangle ABC, $AB = 12.5$ cm, angle $ABC = 40^o$, angle $ACB = 55^o$ and $AC = AD$.

(a) Calculate AC.

$$\frac{AC}{\sin 40^o} = \frac{12.5}{\sin 55^o}$$
$$AC = \frac{12.5 \sin 40^o}{\sin 55^o}$$
$$AC = 9.808$$
$$AC = 9.81 \text{ cm (3sf)}$$

Answer $AC =$ ……..…….. cm [2]

(b) Calculate the area of triangle ADB.

ΔACD is isosceles with $\angle ACD = \angle ADC$

$\angle ADC = 55^o$

$\therefore \angle ADB = 180^o - 55^o = 125^o$ (adj $\angle s$ on a str line)

$\therefore \angle DAB = 180^o - 125^o - 40^o = 15^o$ ($\angle$ sum of Δ)

or $\therefore \angle DAB = 55^o - 40^o = 15^o$ (ext $\angle$ of Δ)

$$\text{Area of } \Delta ADB = \frac{1}{2}(9.808)(12.5)\sin 15^o$$

$$\text{Area of } \Delta ADB = 15.9 \text{ cm}^2 \text{ (3sf)}$$

Answer ……..…….. cm^2 [3]

18 The first four diagrams in a sequence are shown below.

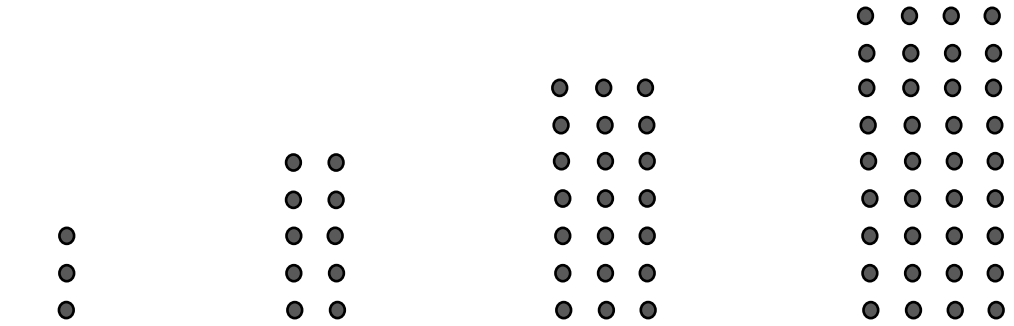


Diagram 1
3 dots

Diagram 2
10 dots

Diagram 3
21 dots

Diagram 4
36 dots

(a) Find the number of dots in Diagram 5.

$5 \times 11 = 55$

Answer ……..……...…… [1]

(b) Find an expression, in terms of n, for the number of dots in Diagram n.

$n(2n+1)$

Answer ……..……...…… [2]

(c) Find the difference in the number of dots between Diagram 7 and Diagram 6.

Diagram 7 has 105 dots
Diagram 6 has 78 dots
difference = $105 - 78 = 27$

Answer ……..……...…… [1]

(d) Show that the difference in the number of dots between Diagram (n + 1) and Diagram n is $4n+3$.

Answer

Diagram $(n+1)$ has $(n+1)(2n+3)$

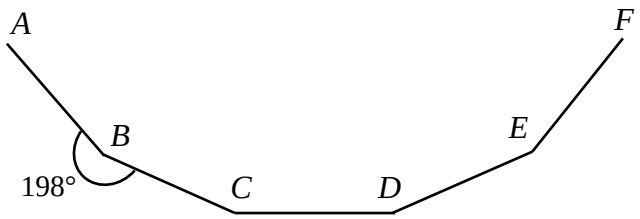
$(n+1)(2n+3) - n((2n+1)$

$= 2n^2 + 5n + 3 - 2n^2 - n$

$= 4n + 3$

[2]

19



The diagram shows part of a regular n-sided polygon, $ABCDEF\ldots$.
Given that the reflex angle $ABC = 198°$

(a) Prove that triangles ABC and triangle CDE are congruent.

Answer

$AB = CD$ (sides of regular polygon)
$BC = DE$ (sides of regular polygon)
$\angle ABC = \angle CDE$ (interior angle of regular polygon)
$\Delta ABC \equiv \Delta CDE$ (SAS)

[2]

(b) Find the size of each exterior angle.

Interior Angle $= 360° - 198°$ ($\angle$ at a point)
$= 162°$
Exterior Angle $= 180° - 162°$ (adj $\angle$ on str line)
$= 18°$

or Exterior Angle $= 198° - 180°$
$= 18°$

(c) Find the value of n.

$$n = \frac{360°}{18°}$$
$$= 20$$

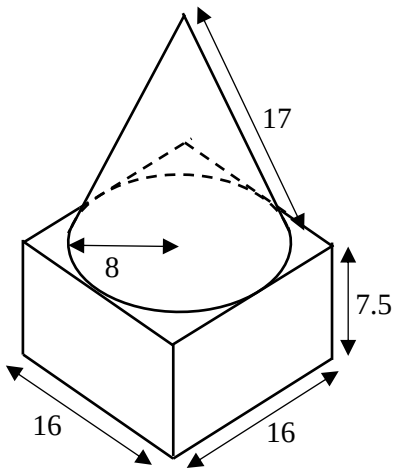
Answer n =……..……..…… [1]

(d) Find the angle BEF.

$\angle BED = 180° - 162°$ (int $\angle$, $BE // CD$)
$= 18°$
$\angle BEF = 162° - 18°$
$= 144°$

Answer Angle BEF =……..……..…… [2]

20



The diagram shows a solid formed from a cone and a cuboid.
The cone has a radius of 8 cm and slant height of 17 cm.
The cuboid has a square base of length 16 cm and a height of 7.5 cm.

(a) Calculate the volume of the solid figure.

$$\text{height} = \sqrt{17^2 - 8^2}$$
$$= 15 \text{ cm}$$
$$\text{Volume} = \frac{1}{3}\pi(8)^2(15) + (16 \times 16 \times 7.5)$$
$$= 2925.3$$
$$= 2930 \text{ cm}^3 \quad (3\text{sf})$$

Answer ……..…… cm^3 [3]

(b) Calculate the total surface area of the solid figure.

Total SA
$$= \pi(8)(17) + \left[(16 \times 16) - \pi(8)^2\right] + \left[(16 \times 16) + (4 \times 16 \times 7.5)\right]$$
$$= 1218.1$$
$$= 1220 \text{ cm}^2 \quad (3\text{sf})$$

Answer ……..…… cm^2 [3]

21 At bakery A, a chocolate bread loaf costs \$2.50, a wholemeal bread loaf costs \$3 and a raisin bread loaf costs \$3.80 .

At bakery B, a chocolate bread loaf costs \$3.50, a wholemeal bread loaf costs \$3.20 and a raisin bread loaf costs \$3.

This information can be represented by the matrix $\mathbf{P} = \begin{matrix} \\ \text{C} \\ \text{W} \\ \text{R} \end{matrix} \begin{matrix} \begin{matrix} \text{A} & \text{B} \end{matrix} \\ \begin{pmatrix} 2.50 & 3.50 \\ 3.00 & 3.20 \\ 3.80 & 3.00 \end{pmatrix} \end{matrix}$

(a) Janet buys 2 loaves of chocolate bread, 4 loaves of wholemeal bread and 3 loaves of raisin bread.
Karen buys 5 loaves of chocolate bread, 3 loaves of wholemeal bread and x loaves of raisin bread.
Represent this information in a 2×3 matrix, **N**.

$$\mathbf{N} = \begin{pmatrix} 2 & 4 & 3 \\ 5 & 3 & x \end{pmatrix}$$

Answer $\mathbf{N} = \begin{pmatrix} & & \\ & & \end{pmatrix}$ [1]

(b) Find, in terms of x, the matrix $\mathbf{S} = \mathbf{NP}$.

$$\mathbf{S} = \mathbf{NP}$$
$$= \begin{pmatrix} 2 & 4 & 3 \\ 5 & 3 & x \end{pmatrix} \begin{pmatrix} 2.50 & 3.50 \\ 3.00 & 3.20 \\ 3.80 & 3.00 \end{pmatrix}$$
$$= \begin{pmatrix} 28.4 & 28.8 \\ 21.5 + 3.8x & 27.1 + 3x \end{pmatrix}$$

Answer $\mathbf{S} = \begin{pmatrix} & & \\ & & \end{pmatrix}$ [2]

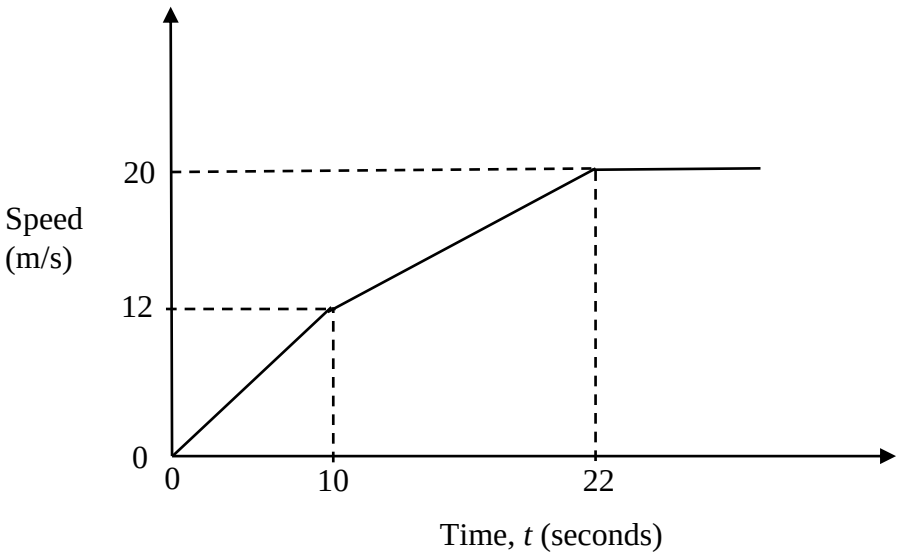
(c) Karen has a budget of \$30.
By writing an inequality in x, determine the total number of loaves of bread she can buy from bakery B.

For bakery B,
$27.1 + 3x \le 30$
$3x \le 2.9$
$x \le 0.966$
$\therefore 8$ loaves

Answer ……..…..…..… loaves [2]

22 The diagram shows the speed-time graph of a car journey.

The car accelerates for 10 seconds reaching a speed of 12 m/s. It then accelerates for another 12 seconds and then travels at a constant speed of 20 m/s.



(a) Calculate the speed of the car when $t = 13$.

$$\text{acceleration} = \frac{v-12}{13-10} = \frac{20-12}{22-10}$$

$$\frac{v-12}{3} = \frac{8}{12}$$

$$v - 12 = 2$$

$$\therefore v = 14$$

Answer ……..…….. m/s [2]

(b) Calculate the distance travelled in the first 22 seconds.

$$\text{distance} = \frac{1}{2} \times 10 \times 12 + \frac{1}{2}(20+12) \times 12$$

$$= 60 + 192$$

$$= 252$$

Answer ……..…….. m [2]

(c) A motor cyclist starts 13 seconds later from the same place as the car, and accelerates uniformly for 9 seconds until it reaches a speed of 25 m/s.

Is the motor cyclist behind the car at $t = 22$?
Justify your decision.

Answer

distance by car = 252 m

distance by motor cyclist $= \frac{1}{2} \times 9 \times 25$

distance by motor cyclist $= 112.5$ m < 252 m

$\therefore$ motor cyclist is behind the car at t = 22

[2]

23 The table shows the time taken by 100 females to complete the 2.4 km run.

Time (t minutes)	Frequency
$8 \le t < 10$	6
$10 \le t < 12$	25
$12 \le t < 14$	36
$14 \le t < 16$	29
$16 \le t < 18$	4

(a) Calculate an estimate for

(i) the mean time for the females,

Midpoint	Frequency
9	6
11	25
13	36
15	29
17	4

$$\text{Mean} = \frac{1300}{100} = 13$$

Answer ……..……..…. minutes [1]

(ii) the standard deviation of their timing.

$$\text{SD} = \sqrt{\frac{17276}{100} - 13^2} = 1.94$$

Answer ……..……..…. minutes [1]

(iii) The mean time for another group of 100 males was 12.6 minutes and the standard deviation was 2.5 minutes.

Make two comparisons between the times for males and the times for females.

Answer ……...

The males ran faster as the mean (of 12.6 minutes) is lower then the females mean time (of 13 minutes).
The timing for the females shows more consistency as its SD (1.94 minutes) is lower as compared to the SD for the males (of 2.5 minutes).

……
……
……
……
……..….. [2]

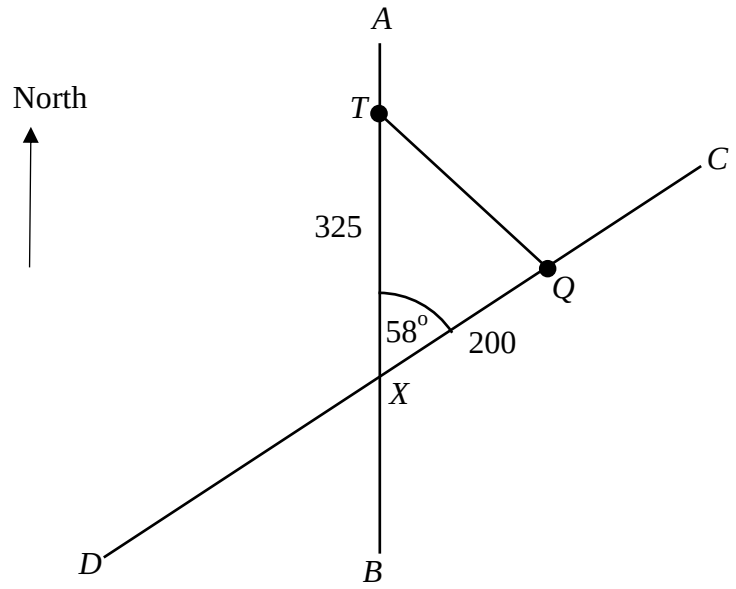
(b) Two female runners are chosen at random.

Find the probability that both runners complete the 2.4 km run in less than 12 minutes.

$$\frac{31}{100}\times\frac{30}{99}=\frac{31}{330}$$

Answer ……..……...…….. [2]

24



AB and *CD* are two roads which meet at point *X* at an angle of 58^{o}, with *A* due north of *X*.
Point *Q* is 200 km from point *X*.
At 1040, a truck *T* is 325 km north of point *X* and is travelling at 75 km/h along the path *TQ* to reach point *Q*.

(a) At what time would the truck reach point Q?

$TQ = \sqrt{325^2 + 200^2 - 2(325)(200)\cos 58^o}$

$TQ = 277.0117 \text{ km}$

$\text{Time taken} = \dfrac{277.0117}{75} \text{ h}$

$= 3.6934 \text{ h}$

$= 3 \text{ h } 41.6 \text{ min}$

$\therefore$ at time 1421 or 1422

Answer ……..……..…… [3]

(b) Calculate the bearing of *Q* from *T* .

$\dfrac{\sin \angle XQT}{325} = \dfrac{\sin 58^o}{277.0117}$

$\sin \angle XQT = \dfrac{325 \sin 58^o}{277.0117}$

$\angle XQT = 84.24^o$

$\therefore$ bearing of Q from $T = 58^o + 84.24^o$

$\therefore$ bearing of Q from $T = 142.2^o$ (1dp)

$\dfrac{\sin \angle XTQ}{200} = \dfrac{\sin 58^o}{277.0117}$

$\sin \angle XTQ = \dfrac{200 \sin 58^o}{277.0117}$

$\angle XTQ = 37.75^o$

$\therefore$ bearing of Q from $T = 180^o - 37.75^o$

$\therefore$ bearing of Q from $T = 142.2^o$ (1dp)

……..…… [2]